Oblivious Pseudorandom Functions and Some (Magical) Applications

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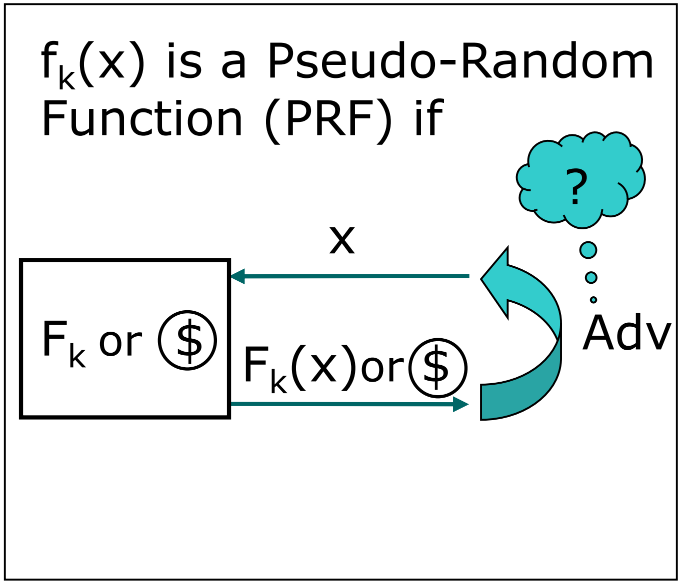
This presentation is based on the following research papers:

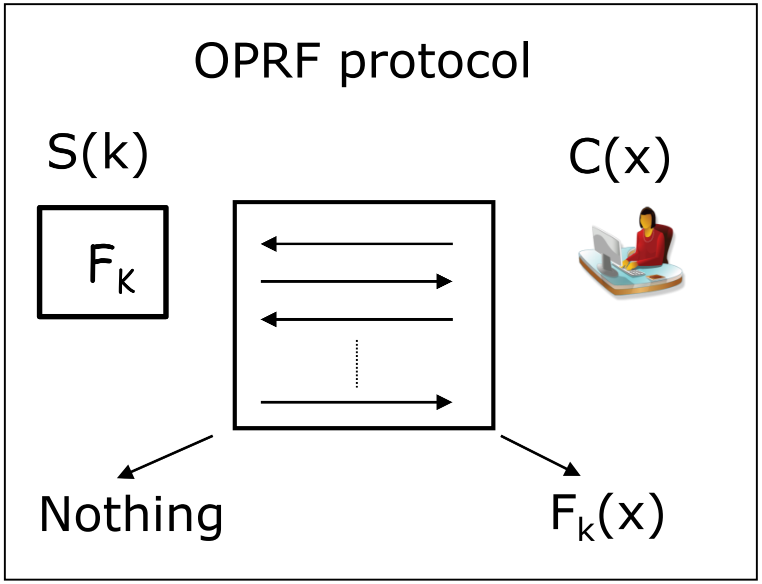
https://eprint.iacr.org/2019/1275

https://eprint.iacr.org/2018/163

https://eprint.iacr.org/2017/363

Oblivious PRF (OPRF)





- OPRF: An interactive PRF "service" that returns PRF results *without learning the input or output of the function*
- *A POWERFUL primitive*

DH-OPRF

The Diffie-Hellman Problem

- Cyclic group G of *prime order* q with generator g
  - $G = \{1, g, g^2, \ldots, g^{q-1}\}$
  - Crucial property: for all x, y in {0…q-1}: $g^{xy} = (g^x)^y = (g^y)^x = g^{yx}$
- "Diffie-Hellman problem": Given g^x and g^y, it's hard to compute g^{xy}
- "One-More DH Assumption":
  - Given $(g, g^k, g_1, g_2, \ldots, g_m)$ and Q calls to a k-exponentiation oracle $(\cdot)^k$
  - Cannot output g_i^k for more than Q elements in $\{g_1, g_2, \ldots, g_m\}$
- We will also need: Hash function H that maps arbitrary strings to random elements in G ("random oracle model")

DH-OPRF

- **PRF**: $F_K(x) = H(x)^k$; *input x, key k from 0…q-1*
- Oblivious computation via Blind DH Computation (S has k, C has x)

