eBay in the Sky: StrategyProof Wireless Spectrum Auctions

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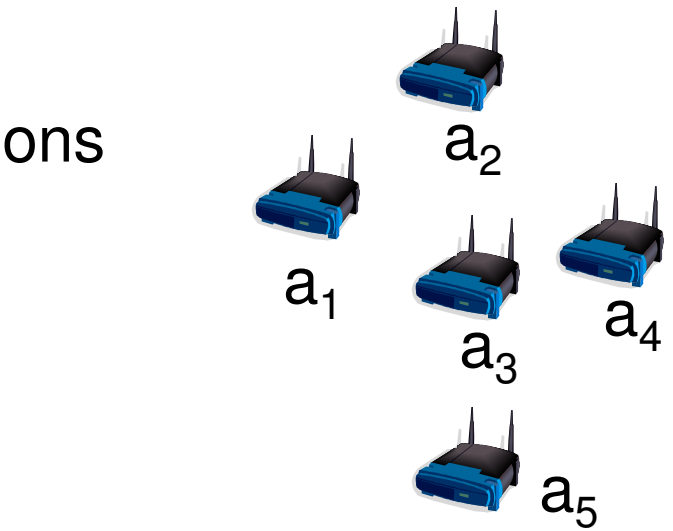
IUSTITIA
(Goddess of Justice)

VERITAS
(Goddess of Truth)

Need for On-Demand Spectrum Auctions

- Explosion in the number of wireless devices
- FCC: static long term licenses -> artificial scarcity
- Solution:
  - Dynamic spectrum redistributions
  - Exploit spatial reusability



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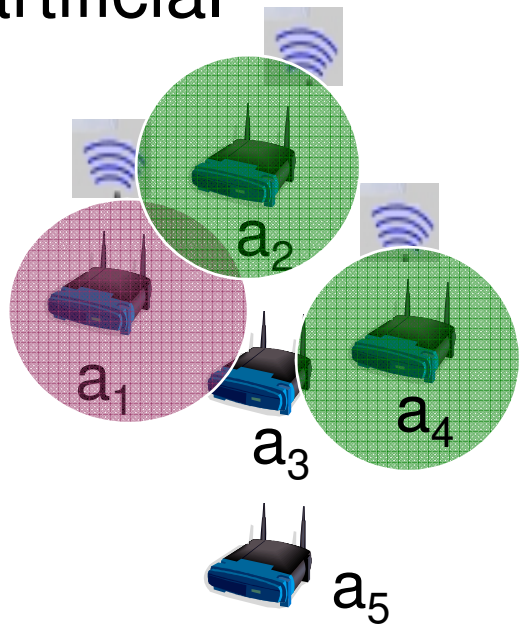

# of channels = 2

Channel1 Channel2

Need for On-Demand Spectrum Auctions

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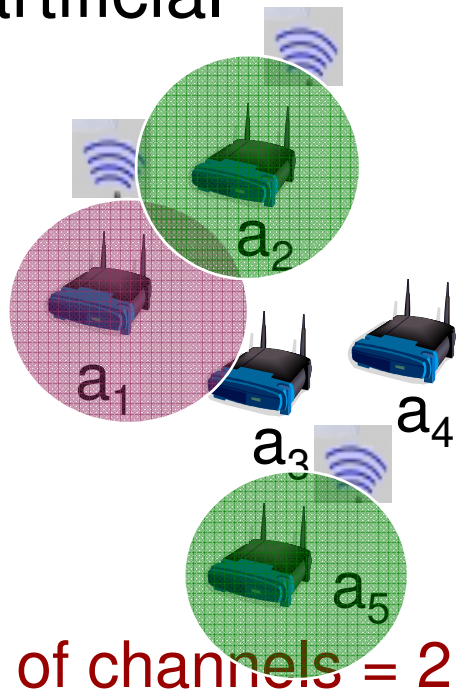
- Auctions widely used to distribute scarce resources
  - Fair and open
  - Economic Efficiency



