Lecture 13

Midterm: Monday Oct 23 in class

Midterm Review Session: Saturday 1-2 pm CSB451

Today: Midterm Review
Intro to Turing Machines / Computability

<u>Review:</u> Test 1

I. Regular Lanuages

DFA

NFA

Regular Expressions

Equivalence: Languages recognized by DFAs

$\equiv$ Languages recognized by NFAs

$\equiv$ Languages described by regular expressions

Closure Properties of Regular Languages

Proving Languages not Regular: Pumping Lemma

Review: Test 1

II. Context Free Languages

PDA

CFGs

Equivalence: Languages accepted by PDA
= Languages generated by CFG's

Closure Properties of CFLs

Proving Languages not context free:
Pumping lemma for CFL's

Review: Test 1

Regular L's examples

All finite languages

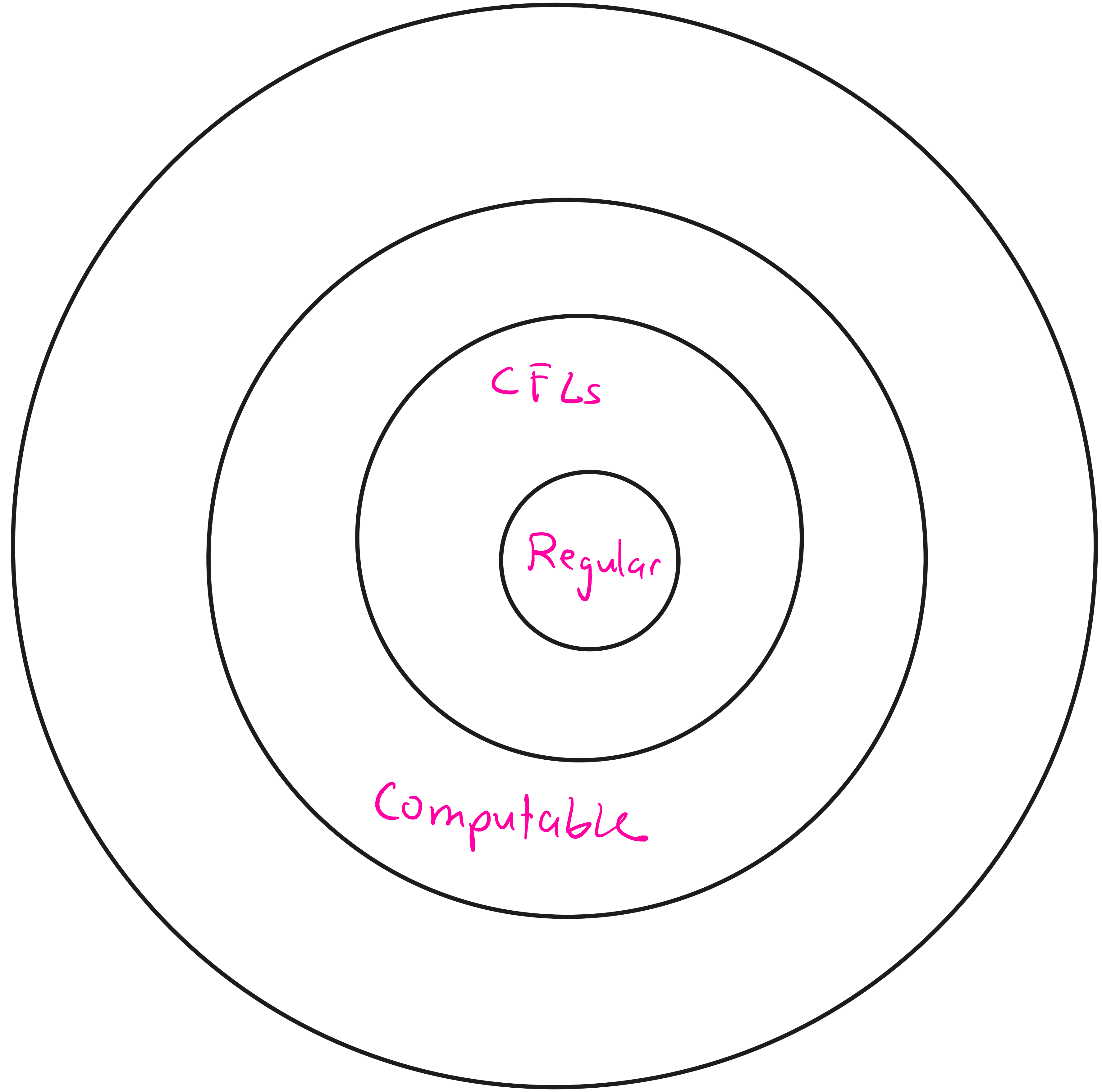
$L = \{w \in \{0,1\}^* \mid w \text{ has an odd \# of 0's}\}$

