Deterministic Receptive Processes are Kahn Processes

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Motivation

SHIM project: “Software/Hardware Integration Medium”

Want an asynchronous concurrent deterministic formalism for embedded systems.

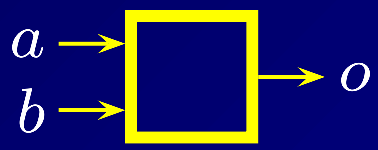
I found two:

Kahn’s process networks (1974)

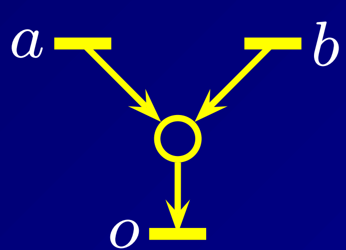
Josephs’s Deterministic Receptive Processes (2003)

Are they “the same”?

Deterministic Merge



Each a or b input produces an o output $\Leftrightarrow$ The number of o's is the sum of the number of a's and b's.



ϵ	$aaaooo$	$abaooo$	$baaooo$	$bbaooo$
ao	$aaoaoo$	$aboaoo$	$baoaoo$	$bboaoo$
bo	$aaooao$	$abooao$	$baooao$	$bbooao$
$aaoo$	$aoaaoo$	$aobaoo$	$boaaoo$	$bobaoo$
$aoao$	$aoaoao$	$aoboao$	$boaoao$	$boboao$
$aboo$				
$aobo$	$aabooo$	$abbooo$	$babooo$	$bbbooo$
$baoo$	$aaoboo$	$aboboo$	$baoboo$	$bboboo$
$boao$	$aaoobo$	$aboobo$	$baoobo$	$bboobo$
$bboo$	$aoaboo$	$aobboo$	$boaboo$	$bobboo$
$bobo$	$aoaobo$	$aobobo$	$boaobo$	$bobobo$

Deterministic Receptive Processes

In Mark Josephs's formalism,

ϵ $\leftarrow$ these traces are *failures* because the process fails to produce more outputs afterwards.

ao
bo
$aaoo$
$aoao$
$aboo$
$aobo$
$baoo$
$boao$
$bboo$
$bobo$

The set of failure traces characterizes one of Josephs's deterministic receptive processes.

This process is deterministic and receptive according to Josephs

Josephs's Receptive Processes

Receptive Process Theory. *Acta Informatica*, 1992.

Process: (I, O, F)

$I \cap O = \emptyset$ input/output alphabets

Set of *failure traces*: $F \subseteq (I \cup O)^*$

Divergences: $F\uparrow = \{s : \{t \in O^* : st \in F\}$ is infinite$\}$

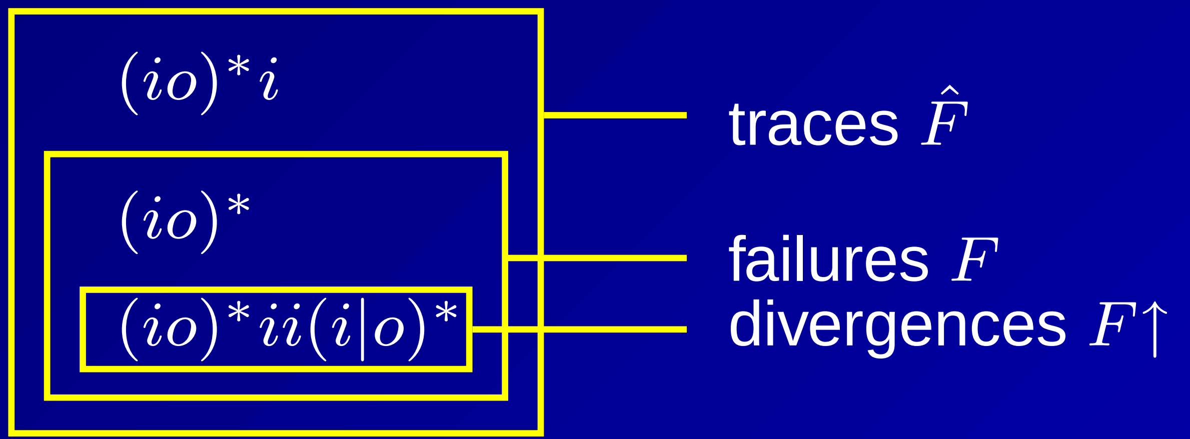
"When an infinite sequence of outputs is possible"

