

## COMS W3261: Theoretical Computer Science.

Instructor: Tal Malkin

### Problem Set 5

Due: Thur, 03/24/11. **No late Homework will be accepted**

**Instructions:** Solve problem 1 and **two** out of problems 2,3,4,5. If you solve more than that, problem 1 and the first two problems out of  $\{2, 3, 4, 5\}$  will count for the problem set score, and the rest will count towards extra credit.

1. Consider the language  $L = \{a^n b^m \mid n \geq 0, m \geq 0, n \neq m\}$  over  $\Sigma = \{a, b\}$ . In both the following you do not need to prove the correctness of your construction, but you do need to write in a clear manner, indicating the main idea where necessary to understand the construction.

- (a) Give a context free grammar for  $L$ .
- (b) Give a state diagram for a push-down automaton for  $L$ .

2. Consider the following context-free grammar (variables are marked with  $\langle \dots \rangle$  and the start variable is  $\langle \text{sent} \rangle$ ).

|                                   |                   |                                                                                                                                                                             |
|-----------------------------------|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\langle \text{sent} \rangle$     | $\longrightarrow$ | $\langle \text{subj} \rangle \langle \text{activity} \rangle \mid$<br>$\langle \text{subj} \rangle \langle \text{activity} \rangle \text{with} \langle \text{subj} \rangle$ |
| $\langle \text{subj} \rangle$     | $\longrightarrow$ | $\langle \text{noun} \rangle \mid \langle \text{noun} \rangle \text{and} \langle \text{subj} \rangle$                                                                       |
| $\langle \text{activity} \rangle$ | $\longrightarrow$ | $\langle \text{verb} \rangle \langle \text{obj} \rangle$                                                                                                                    |
| $\langle \text{obj} \rangle$      | $\longrightarrow$ | $\langle \text{noun} \rangle \mid \langle \text{noun} \rangle \text{with} \langle \text{obj} \rangle$                                                                       |
| $\langle \text{noun} \rangle$     | $\longrightarrow$ | Alice $\mid$ Bob $\mid$ Carol $\mid$<br>bread $\mid$ cheese $\mid$ cups                                                                                                     |
| $\langle \text{verb} \rangle$     | $\longrightarrow$ | bought $\mid$ ate                                                                                                                                                           |

Show that this grammar is ambiguous by showing two different derivation trees for the same sentence.

3. Give an informal description and a state diagram of a pushdown automata for the following language over the alphabet  $\{a, b\}$ :  
 $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is a } b\}$
4. Problem 2.11 (converting a CFG to a PDA)
5. Problem 2.13(a) (describing the language generated by a certain CFG). Note that you are not asked to do part (b).