CSEE W3827

Fundamentals of Computer Systems

Homework Assignment 1

Solutions

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Due September 20th, 2011 at 10:35 AM

Show your work for each problem; we are more interested in how you get the answer than whether you get the right answer.

This document is formatted for on-screen viewing.

1. What are the values, in decimal, of the bytes

$$10011100$$

and

$$01111000,$$

if they are interpreted as 8-bit

(a) Binary numbers?

$10011100_2 = 128 + 16 + 8 + 4 = 156;$
$01111000_2 = 64 + 32 + 16 + 8 = 120$

(b) One's complement numbers?

$-(1100011_2) = -(64 + 32 + 2 + 1) = -99;$
$01111000_2 = 64 + 32 + 16 + 8 = 120$

(c) Two's complement numbers?

$10011100_2 = -128 + 16 + 8 + 4 = -100$ or
$01100011 + 1 = 01100100 = 64 + 32 + 4 = -100;$
$01111000_2 = 64 + 32 + 16 + 8 = 120$

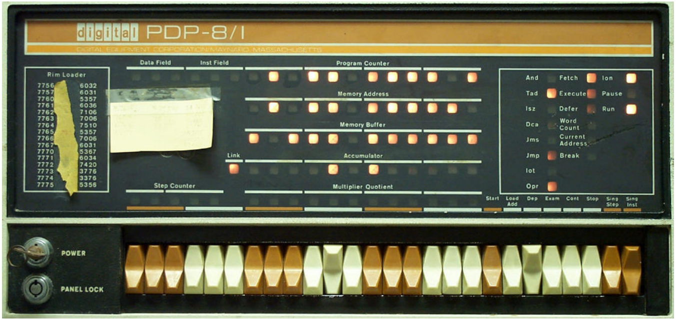
2. The DEC PDP-8 used 12-bit words.

   (a) What were the most negative and most positive decimal numbers one of its words could represent using two's complement?

   $-2^{11} = -2048$ and $2^{11} - 1 = 2047$

   (b) Assuming a word represented an address in memory, how many different locations could the PDP-8 address?

   $2^{12} = 4096$



3. Convert the hexadecimal number “DEAD” into

(a) Binary

1101 1110 1010 1101

(b) Octal

157255 (interpret groups of three bits)

(c) Decimal

$13 \cdot 16^3 + 14 \cdot 16^2 + 10 \cdot 16^1 + 13 \cdot 16^0 = 57005$

(d) Binary-Coded Decimal

$57005_{10} = 0101\,0111\,0000\,0000\,0101_{BCD}$

4. Show that $2 + -7 = -5$ is also true when done in binary using

(a) Signed-magnitude numbers

$0010 + 1111 = -(111 - 010) = -(101) = 1101$

Make sure you strip off the sign bits

(b) One's complement numbers

$0010 + 1000 = 1010 = -(0101)$ (normal binary addition)

(c) Two's complement numbers

$0010 + 1001 = 1011 = -(101)$ (normal binary addition)

5. Show $42 + 49 = 91$ in BCD. Make sure you show when corrections are necessary to normal binary addition.

```
    0100 0010
  + 0100 1001
  -----------
    1000 1011  The result of normal binary addition
       + 0110  Add 6 since this digit exceeded 9
  -----------
    1001 0001
```

6. Complete the truth table for the following Boolean functions:

(a) $XY\overline{Z} + X\overline{Y}Z + \overline{X}YZ$

(b) $(X+Y)(Y+Z)(X+\overline{Z})$

X	**Y**	**Z**	**a**	**b**
0	0	0	0	0
0	0	1	0	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1

7. Consider the function F, whose truth table is below.

X	**Y**	**Z**	**F**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

(a) Write F as a sum of minterms and draw the corresponding circuit.

$$\overline{X}\,\overline{Y}Z + \overline{X}\,Y\,\overline{Z} + X\,\overline{Y}\,\overline{Z} + X\,\overline{Y}\,Z + X\,Y\,Z$$

(b) Write F as a product of maxterms and draw the corresponding circuit.

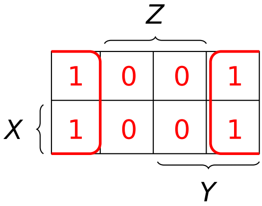
$$(X + Y + Z)(X + \overline{Y} + \overline{Z})(\overline{X} + \overline{Y} + Z)$$

(c) Complete the Karnaugh map for F as shown below.

		Z	Z	
	0	1	0	1
X	1	1	1	0
			Y	Y

8. Consider the function $F = \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}\,Y\,\overline{Z} + X\,\overline{Y}\,\overline{Z} + X\,Y\,\overline{Z}$

(a) Simplify the function using a Karnaugh map: draw the map F, circle implicants, and write the simplified function in algebraic form.



$= \overline{Z}$

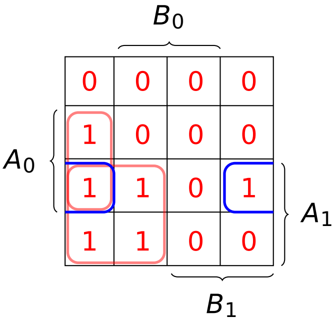
(b) Show how applying the axioms of Boolean algebra can produce the same result.

$$
\begin{aligned}
F &= \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}\,Y\,\overline{Z} + X\,\overline{Y}\,\overline{Z} + X\,Y\,\overline{Z} \\
&= \overline{Z}(\overline{X}\,\overline{Y} + \overline{X}\,Y + X\,\overline{Y} + X\,Y) \\
&= \overline{Z}(\overline{X}(\overline{Y} + Y) + X(\overline{Y} + Y)) \\
&= \overline{Z}(\overline{X}1 + X1) \\
&= \overline{Z}(\overline{X} + X) \\
&= \overline{Z}1 \\
&= \overline{Z}
\end{aligned}
$$

9. Design a circuit that takes two two-bit binary numbers (A_1 and A_0, B_1 and B_0) and produces a true output when, in binary, A is strictly greater than B.

(a) Fill in the truth table

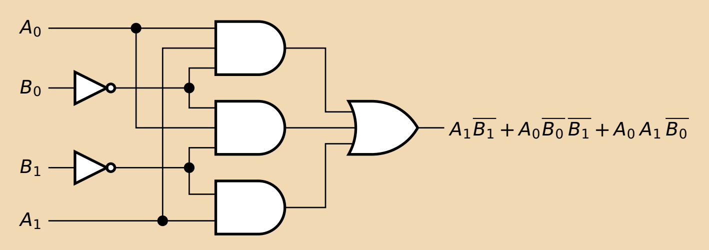
(b) Fill in the Karnaugh map and use it to minimize



$$A_1\overline{B_1} + A_0\overline{B_0}\,\overline{B_1} + A_0 A_1 \overline{B_0}$$

(c) Draw the circuit you derived from the map in part (b).

A_1	A_0	B_1	B_0	$A > B$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0


A_0
B_0
B_1
A_1
$A_1\overline{B_1} + A_0\overline{B_0}\,\overline{B_1} + A_0 A_1\overline{B_0}$