

COMS 4236 – Test 1 Practice Problems

1. LOG-CLIQUE

Define $\text{LOG-CLIQUE} = \{G : \text{an } n\text{-vertex graph } G \text{ has a clique of size at least } \lceil \log n \rceil\}$.

- (a) Give a deterministic $n^{O(\log n)}$ -time algorithm for LOG-CLIQUE.
- (b) Suppose LOG-CLIQUE were NP-complete under polynomial-time many-one reductions. Prove that every language in NP would then have a quasipolynomial-time algorithm, carefully accounting for the polynomial blowup of the reduction. Explain why this is evidence against NP-completeness but not a known contradiction.

2. coNP-completeness of UNSAT

Recall that UNSAT is the language consisting of all CNF formulas F that are unsatisfiable. Prove that UNSAT is coNP-complete. (You may use without proof that SAT is NP-complete.)

3. Decision versus search

Let $R(x, y)$ be a polynomial-time relation with polynomially bounded witnesses, and let

$$L = \{x : \exists y R(x, y) = 1\}.$$

Assuming $P = NP$, prove that on input x one can find a valid witness y in polynomial time (or correctly report that none exists). Your proof should explicitly formulate the language used to recover y bit by bit. Then explain briefly why merely assuming $L \in P$ would not by itself justify the same general argument.

4. Simplifying alternations

Let R and S be polynomial-time predicates.

- (a) Classify

$$L = \{x : \exists u[(\forall v R(x, u, v) = 1) \wedge (\exists w S(x, u, w) = 1)]\}$$

as low in the PH (polynomial hierarchy) as you can. Write an equivalent prenex formula with the fewest alternations.

- (b) Do the same after replacing $\wedge$ by $\vee$. Explain why simply counting the visible quantifier symbols gives the wrong intuition in both cases.

5. Using the deterministic time hierarchy theorem

Recall the time hierarchy theorem: if $t(n) \log t(n) = o(T(n))$, for suitable time-constructible functions, then

$$\text{DTIME}(t(n)) \subsetneq \text{DTIME}(T(n)).$$

Use the time hierarchy theorem to prove $P \neq \text{EXP}$. Your proof must explicitly handle the fact that P is a union of all polynomial-time classes.

6. Configuration graphs and STCON

Let M be a nondeterministic $O(\log n)$ -space machine on input x .

- (a) Construct its configuration graph, prove it has polynomially many vertices, and show that M accepts x iff an accepting configuration is reachable from the start configuration. Explain why the corresponding graph can be output by a logspace TM even though the whole graph cannot be stored.
- (b) Use this to prove $\text{NL} \subseteq \text{P}$.

7. Short Answer

- (a) If A is NP-complete and $A \in \text{P}$, prove $\text{P} = \text{NP}$.
- (b) If $A \leq_p B$ and $B \in \text{P}$, what follows about A ?
- (c) Why does $B \leq_p \text{SAT}$, together with $B \in \text{NP}$, not prove that B is NP-complete?
- (d) If $\text{UNSAT} \in \text{NP}$, what follows about NP and coNP ?
- (e) If an NP-complete language lies in coNP , what follows?
- (f) What is the complement of an NP-complete language complete for?
- (g) Why does an $O(p(n))$ -space machine have at most $2^{O(p(n))}$ many configurations?