CFL

all regular languages

$L = \{ww^R \mid w \in \{0,1\}^*\}$

$L = \{0^n 1^n \mid n \geq 0\}$



Midterm Format   5-6 questions

- One True-False (~ 6 q's)
- 2-3 short answer Q's

  examples: sketch closure prop. of Regular/CFL
  show some L is or isn't CFL
  question about pumping lemma
  question about DFA ⟺ NFA
  Reg Exp ⟺ NFA
- 1 Q: Construct NFA / DFA / Regular Exp. for some Language
- 1 Q: Construct PDA/CFG
- 1 Q: Pumping Lemma for Reg. Lang

Review Problems

1. • give a Reg Expression for

$L_1 = \{ w \in \{0,1\}^* \mid w$ ends in a 1 and does not contain substring '00' $\}$

• give a Reg exp. for $\overline{L_1}$

• give a DFA for L_1

Reg Expression for L_1:

Try 1: $(1+01)^* 1$ doesn't accept 01

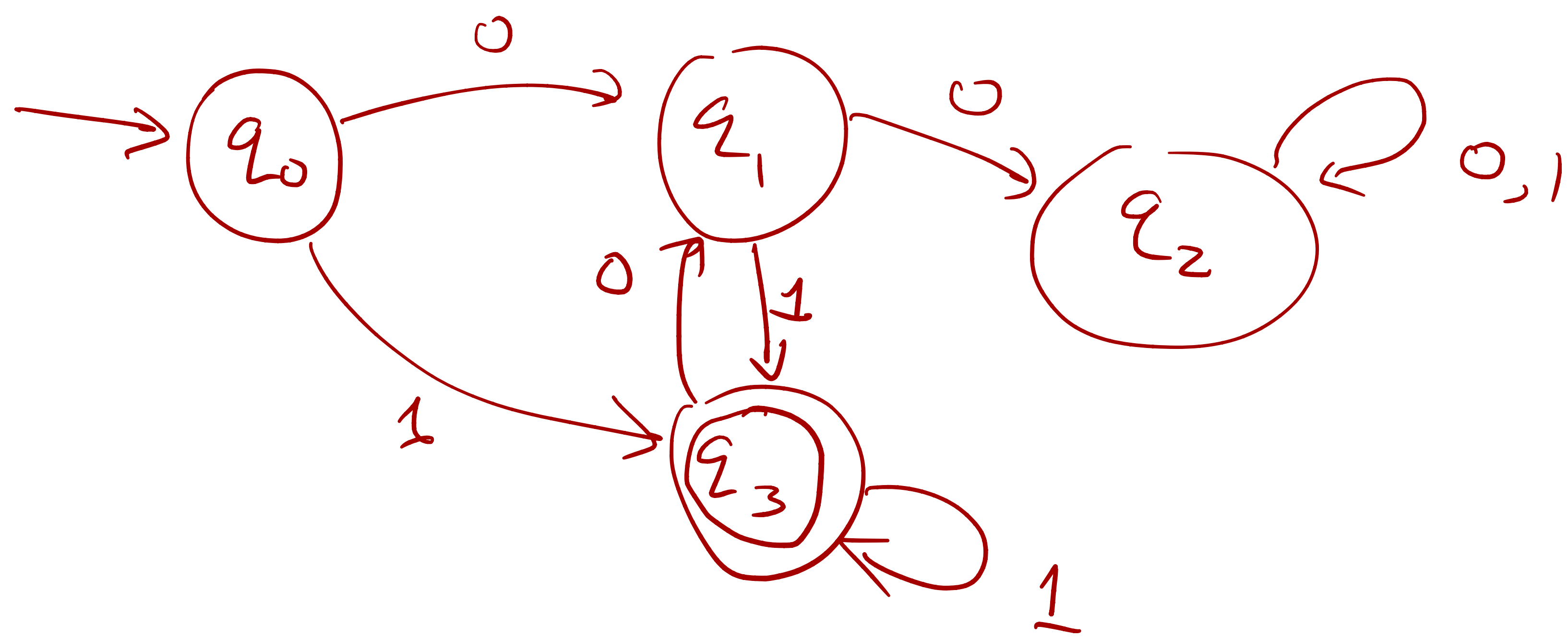
Try 2 $(1+01)^* 1 + (1+01)^*$ ← accepts ε

✓ Try 3 $(1+01)^* 1 + (1+01)^* 01$

$$L_1 : \underbrace{\varepsilon + (0+1)^* 0}_{\text{all string not ending in a 1}} + \underbrace{(0+1)^* 00 (0+1)^*}_{\text{all string contain 2 consecutive 0's}}$$

Review Problems

DFA for $L_1 = \{ w \in \{0,1\}^* \mid w$ ends in a 1 and does not contain substring '00' $\}$



DFA for $\overline{L_1}$: as above but accept states are $\{q_0, q_1, q_2\}$ and not q_3