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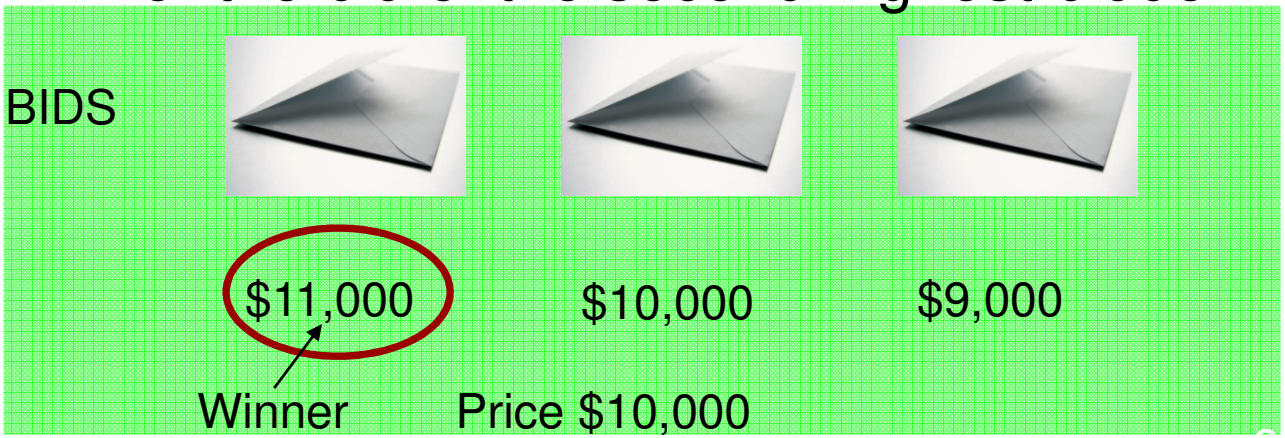
Need for Truthful Mechanisms

- Selfish Bidders lead to challenges and overheads
  - Auction: Game among strategic players
  - Goal: Maximize individual utility
    - Utility = (True Value – Price Paid)
  - Players strategize over others
  - Counter productive for the auctioneer
- Truthful Auctions help overcome these challenges and overheads
  - Truthful auction: Every bidder maximizes its utility by bidding its true value

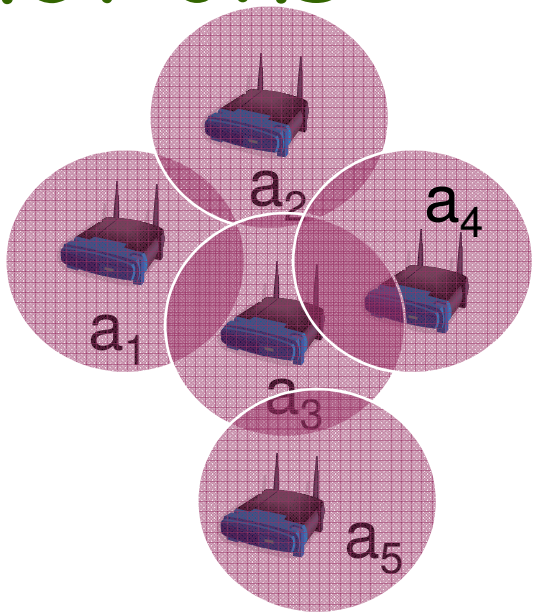
Vickery: A Classical Truthful Auction

- Consider an auction for single item
- Vickery (Nobel Prize Winner)
  - Bidders submit bids in sealed envelopes
  - Auctioneer
    - Awards the item to the highest bidder
    - Charges winner the bid of the second highest bidder



Truthful Spectrum Auctions

- Items: Channels (k)
- Interference Graph
  - Nodes: Bidders
  - Edges: Interference Constraints



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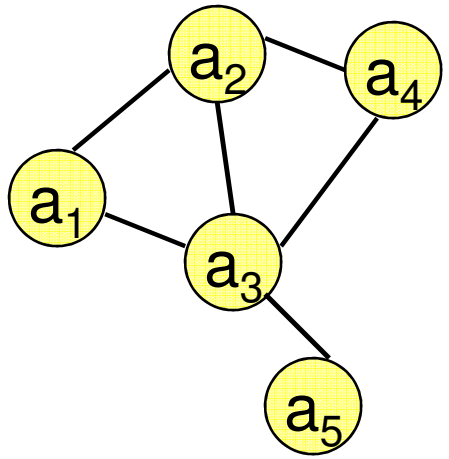

