Lecture 3

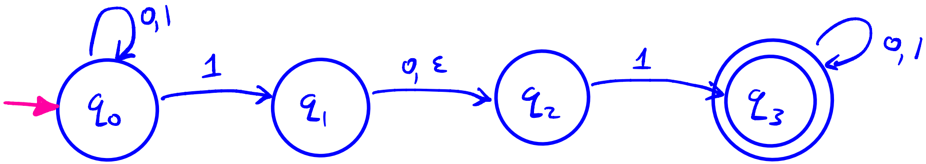
Announcements:

Office hours start next week (see calendar)

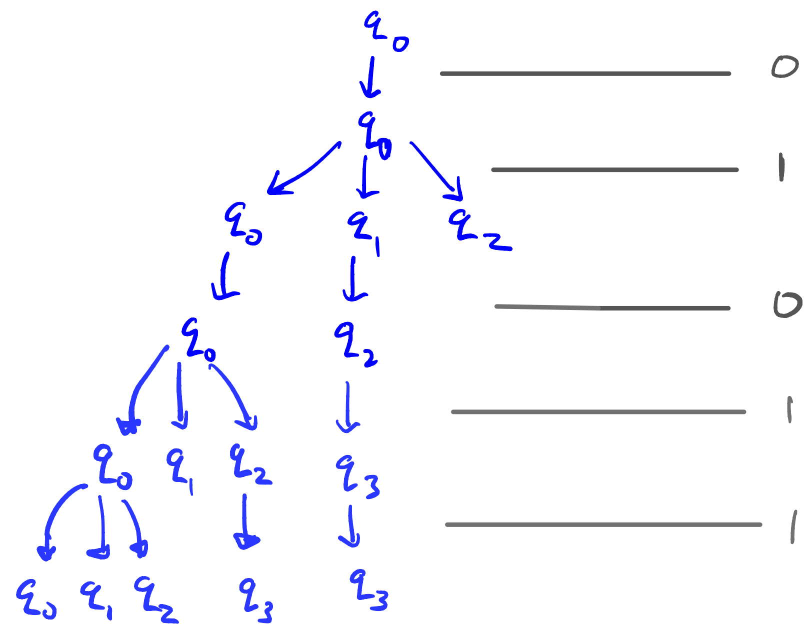
HW 1 out Monday

Nondeterministic Finite Automata (NFA's)

Example 2:



$w = 01011$:



Nondeterministic Finite Automata (NFA's)

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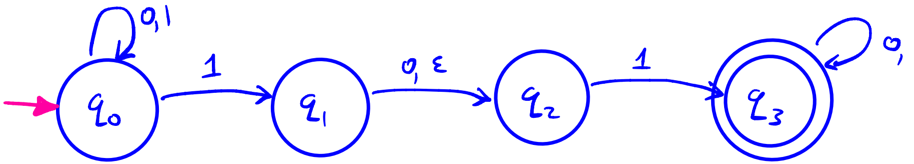