Traces: $\hat{F} = \{s : \exists t \in O^* \,.\, st \in F\}$

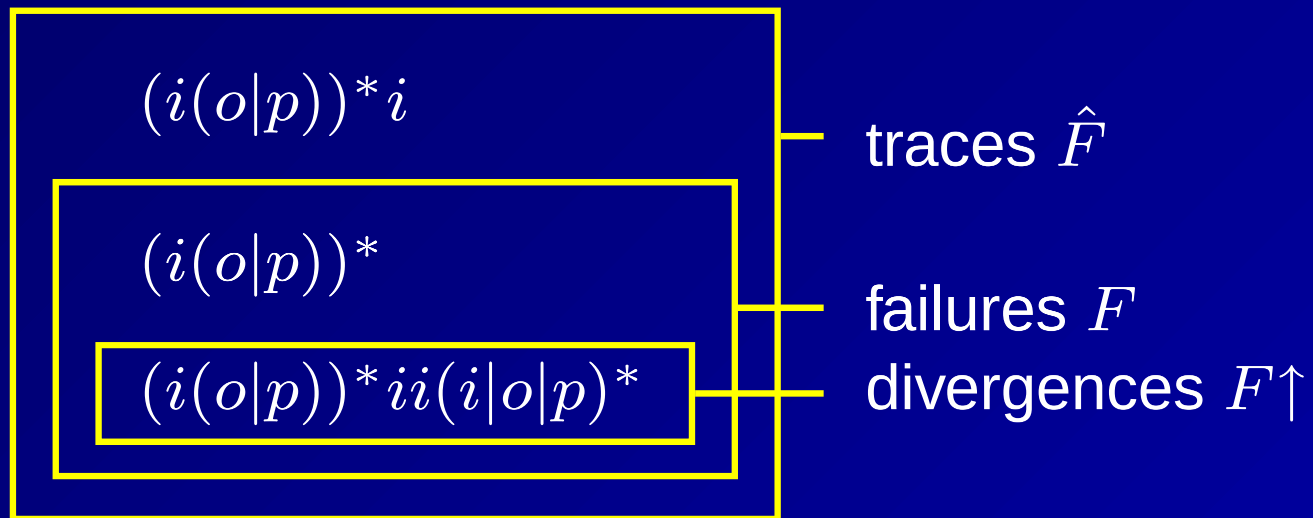
"When zero or more outputs are pending"

Receptive Process Axioms

$s \in F\uparrow \Rightarrow st \in F\uparrow$	Anything follows a divergence
$F\uparrow \subseteq F$	Divergences are failures
$\epsilon \in \hat{F}$	Traces start from nothing
$st \in \hat{F} \Rightarrow s \in \hat{F}$	Traces prefix-closed
$s \in \hat{F} \wedge t \in I^* \Rightarrow st \in \hat{F}$	Input always possible = receptive



Nondeterministic Receptive Process



Problem: process can choose whether to output o or p.

Deterministic Receptive Processes

Josephs, An analysis of determinacy. . . , *ASYNC 2003*.

Four additional rules: one about inputs, three about outputs.

$$(\forall v, w \,.\, x = vw \Rightarrow svi \notin F\!\uparrow) \;\wedge\; i \in I \;\wedge\; sxiu \in F \quad\Rightarrow\quad sixu \in F$$

“An input that arrives early does not matter unless it causes divergence.”

Deterministic Receptive Processes

$$o \in O \;\wedge\; t \in (I \cup (O \setminus \{o\}))^* \;\wedge\; so \in \hat{F} \;\wedge\; st \in F \setminus F{\uparrow} \quad \Rightarrow \quad \text{false}$$

$$o \in O \;\wedge\; t \in (I \cup (O \setminus \{o\}))^* \;\wedge\; so \in \hat{F} \;\wedge\; st \in \hat{F} \quad \Rightarrow \quad sto \in \hat{F}$$

$$o \in O \;\wedge\; t \in (I \cup (O \setminus \{o\}))^* \;\wedge\; so \in \hat{F} \;\wedge\; stou \in F \setminus F{\uparrow} \quad \Rightarrow \quad sotu \in F$$

“If an output can occur now, it must be emitted before the process stops to wait for inputs.”