S: key k

C: input x

$a = (H(x))^r$ ← random r

$b = a^k$ → Computes $H(x)^k \leftarrow b^{1/r}$

- $b^{1/r} = (a^k)^{1/r} = (((H(x))^r)^k)^{1/r} = (((H(x))^k)^r)^{1/r} = (H(x))^k$
- The blinding factor r works as a one-time encryption key: ***hides*** H(x), x ***and*** $F_K(x)$ ***perfectly from S*** (and from any observer)

DH-OPRF

$H'(x, H(x)^k)$

- **PRF**: $F_K(x) = H(x)^k$; input x, key k from 0…q-1
- Oblivious computation via Blind DH Computation (S has k, C has x)

S: key k

C: input x

random r

$a = (H(x))^r$ ←-------------------

$b = a^k$ -------------------→

Computes $H(x)^k \leftarrow b^{1/r}$

- Computational cost: one round, 2 exponentiations for C, one for S
  - Commodity laptop: > 10,000 exponentiations/second
  - Variant: fixed base exponentiation for C (even faster)

DH-OPRF

- Long history (blinded DH): [..., CP'93, SY'96, HFH'99, FK'00, AES'03, JL'10,...],
- $H'(H(x)^k)$ treated as PRF in [NPR'99] and as OPRF in [JL'10]
- Variants $(H(x))^k$, $H'(H(x)^k)$, $H'(x, H(x)^k)$, ...
- Security [JL'10, JKK'14]: Secure as OPRF in the Random Oracle Mode assuming Gap-One-More-DH [BNPS'03]
- DH-OPRF: Most efficient OPRF implementation (elliptic curves)
- *Defining OPRF: Tricky notion → many definitions (balancing security, utility, performance)*

