COMS 3261, Computer Science Theory (Fall 2023): Assignment 2
Due on Gradescope 11:59pm, Monday October 16, 2023

Instructions

- The total number of points is 60 not counting Problem 2a (which is optional and for extra credit.)
- Submit your solutions in pdf format. Late homeworks will **not** be accepted.
- You can discuss with TAs, the prof, and other students, but please acknowledge them at the beginning of each problem. All of your solutions must be written in your own words.

Problems

1. Give PDA's for the following languages.
   a. (10 marks) The set of all strings over $\{0, 1\}$ with exactly twice as many 0's as 1's.
   b. (10 marks) The set of all strings over $\{a, b, c\}$ of the form $a^i b^j c^k$ such that either $i \neq j$ or $j \neq k$.

2. Give context-free grammars generating the following languages.
   a. (EXTRA CREDIT) The set of all strings over $\{a, b\}$ not of the form ww for some string $w \in \{a, b\}^*$. To receive extra credit points for this question, you must provide a complete and correct solution, including an explicit CFG generating this language, as well as a clear and complete argument/proof showing why your CFG generates exactly the language in question.
   b. (10 marks) The complement of the language $\{a^n b^n \mid n \geq 0\}$

3. Are the following languages context free? Prove or disprove your answer.
   a. (10 marks) $\{a^i b^j \mid j = i^2\}$
   b. (10 marks) $\{a^i \mid i \text{ is a prime}\}$

4. (10 marks) Prove that the class of context-free languages are not closed under complementation.