COMS W3261: Theoretical Computer Science.
Instructor: Tal Malkin

Problem Set 9

Due: Thur, 05/05/11, 3pm **No late Homework will be accepted.**
Instructions: Solve **four** out of the following five problems. If you solve all problems, the last solution that you write will count towards extra credit.

1. Let A be some decidable language, and let B some undecidable language. Your answers should apply to any such choices of A, B, but if it helps you to think of concrete examples, you may take $A = \{\langle M \rangle | M \text{ is a TM with at most 10 states}\}$ and $B = \text{Halt}_{TM} = \{\langle M, w \rangle : M \text{ halts on } w\}$. (If your answer applies only to these examples and not in general, you will get partial credit).

   (a) Prove that A is mapping reducible to B ($A \leq_m B$).

   (b) Prove that B is not mapping reducible to A ($B \not\leq_m A$).

2. (a) Prove that every language A is Turing-reducible to its complement $\overline{A}$.

   (b) Prove that if A is a recognizable language, and A is mapping-reducible to $\overline{A}$, then A is decidable.

   (c) Give an example of a language that is not mapping-reducible to its complement (Hint: use the previous part).

3. 7.8 in text (prove that CONNECTED is in P, by analyzing a given algorithm).

4. 7.11 in text (ISO is in NP).

   As a side note, we do not know whether ISO is in P, nor whether ISO is NP-Complete. It is often believed that it is neither, namely that ISO is a hard problem (not in P), but not "the hardest problem" (not NP-Complete).[1]

5. Read problem 7.29 in text (a scheduling problem for final exams). Formulate this problem as a language, and show that this language is in NP. (That is, solve problem 7.29, except you don't have to show that the problem is NP-hard, just that it is in NP).

[1] Of course, if P=NP, this belief is wrong on both counts.