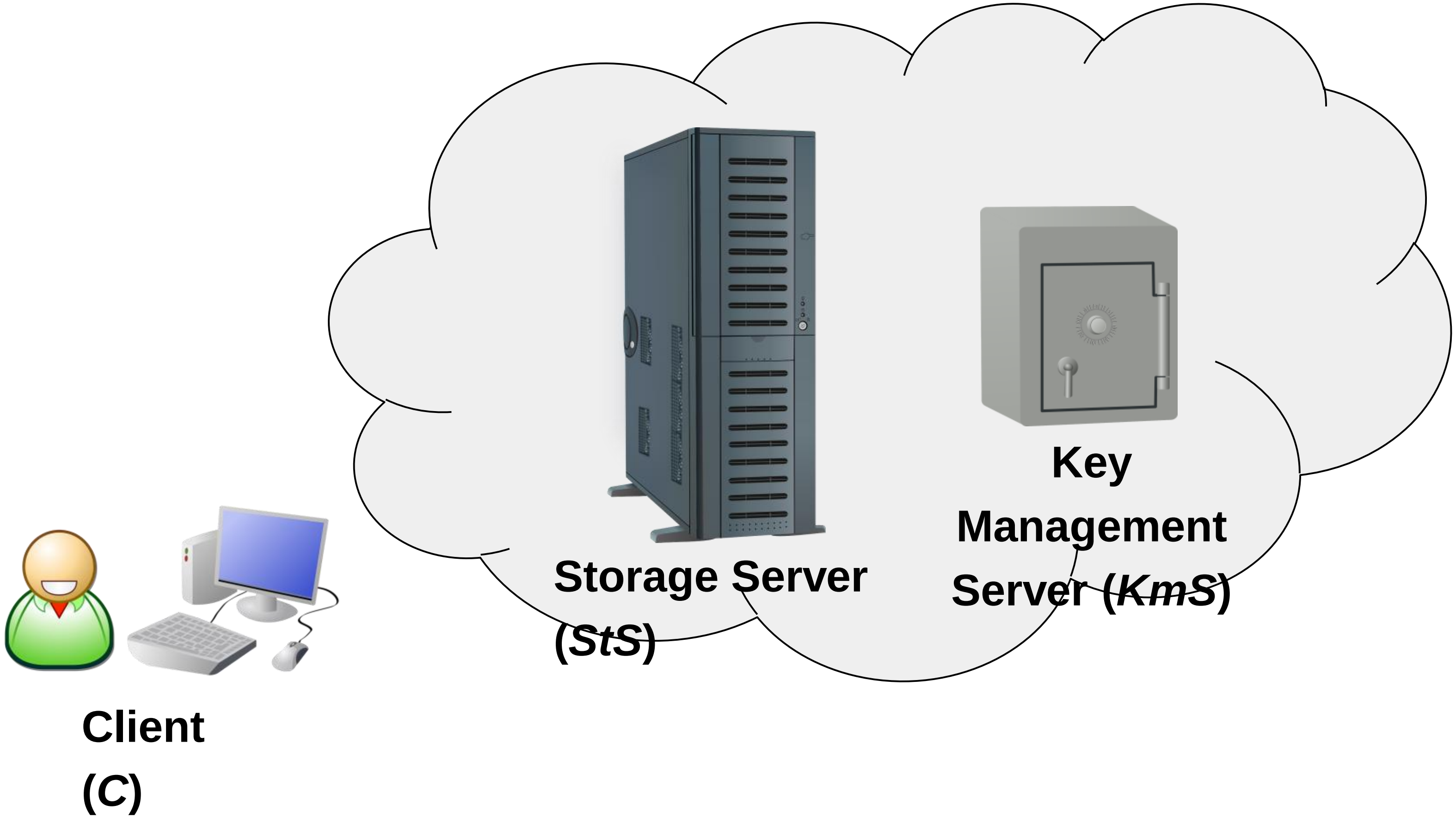
Many applications

- Private set intersection: HFH’99,FIPR’05,JL’10,CT’10,…, PSZ’14’15,KRRT’16,..
- Private Keyword Search (Keyword OT/PIR) [FIPR’05]
- Pattern matching [HL08, FHV13]
- De-duplication (files, medical records, etc.) [BKR’13,BCAPR’17]
- Chameleon pseudonyms, oblivious tokenization [CL’17]
- Search on Encrypted Data [CJJKRS’13, CJKRS’13]: Uses DH-OPRF “non-interactively” by storing blinded copies of the OPRF key