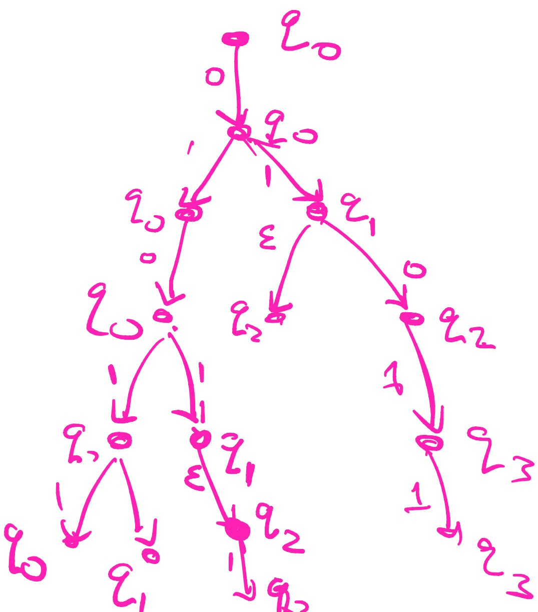

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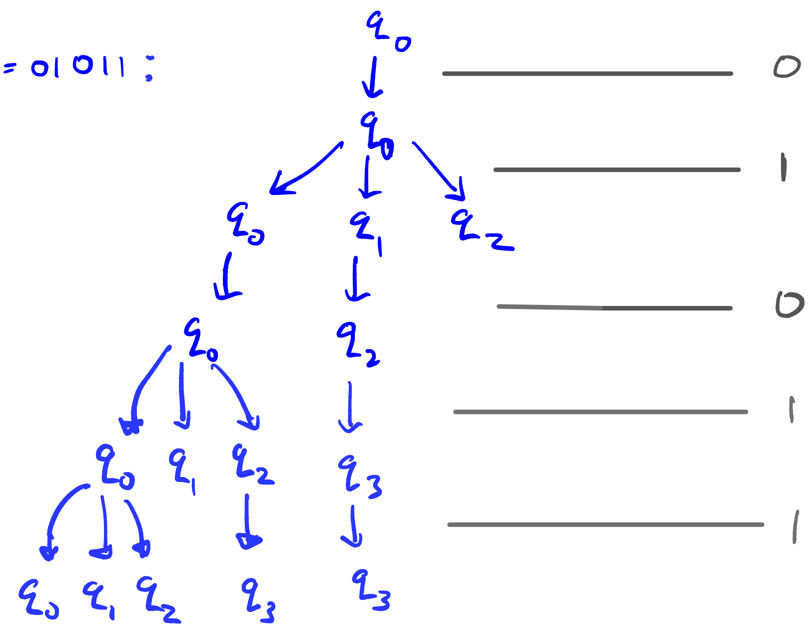

Formal Definition of an NFA

$N = (Q, \Sigma, q_0, F, \delta)$

finite set of states — finite alphabet — start state — accept states

$\delta : Q \times [\Sigma \cup \{\varepsilon\}] \rightarrow \underbrace{\mathcal{P}(Q)}_{\text{power set of } Q}$

$\Sigma = \{0, 1\}$

$\mathcal{P}(Q)$ is the set of all subsets of Q

Example:

$$\mathcal{P}(\{q_0, q_1, q_2\}) = \{\phi, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_2, q_3\}, \{q_0, q_1, q_2\}\}$$

$\phi = \{\}$ (the empty set)

Formal Definition of an NFA

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- Σ: finite alphabet
- q_0: start state
- F: accept states

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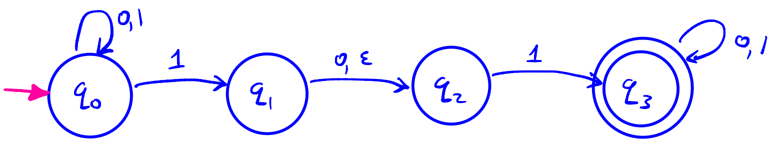
Example



δ:

	0	1	ε
q_0	$\{q_0\}$	$\{q_0, q_1\}$	ϕ
q_1	$\{q_2\}$	ϕ	$\{q_2\}$
q_2	ϕ	$\{q_3\}$	ϕ
q_3	$\{q_3\}$	$\{q_3\}$	ϕ

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$\varepsilon \, \Sigma^n$

An NFA **accepts** a string $w = w_1, w_2, \ldots w_n$ if we can write $w = y_1 y_2 \ldots y_m$ where each $y_i \in \Sigma \cup \{\varepsilon\}$, and there exists a sequence of states $r_0 r_1 \ldots r_m$, $r_i \in Q$ such that:

1. $r_0 = q_0$
2. $r_{i+1} \in \delta(r_i, y_{i+1}) \quad \forall i = 0, 1, \ldots, m-1$
3. $r_m \in F$

$\mathcal{L}(N)$ = set of strings accepted by N

example: $w = 0010$

rewrite as: $\varepsilon 0 \varepsilon \varepsilon 0 1 0 \varepsilon$

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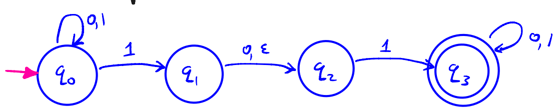
Example



$w = 0111$ $\qquad q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{1} q_3 \xrightarrow{1} q_3$

write $w = 01\varepsilon11$

$w = 0000\ 0$ is not accepted

Are NFAs more powerful than DFAs?

