1 DFA Exercises

1. Determine which of $\varepsilon$, 11, 010, 10, 0101 is accepted by this DFA.

$0 \rightarrow q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3$

$E$: Not accepted

11: Accepted

010: Not accepted

10: Not accepted

0101: Accepted
2. The DFA state diagram below is defined on the alphabet $\Sigma = \{a, b, c\}$. Write out its formal definition (as a 5-tuple). When specifying the transition function $\delta$, draw a table.

$$D = (Q, \Sigma, \delta, q_0, F)$$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b, c\}$

$\delta :$

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$q_1$</td>
<td>$q_3$</td>
<td>$q_3$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$q_1$</td>
<td>$q_2$</td>
<td>$q_3$</td>
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<td>$q_2$</td>
<td>$q_1$</td>
<td>$q_1$</td>
<td>$q_3$</td>
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<td>$q_3$</td>
<td>$q_3$</td>
<td>$q_3$</td>
<td>$q_3$</td>
</tr>
</tbody>
</table>

$F = \{q_1\}$

dead / reject / bad state:

We could also not draw this because for DFAs we have a convention that when there are missing transitions, it means that they all go to the dead/reject/bad state.
3. Draw a DFA that recognizes:

(a) All strings with the prefix 01.

```
q_0 -----> 0 -----> q_1
    |      |      |
    v      v      v
q_2 -----> 1 -----> q_3
```

$q_3$ is a dead / reject / bad state.

(b) $L = \{11, 101, 010, 0110\}$.

```
q_0 -----> 0 -----> q_1
    |      |      |
    v      v      v
q_2 -----> 1 -----> q_3
```

The dead / reject / bad state can be omitted. In this graph, we just omitted it to avoid making the graph messy.

(c) $L = \{w \in \{0, 1\}^* \mid$ the number of 1’s in $w$ is not an integer multiple of 5}$.

```
q_0 -----> 0 -----> q_1
    |      |      |
    v      v      v
q_2 -----> 1 -----> q_3
```

0 is an integer multiple of 5!
2 NFA Exercises

1. Draw an NFA that recognizes:
   
   (a) All strings that contain 101.

   ![NFA Diagram](image)

   (b) $L = \{w \in \{0, 1\}^* \mid w$ has exactly two 0’s or an even number of 1’s$\}$.

   ![Computation Tree](image)

   **Bonus solution:**
3 Miscellaneous Exercises

1. Prove the following languages are regular:

(a) \[ L = \{0^m1^n \mid m, n \geq 0, \text{and } m + n \text{ is odd} \} \]

(b) \[ L = \{x \in \{0, 1\}^* \mid x \text{ contains a substring of two 1's separated by an odd number of characters} \} \]

Draw either a DFA or an NFA. Be careful about the order!

The grey part is the dead/reject/bad state. In this case, we might want to just omit it.

Note that \(111\) is also accepted by \(L\)!

You can name the states in a way that can help make things clear!
2. Convert this NFA to a DFA using subset construction:

Don't forget to note the accept state!

You could also draw the transition tables to help you understand:

**NFA 8:**

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>${q_0, q_1}$</td>
<td>$q_0$</td>
<td>$q_0$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$q_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**DFA 8:**

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>${q_0, q_1}$</td>
<td>$q_0$</td>
</tr>
<tr>
<td>$q_0$</td>
<td>${q_0, q_1}$</td>
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</tr>
<tr>
<td>$q_0$</td>
<td>${q_0, q_1}$</td>
<td>$q_0$</td>
</tr>
</tbody>
</table>
3. (a) What is the language recognized by this NFA?

\[ \text{\textcircled{q}} \]

\[ q_0 \text{ is not accepting : } \emptyset \]

\[ \{ j \neq \epsilon \} \]

The complement of \( \emptyset \) is \( \Sigma^* \).

(b) What is the language recognized by this NFA?

\[ \text{\textcircled{q}} \]

\[ q_1 \text{ is accepting : } \{ \epsilon \} \]

The complement of \( \{ \epsilon \} \) is \( \{ w \in \Sigma^* | |w| > 1 \} \).