Computer Science Theory, Test 2 Review Problems
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1. Answer True or False for each statement. No justification is needed.
   (a) $n = O(n^2)$
   (b) $n \log n = O(n)$
   (c) $n^n = O(2^n)$
   (d) Let A be mapping reducible to B. If B is decidable then A must be decidable.
   (e) Let A be mapping reducible to B. If A is decidable then B must be decidable.
   (f) If the complement of a language L is not recognizable then both L and $\neg L$ are not recognizable.
   (g) If A is NP-complete, $A \subseteq B$, and B is in NP then B is NP-complete.
   (h) If B is NP-complete and $A \subseteq B$ and A is in NP then A is NP-complete.

2. Let Double-CLIQUE denote the language consisting of all pairs (G, k) such that G is an undirected graph containing two disjoint cliques each of size k. Prove that Double-CLIQUE is NP-complete.

3. Prove that the following set is countable.
$$S = \{(i, j) \mid i \geq 0 \text{ and } j > i\}$$

4. Prove that the following set is countable.
$$S = \{L \subseteq \{0,1\}^* | \text{the number of strings in } L \text{ is finite}\}$$

5. Prove that NP is closed under union. That is, for every $L_1, L_2 \in$ NP, $L_1 \cup L_2$ is also in NP.

6. Prove that NP is closed under concatenation.

7. Let L be the language consisting of all pairs $< M >$ such that M encodes a Turing machine and M accepts at least two inputs.
   (a) Prove that L is recognizable.
   (b) Prove that L is not decidable.

8. Recall that 3SAT is the set of all 3-CNF formulas ϕ such that ϕ is satisfiable. Let Search-3SAT be the following *search* problem: Given a 3CNF formula ϕ, output a satisfying assignment for ϕ if one exists, and otherwise output "ϕ is unsatisfiable". Prove that if 3SAT $\in$ P, then Search-3SAT can be solved in polynomial-time by a deterministic TM.