First any L accepted by a DFA is also accepted by an NFA
so the set of all Languages accepted by NFAs <u>includes</u>
all regular Languages

But are there Languages accepted by an NFA but
Not accepted by any DFA?

Are NFAs more powerful than DFAs? NO!

Theorem If $L \subseteq \Sigma^*$ is accepted by a NFA, then L is regular (that is, L is accepted by some DFA)

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Idea of Proof given a NFA N,

- create a DFA M where the states of M are all possible subsets of states from N
- On a string w, the state of M after reading w corresponds to all possible states that N could be in after reading w

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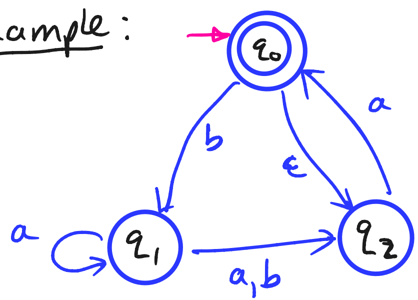

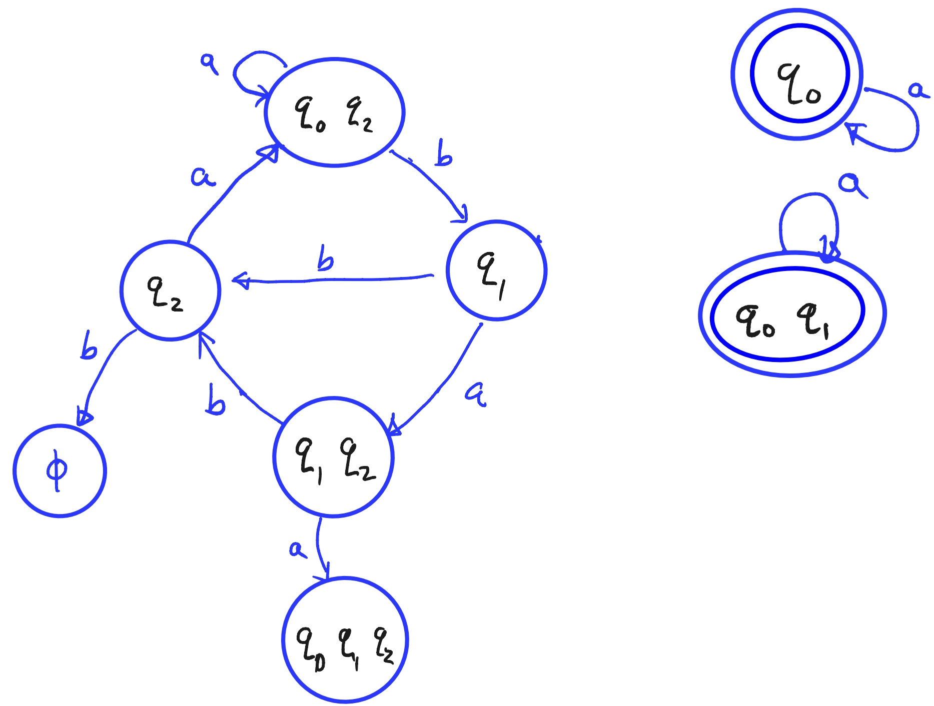

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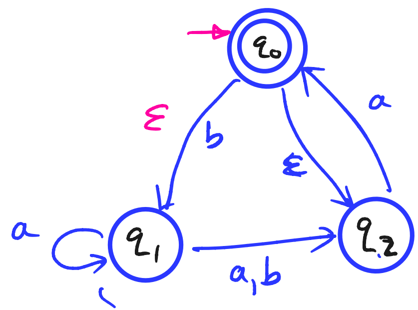