Review Problems

2: give a Reg. Expression for

$$L_2 = \{0^i 1^j \mid i \text{ is even}, j \text{ is odd}\}$$

$L_2 = \{0^i 1^j \mid i \text{ even}, \; j \text{ odd}\}$

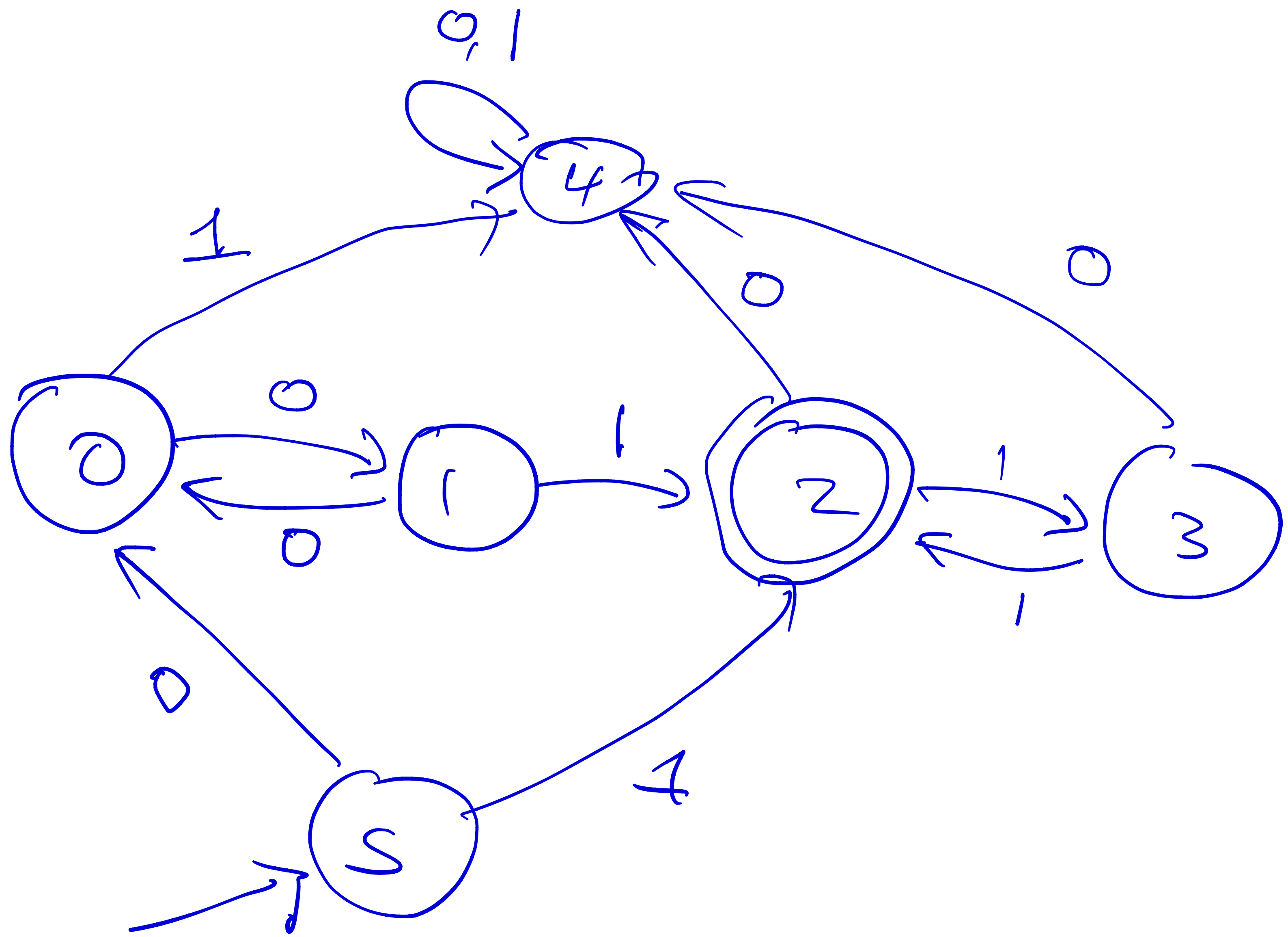
Reg expression : $(00)^* 1 (11)^*$

$\overline{L_2} = \{w \mid w \text{ is not of the form } 0^i 1^j, \; i \text{ even}, \; j \text{ odd}\}$

Reg exp : $0(00)^* 1 (0+1)^* + 0^* (11)^*$

$+ (0+1)^* 10 (0+1)^*$

DFA for $L_2 = \{ w \mid w = 0^i 1^j, \ i \text{ is even}, \ j \text{ is odd} \}$



DFA for $\overline{L_2}$: just make all states but 2 accept states

Review Problems

(3) Is this Language regular?

$$L_3 = \{0^m 1^n \mid m = 5n\}$$

Review Problems

③. Is this Language regular?

$$L_3 = \{0^m 1^n \mid m = 5n\}$$

No.

Proof: use PL for CFL's.

