1.a) $0^p10^p \in L$. Since $|xy| \le p$, $xy = 0^m0^n$ s.t. $m+n \le p$

then $\underbrace{0^m}_{x}\underbrace{0^n}_{y}\underbrace{0^{p-(n+m)}10^p}_{z}$ where $n > 0$

choose $i=2$, $xy^2z = 0^m0^{2n}0^{p-(m+n)}10^p = 0^{p+n}10^p$

Any way to split 0^p into $w0^k$ has $k \le p$, so

$p+n > k$, and so $0^{p+n}10^p \notin L$ and L is not regular

b) $0^{2^p} \in L$. $|xy| \le p$ so $xy = 0^m0^n$ s.t. $m+n \le p$, $n > 0$

$\underbrace{0^m}_{x}\underbrace{0^n}_{y}\underbrace{0^{2^p-(m+n)}}_{z}$ choosing $i=2$, $xy^2z = 0^{2^p+n}$

for $p > 0$, $2^p > p$ so $|0^{2^p+n}| < |0^{2^p+2^p}| = |0^{2^{(p+1)}}|$

since $n > 0$, $|0^{2^p+n}| > 2^p$ therefore $0^{2^p+n} \notin L$

and L is not regular

c) $0^p1^{p-1} \in L$, $|xy| \le p$ so $xy = 0^m0^n$ s.t. $m+n \le p$, $n > 0$

$\underbrace{0^m}_{x}\underbrace{0^n}_{y}\underbrace{0^{p-(m+n)}1^{p-1}}_{z}$ choosing $i=0$, $xy^0z = xz = 0^{p-n}1^{p-1}$

Since $n > 0$, $p-n \le p-1$ and so $0^{p-n}1^{p-1} \notin L$

and L is not regular

d) $0^p1^p \in L$, $\underbrace{0^m}_{x}\underbrace{0^n}_{y}\underbrace{0^{p-(m+n)}1^p}_{z}$, $m+n \le p$, $n > 0$

choose $i=2$, $xy^2z = 0^{p+n}1^p$. $p+n \ne p$ so

$0^{p+n}1^p \notin L$, so L is not regular

2. a) choose $p = 2$

Case 1: $w = b^j c^k$

choose $x = \varepsilon$; then choose $y = b$ or $y = c$ if $j = 0$.
then, $xy^iz = y^iz = b^{j-1+i}c^k$ or c^{k-1+i} which are both in L.

Case 2: $w = ab^nc^n$

choose $x = \varepsilon$, $y = a$. Then, $xy^iz = a^ib^nc^n \in L$

Case 3: $w = aab^jc^k$

choose $x = \varepsilon$, $y = aa$. Then, $xy^iz = a^{2i}b^jc^k$.
since $a^{2i} \neq a$, j and k do not have to be equal, and $a^{2i}b^jc^k \in L$

Case 4: $w = a^nb^jc^k$ where $n \geq 3$

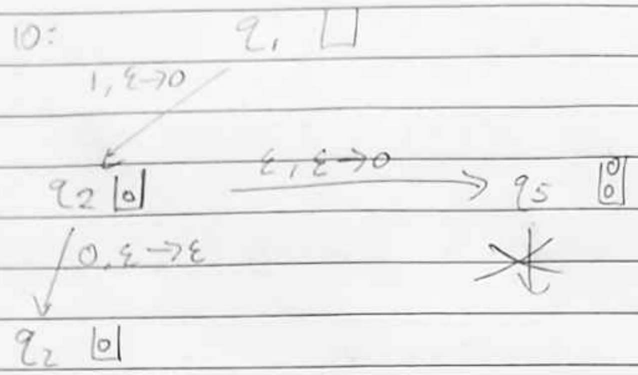
Choose $x = aa$, $y = a$. Then, $xy^iz = aaa^ib^jc^k$.
Like case 3, there will not be exactly 1 a, and so $aaa^ib^jc^k \in L$

In all cases, the pumping lemma holds, so L satisfies the pumping lemma

b) Assume L is regular. Then, let M be the DFA that recognizes L. Denote the start state of M as q_i, and the state corresponding to $\delta(q_i, a) = q_i'$
Let M' be the DFA constructed from M with all transitions $\delta(q, a)$ removed, and with initial state q_i'. Then, M' recognizes the language b^nc^n, which is not regular, a contradiction. Therefore, L is not regular