INTERFERENCE GRAPH

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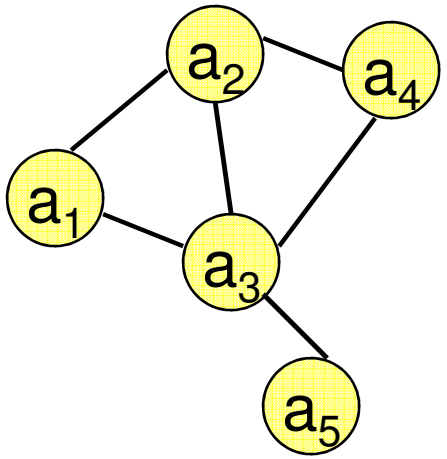
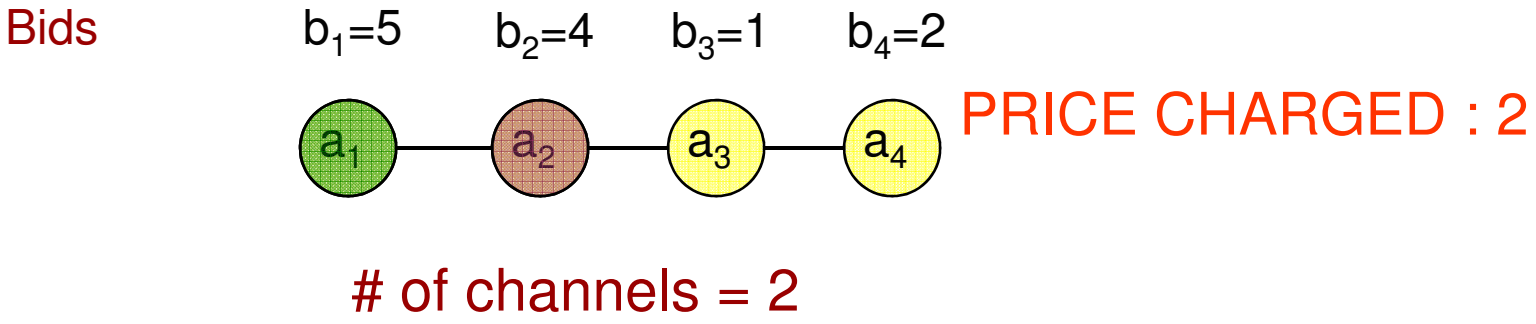


INTERFERENCE GRAPH

Possible Solution 1: Extending Vickery

- Algorithm (For allocating k channels)
  - Allocate channels to k highest bidders
  - Price: Bid of (k+1)th highest bidder



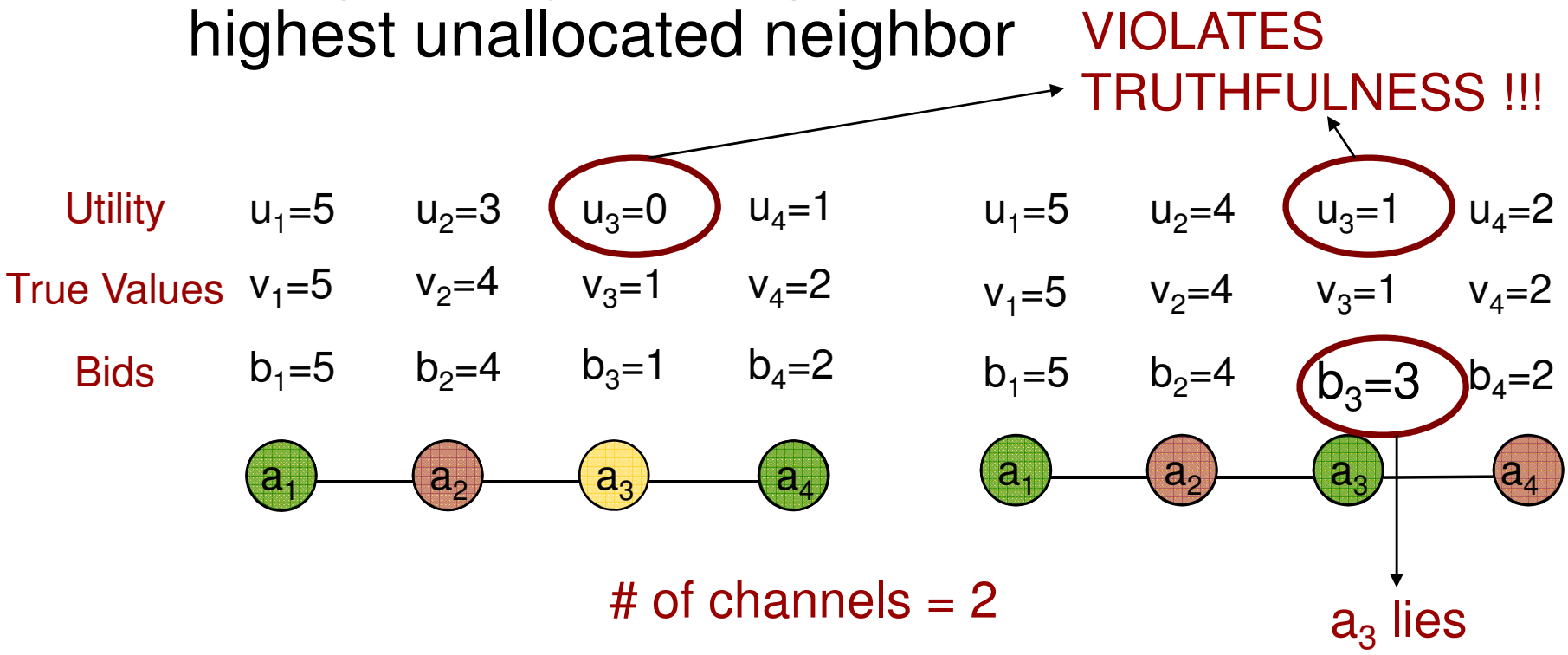
- Inefficient spectrum utilization: spatial reuse not exploited