Review Problems

(4) Is this language regular?

$$L_4 = \{0^m \mid \exists n \text{ such that } m = 5n\}$$

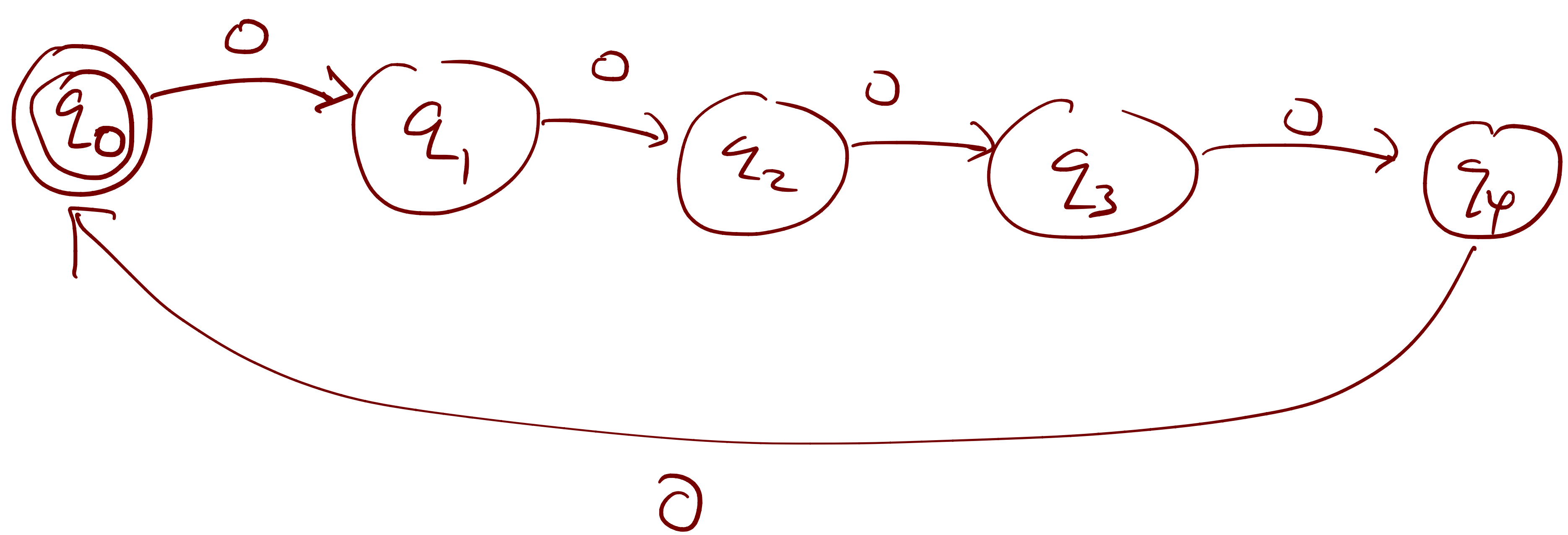
Review Problems

(4.) Is this language regular?

$$L_4 = \{0^m \mid \exists n \text{ such that } m = 5n\}$$

Yes. L_4 = strings 0^m where m is a multiple of 5

NFA:



Reg Exp: $S \rightarrow \varepsilon \mid 00000S$

Review Problems

5. give CFG for $L_5 = \{ w \in \{a,b\}^* \mid w \text{ has twice as many } a\text{'s as } b\text{'s} \}$

* This question is very hard.
I've provided a solution if you are interested
but this goes way beyond what
you are expected to know.
(So you can ignore this question!)

Review Problems

5. give CFG for $L_5 = \{ w \in \{a,b\}^* \mid w$ has twice as many a's as b's $\}$

Try 1

$S \to aaSb \mid bSaa \mid aSab \mid aSba \mid \varepsilon$

Doesn't accept baaaab

Review Problems

Try 2

$S \to SaSaSbS \mid SaSbSaS \mid SbSaSaS \mid SS \mid \varepsilon$

This only generates strings in L_5

But does it generate <u>all</u> strings in L_5 ?

Does it generate : aaaaaabbb ?