$q_0\ \varepsilon^* \to q_0\ q_2$

$q_0\ a\varepsilon^* \to \phi$

$q_0\ b\varepsilon^* \to q_1$

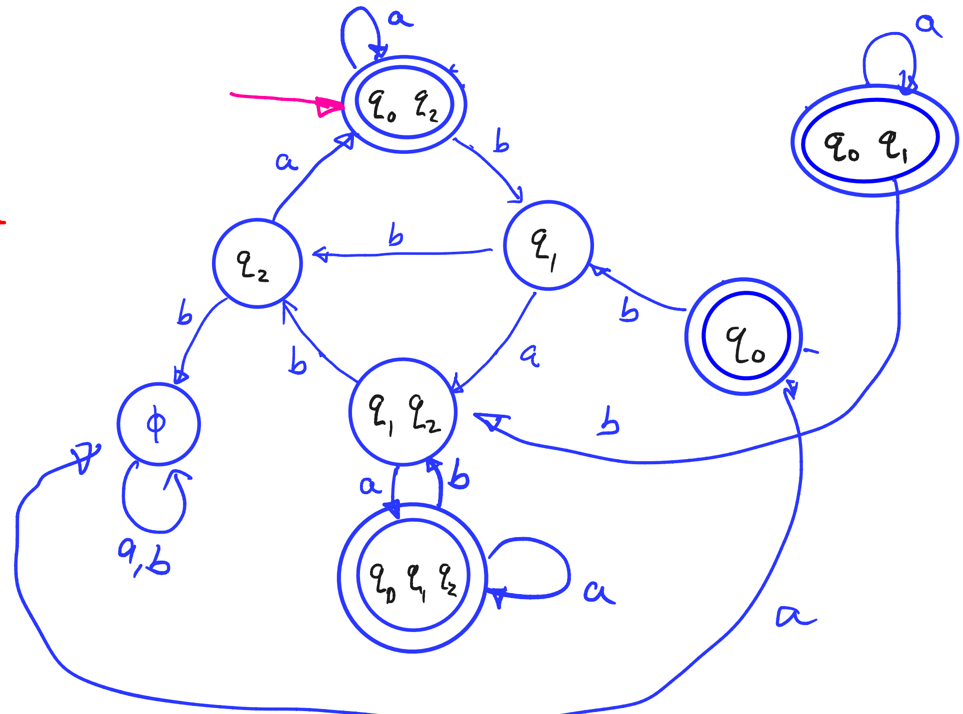
$q_1\ a\varepsilon^* \to q_1\ q_2$

$q_1\ b\varepsilon^* \to q_2$

$q_2\ a\varepsilon^* \to q_0\ q_2$

$q_2\ b\varepsilon^* \to \phi$

$\Rightarrow$



$w = 0\ 0\ 1\ 1$

$\Sigma^* \, 0 \, \Sigma^* \, 0 \, \Sigma^* \, 1 \, \Sigma^* \, 1 \, \Sigma^*$

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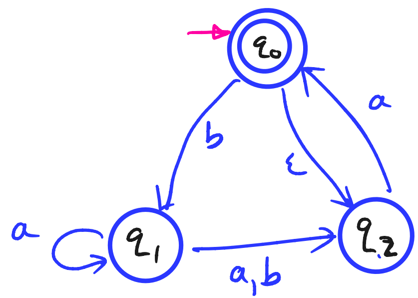

