COMS W3261: Theoretical Computer Science.
Instructor: Tal Malkin

Problem Set 7

Due: Thur, 04/14/11.
Note: from now on, unless otherwise noted, you can use a high-level description for any TM (algorithm) you describe, as we do in class. You still need to argue the correctness of your solutions.

1. Let $NE = \{\langle M \rangle \,|\, M \text{ is a TM that accepts some string}\}$ (namely all $\langle M \rangle$ that recognize a non-empty language).

   Prove that NE is Turing-recognizable.

2. Let $L = \{\langle N \rangle \,|\, N \text{ is an NFA such that all strings accepted by } N \text{ have 000 as a substring}\}$.

   Prove that L is decidable.

3. Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language, and show that it is decidable.

4. Let $S = \{(i, j) : i \text{ and } j \text{ are positive integers}\}$.

   (a) Show that S is a countable set, by showing that the elements of S can all be listed in some order $s_1, s_2, s_3, \ldots$ (You do not have to give an explicit formula for the n-th element s_n. It is sufficient to explain the procedure for how this list is generated, and to argue that each element of S appears exactly once on this list.)

   (b) Let $T = \{U : U \subseteq S\}$ (Note that each such subset $U \subseteq S$ can be defined by specifying for each element of S whether it is in the subset or not). Use diagonalization to show that T is an *uncountable* set.