Yes $\quad S \to aaSb \to aaaaSbb$

$\to aaaaaabbb$

Try 3

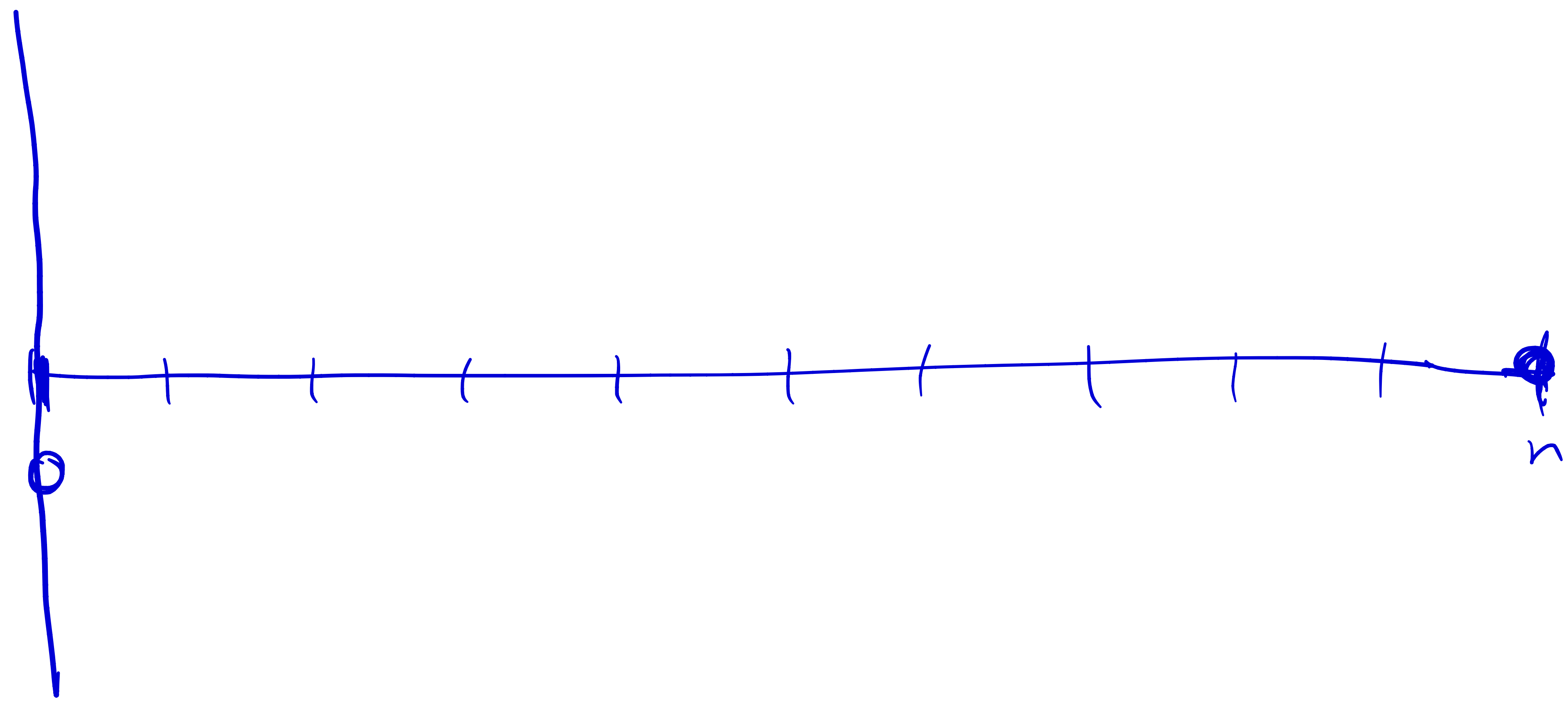
$$S \to SS \mid AbA \mid bSaa \mid aaSb \mid \varepsilon$$

$$A \to aS \mid SA$$

Let $w \in L_5$, and say $|w| = n$. We know: $\left(2 \cdot (\# b\text{'s}) - (\# a\text{'s}) \right) = 0$

b

f^w:



Let $f^w(i) = 2 * (\# b\text{'s in } w_1 \ldots w_i) - (\# a\text{'s in } w_1 \ldots w_i)$