Possible solution 2: VCG

- Optimal Spectrum utilization : NP-Hard
  - VCG is truthful but not polynomial time
- Relax Optimality constraint
  - Pareto Optimal solution: Cannot allocate any more channels without de-allocating at least one bidder
- Given an interference graph G, set of bids B, available number of channels k, design a truthful auction mechanism which run in polynomial time, results in pareto optimal allocations and has a non-trivial pricing scheme

Possible Solution 3: Extending Secondary Price Auctions

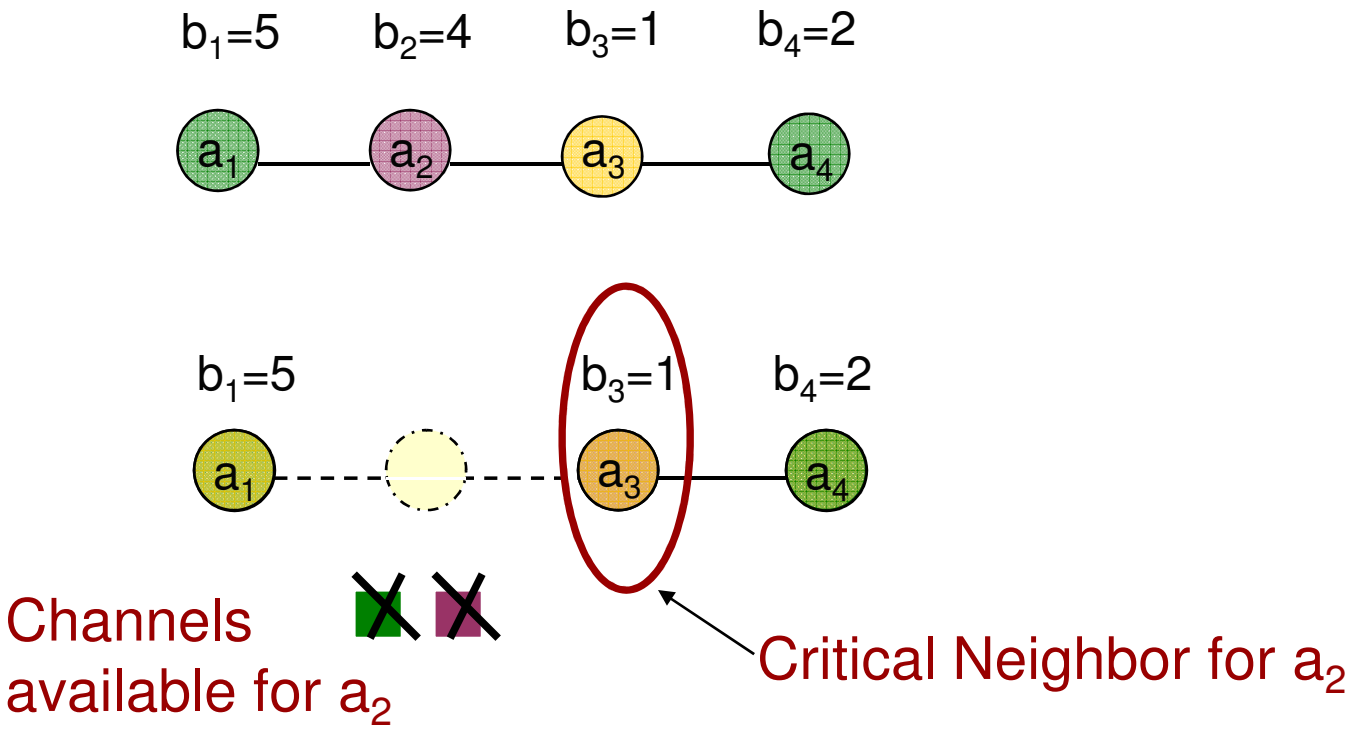
- Sort and Greedily allocate channels
  - Allocate lowest available index
- Charge every winning bidder the bid of the highest unallocated neighbor