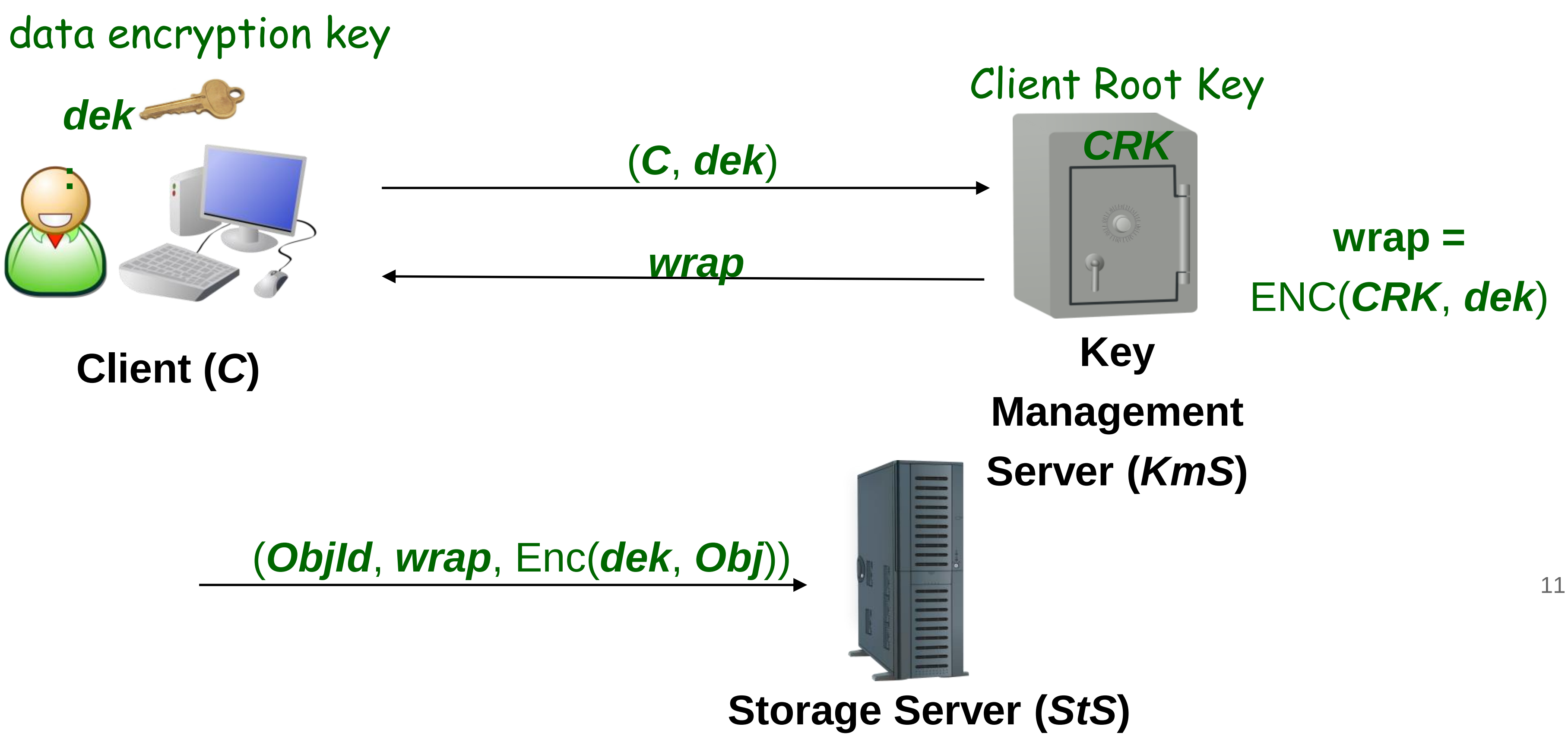
New Applications

- Key management services (esp. cloud storage systems)
- Revamping the world of password protection…

What is a "Cloud KMS"?

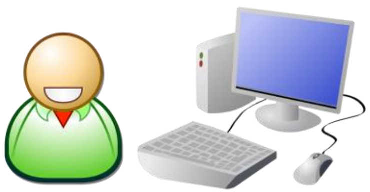


Wrap-Unwrap Method: Wrapping



Wrap-Unwrap Method: Unwrapping

(***ObjId***, ***wrap***, **e** = Enc(***dek***, ***Obj***))



(***C***, ***wrap***) →

← ***dek***

CRK

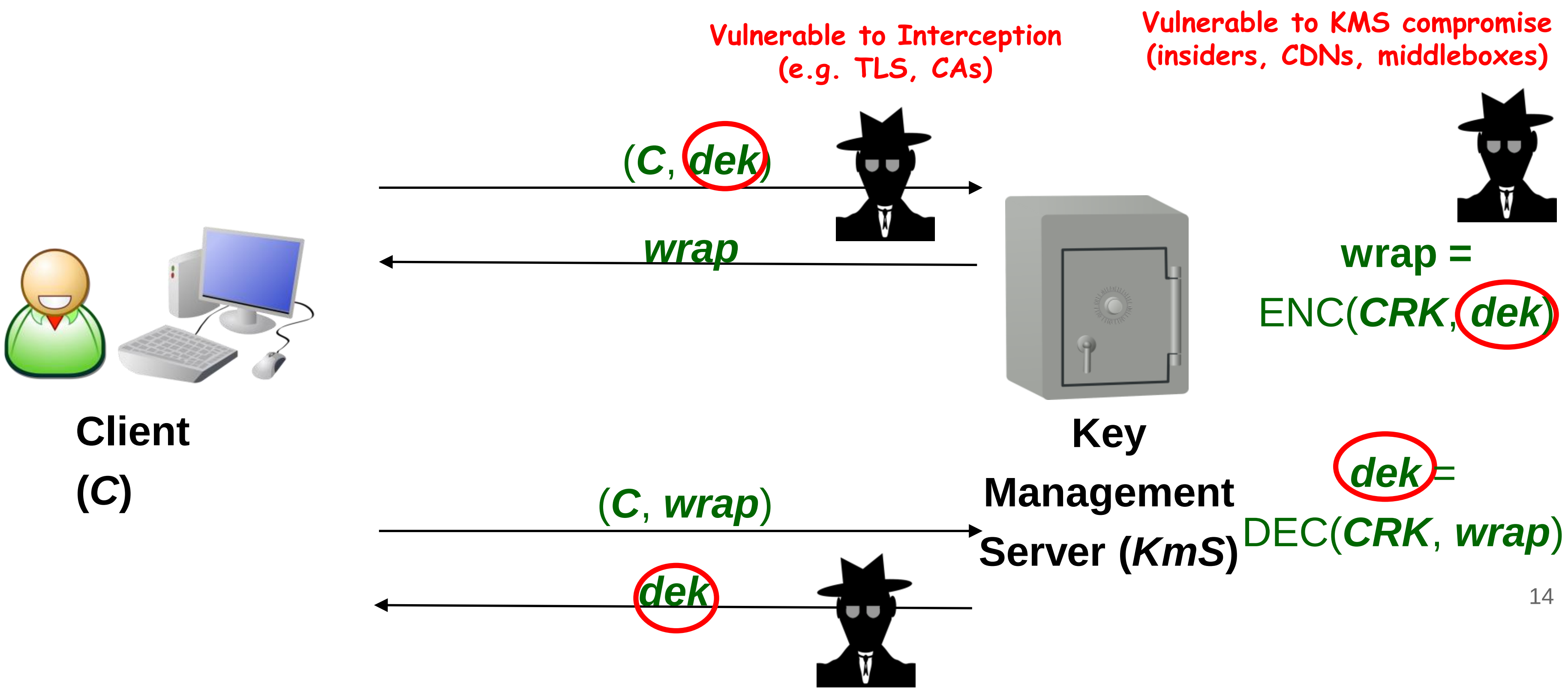
dek = DEC(***CRK***, ***wrap***)

