P_k, we know that

$$P_{k+1} = P_k M - Y.$$

Therefore, using the induction hypothesis to calculate P_k,

$$P_{k+1} = \left[PM^k - Y\left(\frac{M^k - 1}{M - 1}\right)\right] M - Y.$$

Multiplying through by M and rewriting Y yields

$$\begin{aligned} P_{k+1} &= PM^{k+1} - Y\left(\frac{M^{k+1} - M}{M - 1}\right) - Y\left(\frac{M - 1}{M - 1}\right) \\ &= PM^{k+1} - Y\left(\frac{M^{k+1} - 1}{M - 1}\right). \end{aligned}$$

Thus the formula is correct for $t = k + 1$, which proves the theorem.

Problem 0.14 asks you to use the preceding formula to calculate actual mortgage payments.

EXERCISES

0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

- **a.** $\{1, 3, 5, 7, \dots\}$
- **b.** $\{\dots, -4, -2, 0, 2, 4, \dots\}$
- **c.** $\{n \mid n = 2m \text{ for some } m \text{ in } \mathcal{N}\}$
- **d.** $\{n \mid n = 2m \text{ for some } m \text{ in } \mathcal{N}, \text{ and } n = 3k \text{ for some } k \text{ in } \mathcal{N}\}$
- **e.** $\{w \mid w \text{ is a string of 0s and 1s and } w \text{ equals the reverse of } w\}$
- **f.** $\{n \mid n \text{ is an integer and } n = n + 1\}$

0.2 Write formal descriptions of the following sets

- **a.** The set containing the numbers 1, 10, and 100
- **b.** The set containing all integers that are greater than 5
- **c.** The set containing all natural numbers that are less than 5
- **d.** The set containing the string aba
- **e.** The set containing the empty string
- **f.** The set containing nothing at all

0.3 Let A be the set $\{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$ and B be the set $\{\mathtt{x}, \mathtt{y}\}$.

- **a.** Is A a subset of B?
- **b.** Is B a subset of A?
- **c.** What is $A \cup B$?
- **d.** What is $A \cap B$?
- **e.** What is $A \times B$?
- **f.** What is the power set of B?

0.4 If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

0.5 If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

0.6 Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f\colon X \longrightarrow Y$ and the binary function $g\colon X \times Y \longrightarrow Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

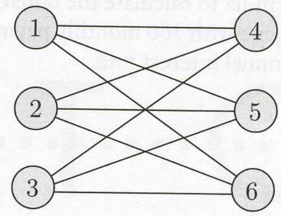
- **a.** What is the value of $f(2)$?
- **b.** What are the range and domain of f?
- **c.** What is the value of $g(2, 10)$?
- **d.** What are the range and domain of g?
- **e.** What is the value of $g(4, f(4))$?

0.7 For each part, give a relation that satisfies the condition.

- **a.** Reflexive and symmetric but not transitive
- **b.** Reflexive and transitive but not symmetric
- **c.** Symmetric and transitive but not reflexive

0.8 Consider the undirected graph $G = (V, E)$ where V, the set of nodes, is $\{1, 2, 3, 4\}$ and E, the set of edges, is $\{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{2, 4\}, \{1, 4\}\}$. Draw the graph G. What is the degree of node 1? of node 3? Indicate a path from node 3 to node 4 on your drawing of G.

0.9 Write a formal description of the following graph.



PROBLEMS

0.10 Find the error in the following proof that $2 = 1$.
Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a+b)(a-b) = b(a-b)$, and divide each side by $(a-b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

0.11 Find the error in the following proof that all horses are the same color.
CLAIM: In any set of h horses, all horses are the same color.
PROOF: By induction on h.

Basis: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

Induction step: For $k \geq 1$ assume that the claim is true for $h = k$ and prove that it is true for $h = k+1$. Take any set H of $k+1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore all the horses in H must be the same color, and the proof is complete.

0.12 Show that every graph with 2 or more nodes contains two nodes that have equal degrees.

A***0.13** **Ramsey's theorem.** Let G be a graph. A ***clique*** in G is a subgraph in which every two nodes are connected by an edge. An ***anti-clique***, also called an ***independent set***, is a subgraph in which every two nodes are not connected by an edge. Show that every graph with n nodes contains either a clique or an anti-clique with at least $\frac{1}{2}\log_2 n$ nodes.