Veritas:Truthful and Efficient Spectrum Auctions

- Greedy Allocation
  - Best known polynomial time channel allocation schemes are greedy
- Veritas-Pricing:
  - Charge every winner i, the bid of its critical neighbor C(i)
  - Finding Critical Neighbor for i
    - run allocations on $\{B/b_i\}$ (B: set of bids)
    - *Critical Neighbor:The neighbor which makes the number of channels available for i go 0*

Veritas Toy Example



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Proof of Veritas's truthfulness

- Theorem: Veritas spectrum auction is truthful, achieves pareto optimal allocations, and runs in polynomial time $O(n^3k)$
- Proof sketch
  - Critical Value: Given a bid-set B, unique critical value exists for every allocated bidder.
  - Monotonicity of allocations: If a bidder bids greater its critical value, it is always allocated.
  - Truthfulness: If we charge every bidder by its critical value, no bidder has an incentive to lie.

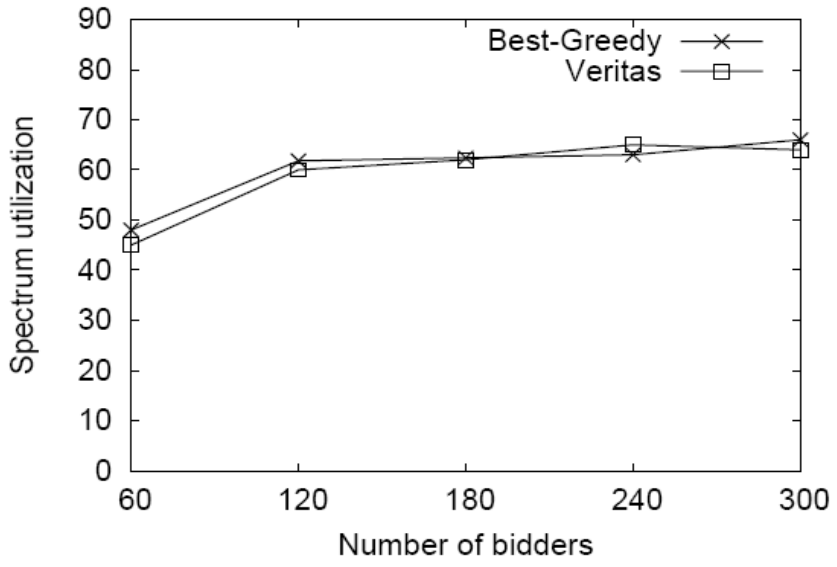
Simulations

- Compare revenue and spectrum utilization of Veritas with other truthful and non-truthful designs.
- Synthetic Data
  - Nodes placed randomly in 1 X 1 square
  - Unit disk interference graph with radius = 0.1
  - Bids are randomly picked from the interval (0, 1]
- All results are averaged over multiple seeds

Spectrum Utilization: Best-Greedy vs Veritas

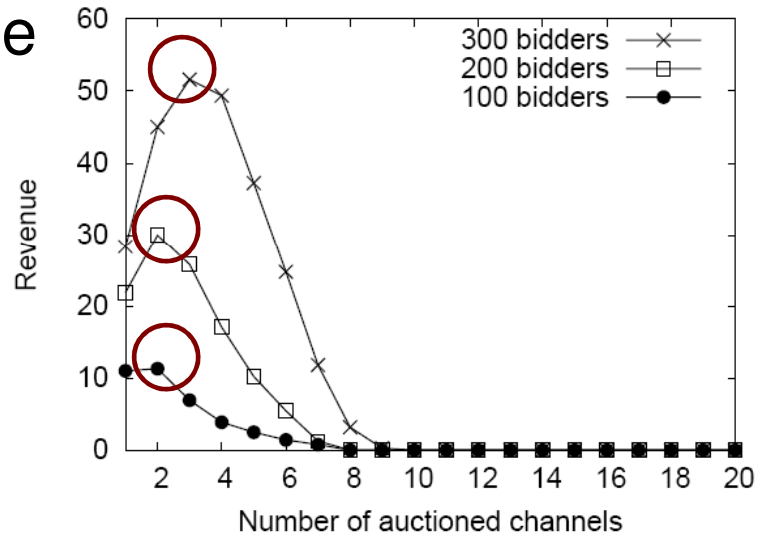
- Best Greedy: best known polynomial time spectrum allocation scheme (non-truthful)
- Veritas: Achieves truthfulness yet comparable spectrum utilization