Client (*C*)

Key Management Server (*KmS*)

Obj = Dec(***dek***, ***e***)

Cloud KMS – Weaknesses and Vulnerabilities



OPRF-based KMS

- OPRF replaces traditional wrap/unwrap approach
- DEK = OPRF(key=CRK, input=DEK-id) ,   i.e., DEK = $(H(\text{DEK-id}))^{CRK}$
  - CRK is the client's OPRF key, replaces the traditional wrapping key
- Keys (DEK) transmitted with perfect secrecy from network and insiders - no reliance on TLS or CA's (even "PQ Secure")
- KMS can't determine which keys the user is accessing

Further Features of OPRF Approach

- Verifiability: If client has g^k ($g \in G$, k the client's OPRF key), it can verify that $H(\text{DEK-ID})^k$ is correct, hence DEK is correct
  - Note that if KMS returns wrong key/wrap data lost forever
- Reduced storage: No need to store wraps in addition to key id's; KMS can derive OPRF keys from a single key (reduces off-HSM storage)
- Implicit authentication: Bearer tokens, passwords, etc., input to OPRF provide authentication w/o KMS having to verify anything
- Threshold security: Can distribute the OPRF into n servers (HSMs) with OPRF key secure as long as no more than t are compromised

Threshold DH-OPRF (n-out-of-n)

- Single server solution: $F_k(x) = (H(x))^k$ (H' omitted for simplicity)
- Multi-server solution: Server S_i has share k_i, $k = k_1 + k_2 + \cdots + k_n$
  - $F_k(x) = (H(x))^{k_1} \cdot (H(x))^{k_2} \cdot \cdots \cdot (H(x))^{k_n} = (H(x))^{\sum k_i}$
- U sends *same* $a = (H(x))^r$ to each server; S_i returns $b_i = \mathrm{a}^{k_i}$;
  U deblinds all b_i and multiplies
- Efficiency: 2 exp's for client (indep of n), 1 per server, 1 round
- Key k is <u>never</u> reconstructed: "function sharing" vs "secret sharing"

Threshold DH-OPRF (t-out-of-n)

- *t-out-of-n* threshold DH-OPRF: Each server S_i has share k_i
- $F_k(x)$ computed from any set of t servers $S_{i1}, \ldots, S_{it}$
  - $F_k(x) = (H(x))^{\lambda_{i1} k_{i1}} \cdot (H(x))^{\lambda_{i2} k_{i2}} \cdot \ldots \cdot (H(x))^{\lambda_{it} k_{it}}$
  - λ_{ij} is a Lagrange interpolation coefficient ("Shamir in the exponent")
- As before: key k is never reconstructed
  - Not even during generation/sharing: Distributed key generation

Threshold DH-OPRF (more goodies)

- Single client message → proxy-based threshold operation
- Verifiability: via ZK or interactive (latter good for proxy-based)
  - Still a single message from C, double the # of exp's, still indep of n, t
- Distributed OPRF key generation (key never exists in one physical place)
- Share rebuilding
- Proactive security