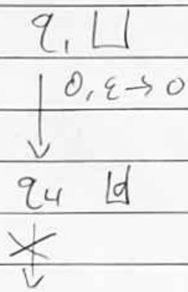
Alternatively, if L is regular, then $L \cap ab^*c^* = ab^nc^n$ is regular by closure of regular languages under intersection. This is a contradiction since ab^nc^n is not regular. Therefore L is not regular

3. 10:



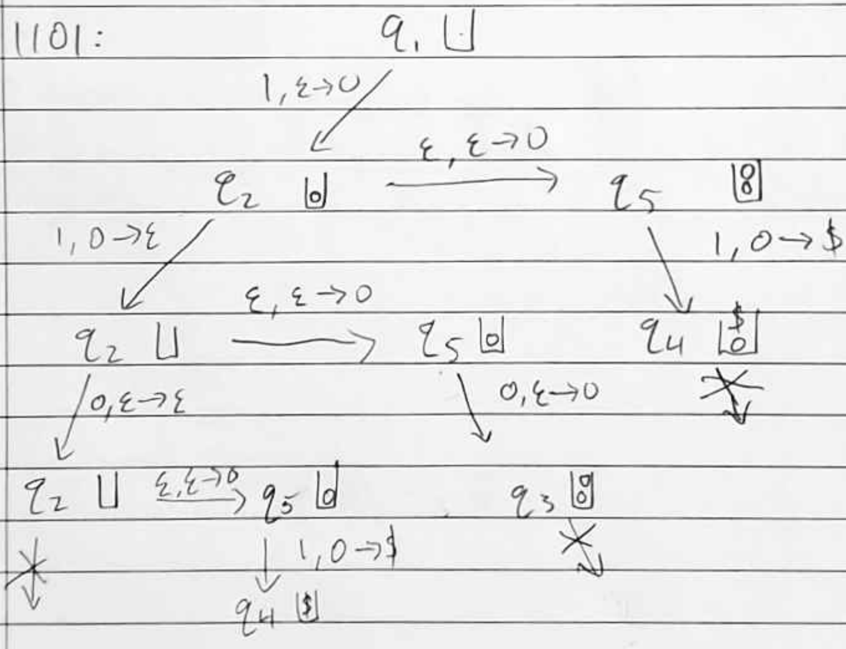
reject 10

010:



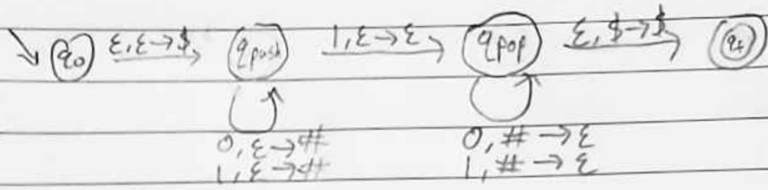
reject 010

1101:

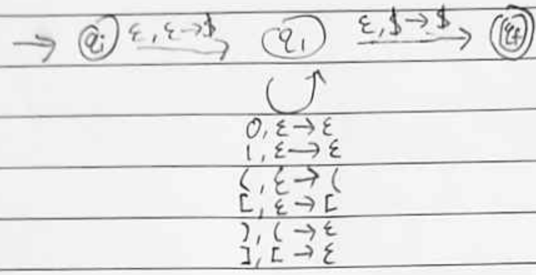


reject 1101

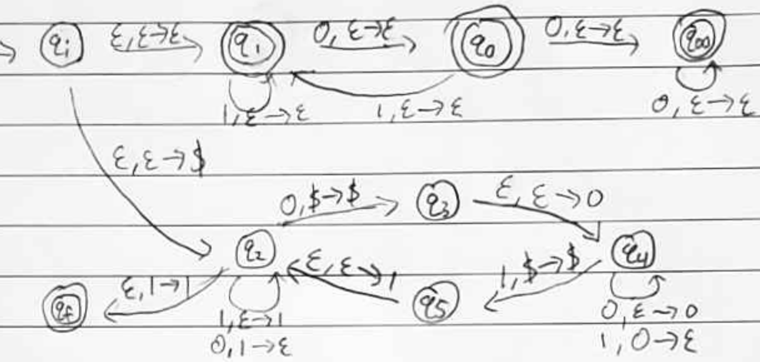
4. a) let $\Gamma = \{\$, \#\}$



b)



c)



top portion is DFA recognizing strings without 001
bottom portion uses stack to count 0's and 1's