# of channels = 8



Veritas Revenue

- **Revenue curve not monotonically increasing when # of channels is increased**
  - Effect of truthful pricing scheme
  - Requires sufficient competition



VERITAS ALGORITHM

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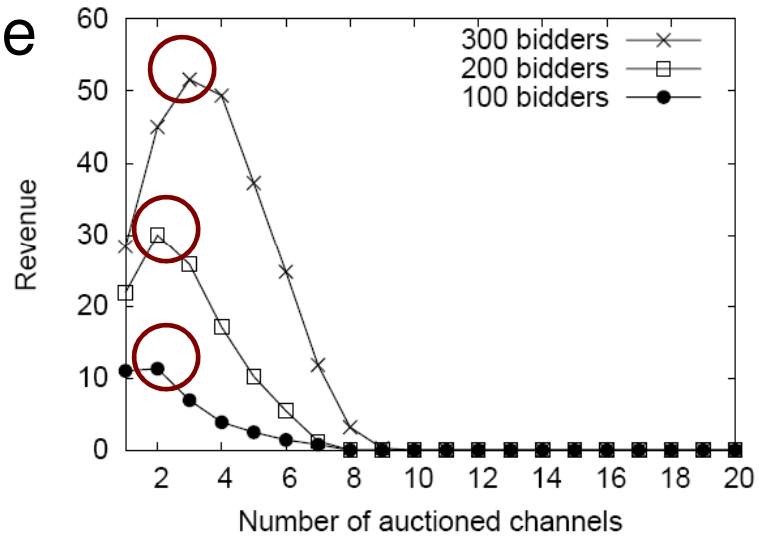
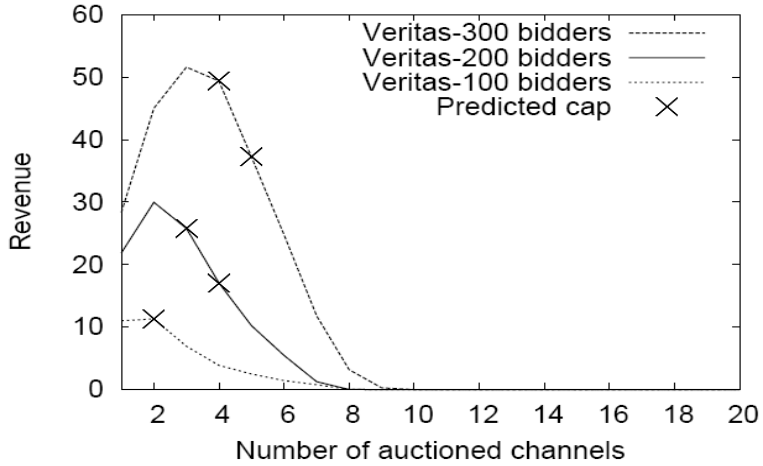


VERITAS ALGORITHM

- **Important to choose the right # of channels**



Heuristic for choosing number of channels

Veritas Extensions

- Objective functions: Veritas allocation scheme can sort on broad class of functions of bids
  - Useful for choosing objective function
- Bidding Formats:
  - Range Format: Every bidder i specifies parameter d_i, and requests any number of channels in the range $(0, d_i)$
  - Contiguous Format: Bidder requests the channels allocations to be contiguous

Conclusion

- We propose Veritas: a polynomial time truthful mechanism for dynamic channel allocation (pareto optimal allocations)
- Related work: VERITAS differs from conventional spectrum allocation/auction designs by achieving both truthfulness and spectrum efficiency (spatial reuse)
- We show that the Veritas mechanism is highly flexible can be extended for many objective functions and bidding formats

Thanks for listening

- Questions ???