“An output may always be delayed.”

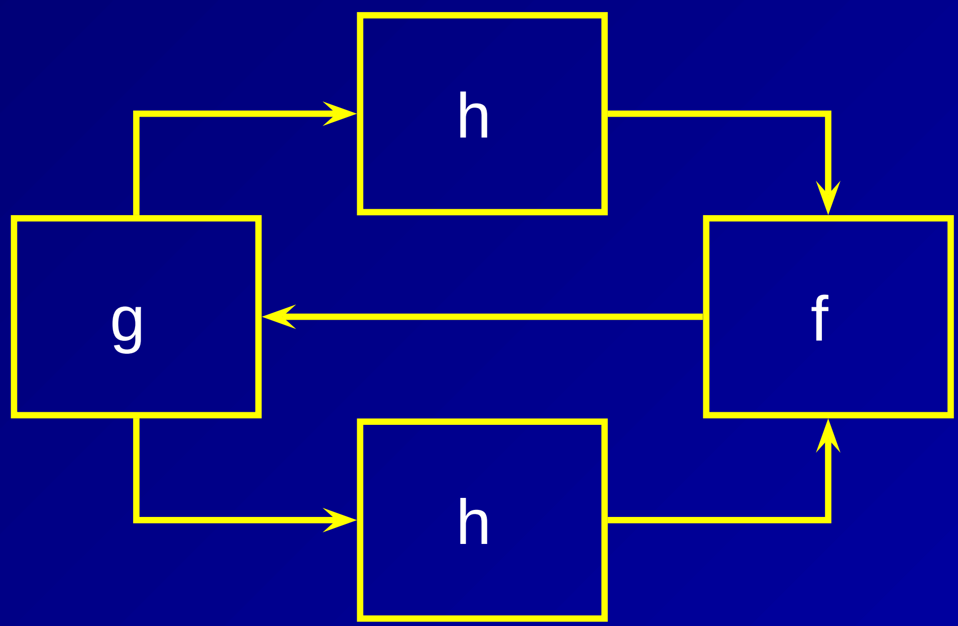
“Delaying an output does not affect long-term behavior.”

Kahn's Networks

Alternating sequence of 0s and 1s along center channel

Emits a 1 then copies input to output

Copies alternately



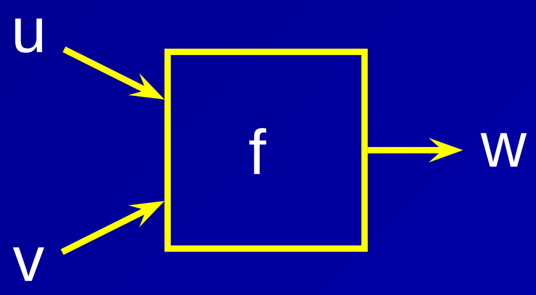
Merges by alternating

Emits a 0 then copies input to output

Kahn’s Processes

“A Simple Language for Parallel Programming”

```
process f(in int u, in int v, out int w)
{
  int i; bool b = true;
  for (;;) {
    i = b ? wait(u) : wait(v);
    printf("%i\n", i);
    send(i, w);
    b = !b;
  }
}
```



Kahn's Formalism

Channels convey sequences of data values.

Sequences partially ordered: $aa \sqsubseteq aaa$, but $aa \not\sqsubseteq ab$.

Each process a function on finite and infinite sequences

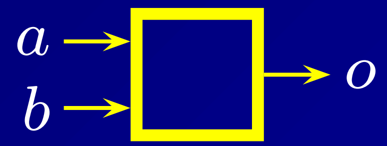
$$f : D_1^\omega \times D_2^\omega \times \cdots \times D_n^\omega \to D^\omega$$

f is monotonic, $x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$, and continuous $f(\sqcup X) = \sqcup f(X)$.

Continuity guarantees the function of a system $F = (f_1, f_2, \ldots, f_k)$ has a unique least fixed point $F(X) = X$. This is the (only) behavior of the system.