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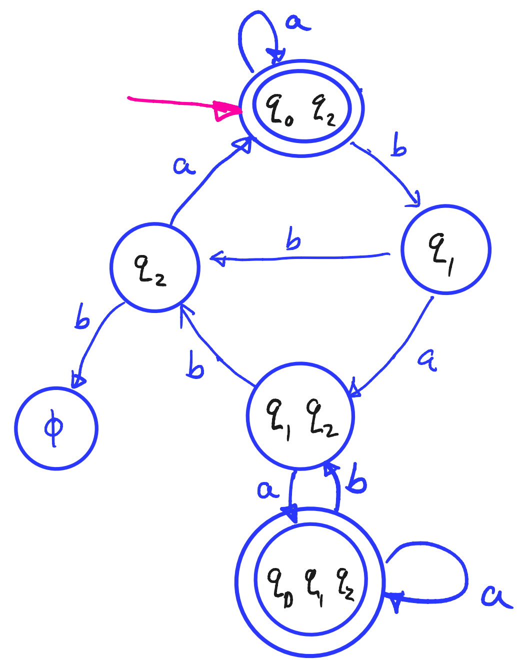
$q_1\ a\varepsilon \rightarrow q_1\ q_2$

$q_1\ b\varepsilon \rightarrow q_2$

$q_2\ a\varepsilon \rightarrow q_0\ q_2$

$q_2\ b\varepsilon \rightarrow \phi$



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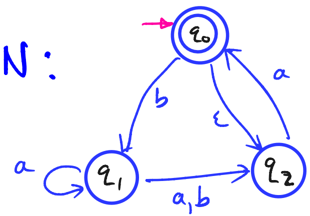
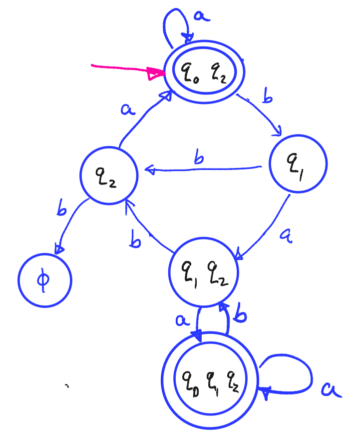
Example:

N:



$\Rightarrow$

M:



$N = \{ Q_N = \{q_0, q_1, q_2\},\ \Sigma = \{a, b\},\ q_0,\ F = \{q_0\},\ \delta_N \}$

$M = \{ Q_M = \{ \phi, \cancel{[q_0]}, [q_1], [q_2], \cancel{[q_0 q_1]}, [q_0 q_2], [q_1 q_2], [q_0 q_1 q_2] \}$

$\Sigma = \{a, b\}$

start state $= \{q_0 q_2\}$

$F = \{ \{q_0 q_1\}, \{q_0 q_1 q_2\}$

$\delta_M \}$

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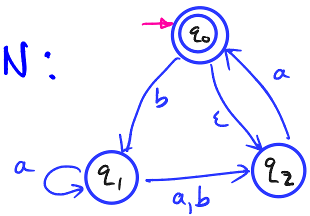
Example: N:



$\Rightarrow$

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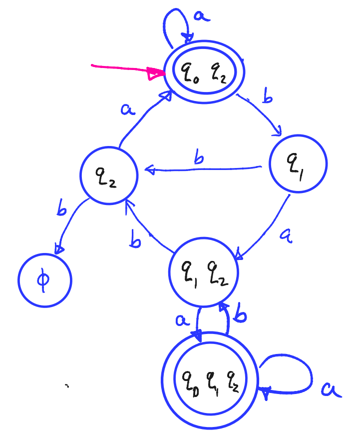

Claim
The construction satisfies the following property:
for every (sub)string w, Let
$Q_N' \subseteq Q_N$ be the set of all possible states that we could be in when running N on w
Then the DFA M on w ends in the state corresponding to the subset Q_N'

Closure Properties of Regular Languages

1. If $L \subseteq \Sigma^*$ is regular, then $\overline{L} = \{ w \in \Sigma^* \mid w \notin L \}$ is also regular

2. If L is regular, then $L^* = \{ w \mid w = v_1 \cdot v_2 \cdot \ldots \cdot v_k \mid v_1, \ldots, v_k \in L \}$ is regular

3. If L_1 and L_2 are regular, then $L_1 + L_2 = \{ w \mid w \in L_1 \text{ or } w \in L_2 \}$ is regular

4. If L_1 and L_2 are regular, then $L_1 \cdot L_2 = \{ w \mid$ w can be written as $u \cdot v$ where $u \in L_1$ and $v \in L_2 \}$