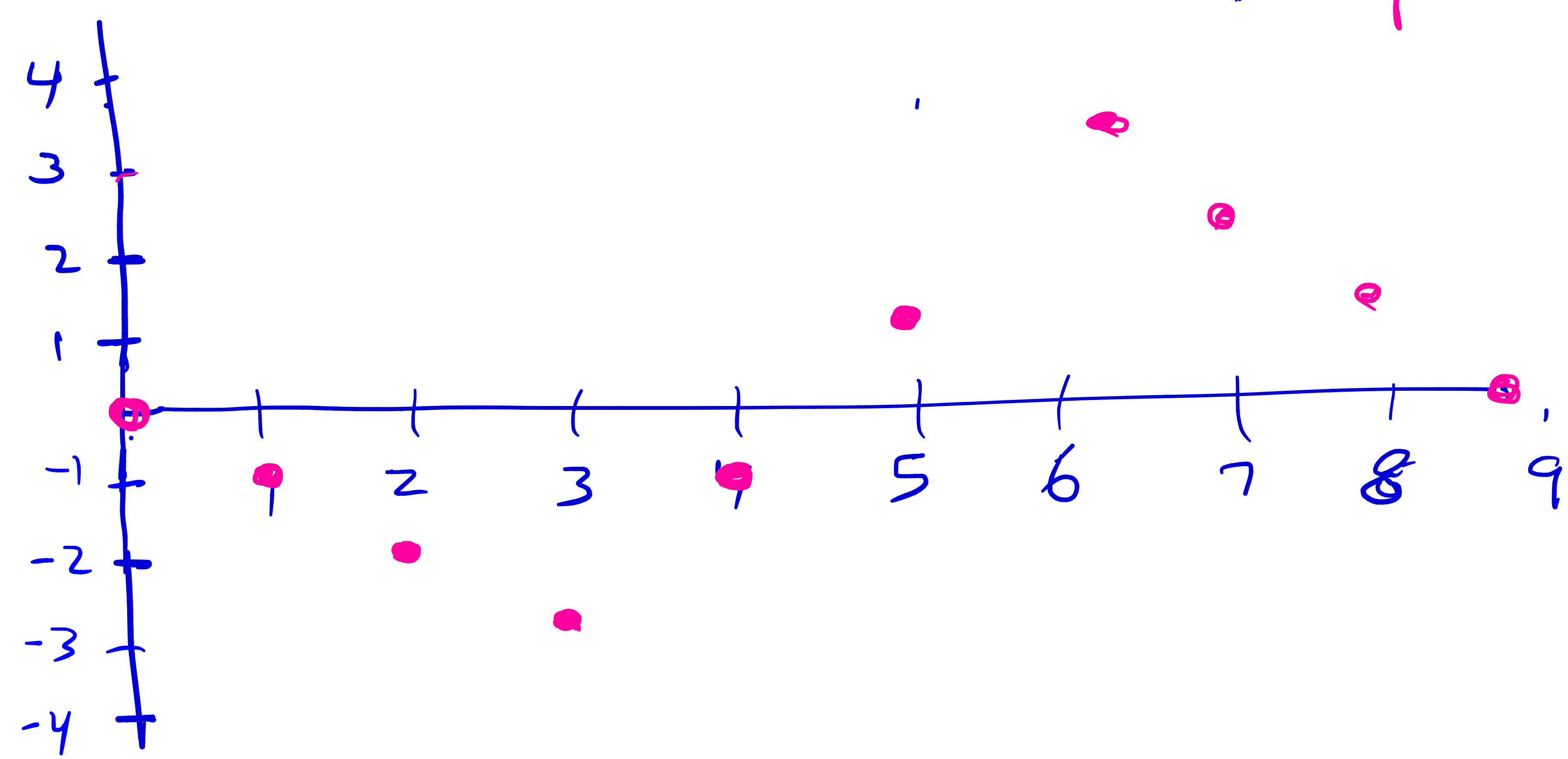
we know $f^w(0) = 0$, and $f^w(n) = 0$

Let $w \in L_5$, and say $|w| = n$. We know: $\left(2 \cdot (\# b\text{'s}) - (\# a\text{'s}) \right) = 0$

example $w = aaabbb\,aaa$

$n = |w| = 9$

f :