Updatable Oblivious KMS

- KMS stores client's CRK k ; Client stores g and $y = g^k$
- To encrypt: Client sets $h=g^s$ (random s), sets DEK = y^s, stores h
  - DEK = $y^s = (g^k)^s = (g^s)^k = h^k$; Client can compute h^k *by itself* w/o knowing k !!
- To decrypt with h: Client sends h^r (random r) to KMS, gets back $(h^r)^k$, deblinds r to obtain h^k, sets DEK = h^k
  - Only decryption is interactive (at the cost of storing h), KMS learns nothing
- Non-interactive key update: KMS rotates k to k', sends Δ= k'/k to C, C sets every DEK h to h^Δ → can decrypt with k' but not with k
  - In regular KMS rotation, server is involved with each DEK update!

BIG MISSING PIECE:

DEFINITIONS and PROOFS

PPSS: Password Protected Secret Sharing

(password-protected distributed storage)

How to store a secret

- We want to protect *secrecy* and *availability* of information while remembering a *single* password
  - Single server = Single point of compromise for secrecy (offline dict attacks)
  - Single server = Single point of failure for availability (server gone, secret gone)
  - ➔ Multi-server solution a must.
- Crypto solution: keep the secret encrypted in multiple locations; *secret share the encryption key* in multiple servers
  - Share among n servers, retrieve from t+1 servers (e.g. n=5, t=2)
- Protects availability and secrecy: *available* as long as t+1 available, *secret* as long as no more than t corrupted

Wait, but how do you authenticate to each server for share retrieval?

- Server needs to authenticate the user before delivering a share
- All we have is a user and a password
  - A strong independent password with each server? Not realistic
  - Same (or slight-variant) password for each server? Not good

➔ *Each server as a single point of compromise!*

  - From one point of compromise to n. We didn't achieve much, did we?

Password Protected Secret Sharing (PPSS)

- Init: User secret shares a secret among n servers; *forgets secret* and keeps a *single password*.
- Retrieval: User contacts t + 1 servers, authenticates using the *single password* and *reconstructs the secret*.
- Security: Breaking into t servers leaks nothing about secret or password
  - Break = All server's secret information leaks (shares, long-term keys, password file)
  - Only adversary option: Guess the password, try it in an online attack.
  - Offline attacks with $\leq$ t corrupted servers are useless.

+ Soundness: User *reconstructs the correct secret* or else rejects (CRUCIAL)

Note: No PKI except for Init, secure even if user forgets initialized servers

PPSS Solution = Threshold OPRF

- n servers share a Threshold OPRF $F_k(x)$
- U's secret defined as $s=F_k(pwd)$
  - If U's secret is not random (e.g., bitcoin), s can be used as an encryption key
- To retrieve s, U runs T-OPRF with any t+1 servers
- In more detail (adding crucial soundness):
  - U's secret defined as H(s,1)
  - In addition to k_i, servers store H(s,2), which they send to U together with OPRF response; if not all servers send H(s,2), U aborts (soundness)
- Security bonus: Even if t+1 servers compromised, a full exhaustive offline attack needed to find password!

PPSS Efficiency (same as Threshold OPRF)

- Computation:
  - Single exponentiation for each server
  - Only two exponentiations *in total* for the client (*independent* of t and n)
  - t multiplications for client and for each server
- Communication: Single parallel message from user to t+1 servers, one msg back from each server. No inter-server communication.
- *No assumed PKI or secure channels* (other than for initialization)
- Any t, n ($t \leq n$)
- Robustness: NIZK, interactive [2x expon], ACNP'16

Password-Authenticated Key Exchange (PAKE)

OPAQUE: Oblivious PAKE

- Asymmetric PAKE: User-Server password authentication **(+ KE)**
  - User has pwd, server stores pwd-related state (*but not pwd!)*
  - Except that in password-over-TLS, server learns password at decryption (as well as anyone that sees, legitimately or not, unencrypted traffic)
- Can we do password authentication so that server (or anyone other than the client) sees the password?
- Goal: *Only feasible attacks are (unavoidable) online guesses*
- Solution: OPAQUE = 1-out-of-1 PPSS !
  Use retrieved secret as private key for a key exchange protocol

You may use it one day…