Deterministic Merge

As a Kahn process,



$$
\begin{array}{lll}
(\epsilon, \epsilon) = \epsilon & (a, \epsilon) = o & (aa, \epsilon) = oo \\
(\epsilon, b) = o & (a, b) = oo & (aa, b) = ooo \\
(\epsilon, bb) = oo & (a, bb) = ooo & (aa, bb) = oooo \quad \cdots \\
(\epsilon, bbb) = ooo & (a, bbb) = oooo & (aa, bbb) = ooooo \\
(\epsilon, bbbb) = oooo & (a, bbbb) = ooooo & (aa, bbbb) = oooooo
\end{array}
$$

$$\vdots$$

Clearly monotonic and continuous, hence deterministic.

Cannot be described in Kahn's sequential language.

A constructive proof that Deterministic Receptive Processes behave like Kahn processes

Projection

Projection selects a single event from a trace:

$$
\begin{aligned}
\epsilon \downarrow e &= \epsilon \\
as \downarrow e &= \begin{cases} a(s \downarrow e) & \text{if } a = e\text{, and} \\ s \downarrow A & \text{otherwise.} \end{cases}
\end{aligned}
$$

$$i_1 i_2 o_1 o_2 i_1 o_2 o_1 i_2 i_1 o_1 o_2 \downarrow i_1 = i_1 i_1 i_1$$

$$i_1 i_2 o_1 o_2 i_1 o_2 o_1 i_2 i_1 o_1 o_2 \downarrow i_2 = i_2 i_2$$

Input and Output Functions

$$\mathcal{I}(f) = (f \downarrow i_1, f \downarrow i_2, \ldots, f \downarrow i_p)$$

$$\mathcal{O}(f) = (f \downarrow o_1, f \downarrow o_2, \ldots, f \downarrow o_q)$$

Example: $f = i_1 i_2 o_1 o_2 i_1 o_2 o_1 i_2 i_1 o_1 o_2$

$\mathcal{I}(f) = (i_1 i_1 i_1, i_2 i_2)$

$\mathcal{O}(f) = (o_1 o_1 o_1, o_2 o_2 o_2)$

The Central Lemma

The input/output relationship of a deterministic receptive process $P = (I, O, F)$ with no divergence is monotonic, i.e., for $f_1, f_2 \in F$, if $\mathcal{I}(f_1) \sqsubseteq \mathcal{I}(f_2)$ then $\mathcal{O}(f_1) \sqsubseteq \mathcal{O}(f_2)$.

Proof by contradiction. Assume $\mathcal{I}(f_1) \sqsubseteq \mathcal{I}(f_2)$ but $\mathcal{O}(f_1) \not\sqsubseteq \mathcal{O}(f_2)$.

Reorder the events in f_1 and f_2 so that inputs appear first and the two share a common prefix.

There must be at least one more output that occurs less often in f_2 and hence in the reordered traces, but this contradicts the axiom of compulsory emission. QED.

An Illustration

$$f_1 = i_1 o_1 i_2 o_2 i_2 o_2 i_1$$

$$f_2 = i_1 i_2 i_2 o_2 i_1 o_2 i_1 i_2$$

$$\mathcal{I}(f_1) = (i_1 i_1, i_2) \quad \sqsubseteq \quad \mathcal{I}(f_2) = (i_1 i_1 i_1, i_2 i_2 i_2)$$

$$\mathcal{O}(f_1) = (o_1, o_2 o_2) \quad \not\sqsubseteq \quad \mathcal{O}(f_2) = (\epsilon, o_2 o_2)$$

Move inputs earlier (safe because no divergence)

$$f'_1 = i_1\ i_1\ i_2\ i_2\ o_1\ o_2\ o_2$$

Must be emitted in f'_2

$$f'_2 = i_1\ i_1\ i_2\ i_2\ i_1\ i_2\ o_2\ o_2$$

f'_2 cannot be a failure because the output o_1 must eventually be emitted. Contradiction.

Technical point

Josephs only talks about finite traces

Kahn needs infinite traces because he takes limits

Unsurprising result: define the behavior of Josephs's process as being its limit and everything works.

Conclusion

Kahn and Josephs deterministic for roughly same reason

Big difference: Josephs models “don’t-cares” as divergences—no obvious analog in Kahn’s model

Josephs’s axioms more complex, but more operational

Not in the paper: we have found a more fundamental definition that makes Josephs’s axioms lemmas.

Ongoing work: developing the SHIM model and system built around a Kahn/Josephs-like model of computation.