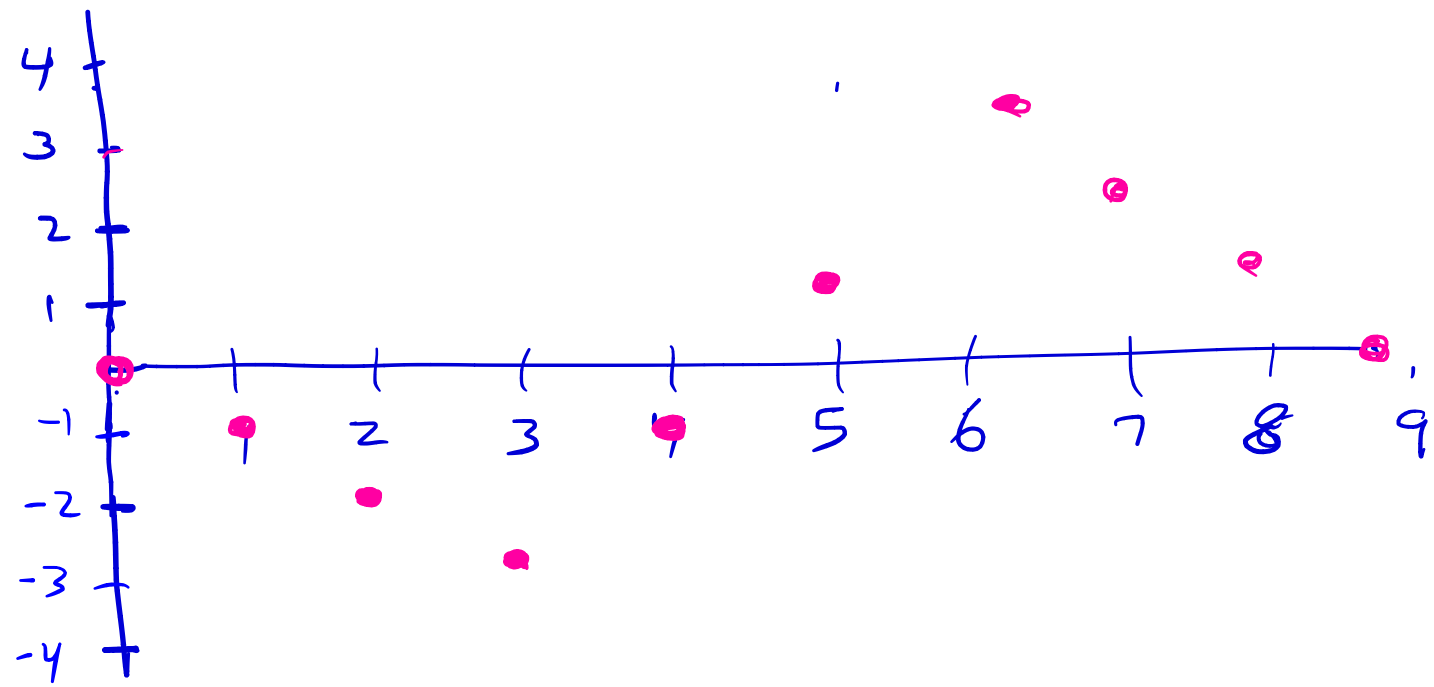


$f(i) = 2 * (\# b\text{'s in } w_1 \dots w_i) - (\# a\text{'s in } w_1 \dots w_i)$

we know $f(0) = 0$, and $f(n) = 0$

Let $w \in L_S$, and say $|w| = n$. We know: $\Big(2\cdot(\#b\text{'s}) - (\#a\text{'s}) \Big) = 0$

example $w = aaabbbaaa$
$n = |w| = 9$



$f(i) = 2 * (\#b\text{'s in } w_1 \ldots w_i) - (\#a\text{'s in } w_1 \ldots w_i)$

we know $f(0) = 0$, and $f(w) = 0$

<u>Case 1</u> $w = xyz$ where $f(x) > 0$ $f(xy) < 0$

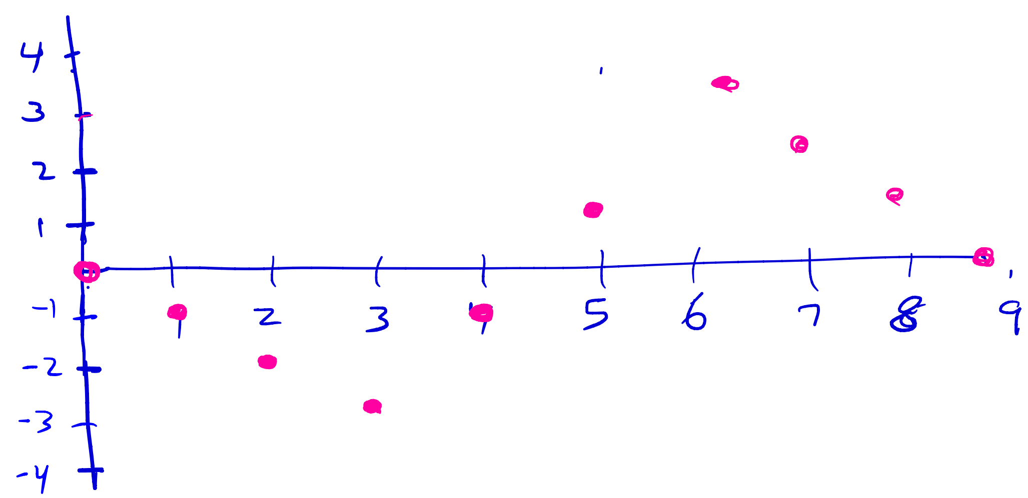
each time we add one character to x we either increase f^w by 2 or decrease f^w by 1, so we can split y into $y_1 y_2$ such that $c(xy_1) = 0$. then can by induction generate xy_1 from S, and $y_2 z$ from S.

<u>Case 2</u> $w = xyz$ where $f^w(x) < 0$, $f^w(xy) > 0$ (like example above)

then can always split y into $y_1 y_2$ such that either (i) $f(xy_1) = 0$ or (ii) $f(xy_1) = -1$ and y_2 starts with b

Let $w \in L_S$, and say $|w| = n$. We know: $\Big(2\cdot(\#b\text{'s}) - (\#a\text{'s}) \Big) = 0$

example $w = aaabbbaaa$, $n = |w| = 9$



$f(i) = 2 * (\#b\text{'s in } w_1 \dots w_i) - (\#a\text{'s in } w_1 \dots w_i)$

we know $f(0) = 0$, and $f(w) = 0$

Case 1 $w = xyz$ where $f(x) > 0$ $f(xy) < 0$

each time we add one character to x we either increase f^w by 2 or decrease f^w by 1, so we can split y into $y_1 y_2$ such that $c(xy_1) = 0$.

then can by induction generate xy_1 from S, and $y_2 z$ from S.

