

Problem Set 8

Due: Thur, 04/21/11 (as usual, by the beginning of lecture)

Note: Due to Passover, late submissions will be accepted by 4/26 without late penalty. However, note that there is likely to be another homework due on 4/28.

Note regarding Turing reductions in the textbook: Sipser introduces Turing reductions in the beginning of Chapter 5, by describing them intuitively and just calling them 'reductions'. It defines them and starts calling them 'Turing reductions' in section 6.3

1. Recall that we defined (in class and in a homework problem) the languages

$$X_{TM} = \{\langle M \rangle : M \text{ does not accept } \langle M \rangle\}$$

$$NE_{TM} = \{\langle M \rangle : L(M) \text{ is not empty}\}.$$

We proved that X_{TM} is not decidable, and not even recognizable, and that NE_{TM} is recognizable.

- (a) Prove that NE_{TM} is undecidable, by showing that $X_{TM} \leq_T NE_{TM}$. Specifically, let S be a decider for NE_{TM} , show a decider T for X_{TM} .
 - (b) Why can't the same proof be used to prove that NE_{TM} is not recognizable? Specifically, if S is a recognizer for NE_{TM} , why wouldn't T (that you built in part (a)) be a recognizer for X_{TM} ?
2. Show that no Java program could be written which, given as input an arbitrary Java program, can determine whether or not there is any input on which the given program prints out "AN ERROR HAS OCCURED". (You may assume Java programs are equivalent to Turing Machines).
 3. Define the language

$$SMALL = \{\langle M \rangle : L(M) = \{0000, 0101, 1110\}\}.$$

Prove that $SMALL$ is undecidable by showing $A_{TM} \leq_T SMALL$.

4. (Extra Credit:) For a class of languages \mathcal{C} , we define $\mathcal{C}_{TM} = \{\langle M \rangle : L(M) \in \mathcal{C}\}$.
 - (a) Let \mathcal{C} be a class of languages such that $\emptyset \in \mathcal{C}$, and such that there exists a TM-recognizable language $L_N \notin \mathcal{C}$.
Prove that \mathcal{C}_{TM} is undecidable, by showing that $A_{TM} \leq_T \mathcal{C}_{TM}$. Note: in class (and Theorem 5.3 in text) we proved this for the special case of \mathcal{C} being the class of regular languages.
 - (b) Let \mathcal{C} be a class of languages, and L_N, L_Y be two TM-recognizable languages, such that $L_Y \in \mathcal{C}$, and $L_N \notin \mathcal{C}$. Prove that \mathcal{C}_{TM} is undecidable, by showing that $A_{TM} \leq_T \mathcal{C}_{TM}$.
Note: Obviously, the previous part (4a) is a special case of this one. Another special case is the previous problem (3).