

CS Theory (Fall '26)

Challenge Problem II

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This problem is **not worth any extra credit**, we do not expect you to solve it, as it is harder than any graded problem we will give you in this class. However, please feel free to attempt it! If you present your solution (or a partial solution!) to one of the Head TAs or the Instructors during office hours, we will give you a **sticker and candy!**

You can come to office hours to work on this problem, with friends or by yourself! The teaching staff will be happy to help you (but not too much)!!

1 The Traveler and the Tablet

Emerging from the desert, the Traveler still hungers for stickers and candy. They set off to explore a jungle they heard holds rewards for brave CST students. In this jungle, they come upon a tablet. It reads

“If you seek my treasure, then know that I loathe progressions in regular increments. Thus, I beseech you to show your mastery over regular languages: determine which of the following languages may be recognized by regular expressions, and for those that cannot, convince me of their impossibility.

First, can you recognize the strings of numbers containing no three 0's whose positions are equally spaced?

Second, can you recognize the strings of numbers containing no three equal bits whose positions are each exactly two apart?

Third, can you recognize the strings of numbers containing no three equal bits whose positions are equally spaced?”

The tablet shudders and retreats into the ground, while the traveler gets to work.

2 The Traveler and the Tablet (formally)

Determine for which values of integer $c \geq 0$ the following languages are regular or not. If they are regular, describe how you might construct a regular expression for L_c or its complement (you can use c in the regular expression). All languages are over the alphabet $\Sigma_c = \{1, \dots, c, \#\}$.

Q1.

$$L_c = \left\{ w_1\# \dots \#w_n \mid w_1, \dots, w_n \in \{0, 1, \dots, c\}, \text{ and} \right. \\ \left. \text{there are no } i > j > k \text{ s.t. } w_i = w_j = w_k = 0 \text{ and } i - j = j - k \right\}$$

Q2.

$$L_c = \left\{ w_1\# \dots \#w_n \mid w_1, \dots, w_n \in \{0, 1, \dots, c\}, \text{ and} \right. \\ \left. \text{there are no } i > j > k \text{ s.t. } w_i = w_j = w_k \text{ and } i - j = j - k = 2 \right\}$$

Q3.

$$L_c = \left\{ w_1\# \dots \#w_n \mid w_1, \dots, w_n \in \{0, 1, \dots, c\}, \text{ and} \right. \\ \left. \text{there are no } i > j > k \text{ s.t. } w_i = w_j = w_k \text{ and } i - j = j - k \right\}$$