Case 2 $w = xyz$ where $f^w(x) < 0$, $f^w(xy) > 0$ (like example above)

then can always split y into $y_1 y_2$ such that either (i) $f(xy_1) = 0$

or (ii) $f(xy_1) = -1$ and y_2 starts with b

if (i) holds then can generate xy_1 from S and $y_2 z$ from S

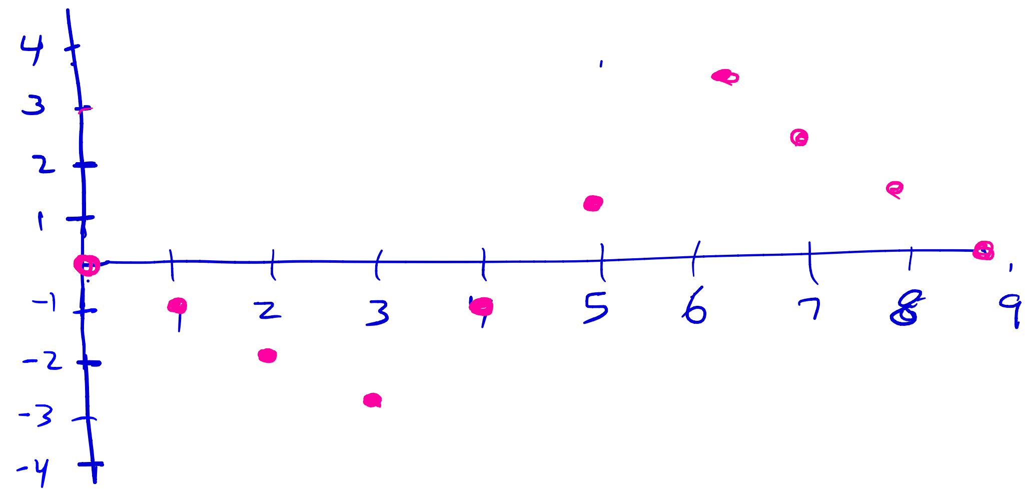
if (ii) holds then $w = w_1 b w_2$ where $f(w_1) = f(w_2) = -1$

(in our example $\underbrace{aab}_{w_1} b \underbrace{baaa}_{w_2} = w$)

then generate w by: $S \to AbA$, where A generates any x with $f(x) = -1$

Let $w \in L_S$, and say $|w| = n$. We know: $\left(2 \cdot (\# b\text{'s}) - (\# a\text{'s}) \right) = 0$

example $w = aaabbbaaa$, $n = |w| = 9$



$f(i) = 2 * (\# b\text{'s in } w_1 \ldots w_i) - (\# a\text{'s in } w_1 \ldots w_i)$

we know $f(0) = 0$, and $f(w) = 0$

Case 1 $w = xyz$ where $f(x) > 0$ $f(xy) < 0$

each time we add one character to x we either increase f^w by 2 or decrease f^w by 1, so we can split y into $y_1 y_2$ such that $c(xy_1) = 0$. then can by induction generate xy_1 from S, and $y_2 z$ from S.

Case 2 $w = xyz$ where $f^w(x) < 0$, $f^w(xy) > 0$ (like example above)

then can always split y into $y_1 y_2$ such that either (i) $f(xy_1) = 0$ or (ii) $f(xy_1) = -1$ and y_2 starts with b

Case 3 If cases (1) + (2) dont hold then either $w = bxaa$ or $aaxb$.

If $w = bxaa$. Then generate w by: $S \to bSaa$, and inductively $S \overset{*}{\Rightarrow} x$

Similarly if $w = aaxb$ then use $S \to aaSb$, and inductively $S \overset{*}{\Rightarrow} x$

Review Problems

6. give a CFG for

$L_6 = \{ x_1 \# x_2 \# \dots \# x_k \mid k \geq 1 \text{ and } x_i = x_j^R \text{ for some } i, j \}$

Note: Solution has been corrected

Review Problems

6. give a CFG for

$L_6 = \{ x_1 \# x_2 \# \ldots \# x_k \mid k \geq 1 \text{ and } x_i = x_j^R \text{ for some } i,j \}$

$S \to LBR \mid LCR$

$C \to aCa \mid bCb \mid a \mid b \mid \varepsilon$ ← generates palindromes

$B \to aBa \mid bBb \mid \#L$ ← generates $w \# w_1 \# w_2 \cdots \# w_k \# w^R$ for any k

$L \to A \# L \mid \varepsilon$ ← $w_1 \# w_2 \# \ldots \# w_k \#$ for any k

$R \to \# A R \mid \varepsilon$ ← $\# w_1 \# w_2 \ldots \# w_\ell$ for any ℓ

$A \to 0A \mid 1A \mid \varepsilon$ ← generates all $w \in \{a,b\}^*$

LBR: generates $x_1 \# x_2 \# \ldots \# w_k$ such that $w_i = w_j^R$ for some $i \neq j$

LCR: " " such that $w_i = w_i^R$ for some i

Review Problems

7. Give PDAs for Questions 6-7

Are these languages regular?