



Syntax and Parsing

COMS W4115

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Lexical Analysis (Scanning)

Lexical Analysis (Scanning)

Translates a stream of characters to a stream of tokens

$f \circ o \sqcup = \sqcup a + \sqcup \text{bar}(2, \sqcup q);$

ID	EQUALS	ID	PLUS	ID	L PAREN	NUM
COMMA	ID	L PAREN	SEMI			

Token	Lexemes	Pattern
EQUALS	=	an equals sign
PLUS	+	a plus sign
ID	a foo bar	letter followed by letters or digits
NUM	0 42	one or more digits

Lexical Analysis

Goal: simplify the job of the parser.

Scanners are usually much faster than parsers.

Discard as many irrelevant details as possible (e.g., whitespace, comments).

Parser does not care that the identifier is “super callifragilisticexpialidocious.”

Parser rules are only concerned with tokens.

Describing Tokens

Alphabet: A finite set of symbols

Examples: { 0, 1 }, { A, B, C, …, Z }, ASCII, Unicode

String: A finite sequence of symbols from an alphabet

Examples: ϵ (the empty string), Stephen, $\alpha\beta\gamma$

Language: A set of strings over an alphabet

Examples: \emptyset (the empty language), { 1, 11, 111, 1111 }, all English words, strings that start with a letter followed by any sequence of letters and digits

Operations on Languages

Let $L = \{ \epsilon, wo \}$, $M = \{ man, men \}$

Concatenation: Strings from one followed by the other

$L.M = \{ man, men, woman, women \}$

Union: All strings from each language

$L \cup M = \{ \epsilon, wo, man, men \}$

Kleene Closure: Zero or more concatenations

$M^* = \{ \epsilon \} \cup M \cup M.M \cup M.M.M \dots = \{ \epsilon, man, men, manman, manmen, menmen, manmanman, manmanmen, manmenmen, \dots \}$

Kleene Closure

The asterisk operator (*) is called the Kleene Closure operator after the inventor of regular expressions, Stephen Cole Kleene, who pronounced his last name “CLAY-neee.”

His son Ken writes “As far as I am aware this pronunciation is incorrect in all known languages. I believe that this novel pronunciation was invented by my father.”

A standard way to express languages for tokens.

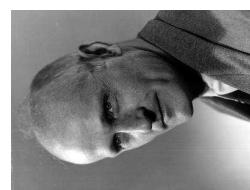
1. ϵ is a regular expression that denotes $\{ \epsilon \}$

2. If $a \in \Sigma$, a is an RE that denotes $\{ a \}$

3. If r and s denote languages $L(r)$ and $L(s)$,

- $(r)(s)$ denotes $L(r) \cup L(s)$
- $(r)^*$ denotes $\{ tu : t \in L(r), u \in L(s) \}$
- $(r)^*$ denotes $\bigcup_{i=0}^{\infty} L^i$ ($L^0 = \{ \epsilon \}$ and $L^i = LL^{i-1}$)

Regular Expression Examples



RE	Language
$a b$	$\{a, b\}$
$(a b)(a b)$	$\{aa, ab, ba, bb\}$
a^*	$\{ \epsilon, a, aa, aaa, aaaa, \dots \}$
$(a b)^*$	$\{ \epsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, \dots \}$
$a a^*$	$\{a, b, ab, aab, acab, acaaab, \dots \}$

Specifying Tokens with REs

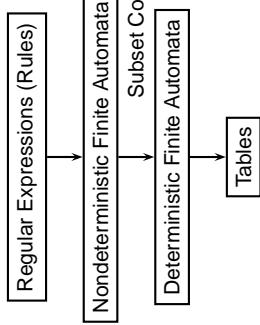
Implementing Scanners Automatically

Typical choice: $\Sigma = \text{ASCII characters, i.e., } \{_, !, \#, \$, \dots, 0, 1, \dots, 9, \dots, A, \dots, Z, \dots, \sim\}$

letters: $A|B|\dots|Z|a|\dots|z$

digits: $0|1|\dots|9$

identifier: letter (letter | digit)*

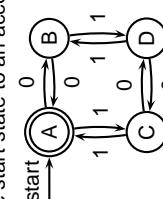


Nondeterministic Finite Automata

“All strings containing an even number of 0’s and 1’s”	1. Set of states $S: \{\textcircled{A}, \textcircled{B}, \textcircled{C}, \textcircled{D}\}$
letters: $A B \dots Z a \dots z$	2. Set of input symbols $\Sigma: \{0, 1\}$
digits: $0 1 \dots 9$	3. Transition function $\sigma: S \times \Sigma \rightarrow 2^S$
identifier: letter (letter digit)*	state ϵ 0 1 A \textcircled{B} \textcircled{C} \textcircled{D} B \textcircled{A} \textcircled{D} C \textcircled{A} \textcircled{B} D \textcircled{A} \textcircled{C}
	4. Start state $s_0: \textcircled{A}$
	5. Set of accepting states $F: \{\textcircled{A}\}$

The Language induced by an NFA

An NFA accepts an input string x iff there is a path from the start state to an accepting state that “spells out” x .

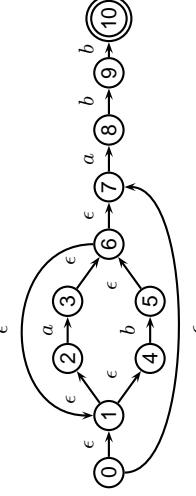


Show that the string “010101” is accepted.



Translating REs into NFAs

Example: translate $(a|b)^*abb$ into an NFA



Show that the string “aabbb” is accepted.
 $\textcircled{0} \xrightarrow{\epsilon} \textcircled{1} \xrightarrow{a} \textcircled{2} \xrightarrow{a} \textcircled{3} \xrightarrow{\epsilon} \textcircled{4} \xrightarrow{b} \textcircled{5} \xrightarrow{\epsilon} \textcircled{6} \xrightarrow{\epsilon} \textcircled{7} \xrightarrow{a} \textcircled{8} \xrightarrow{b} \textcircled{9} \xrightarrow{b} \textcircled{10}$

Simulating an NFA: ·aabbb, Start

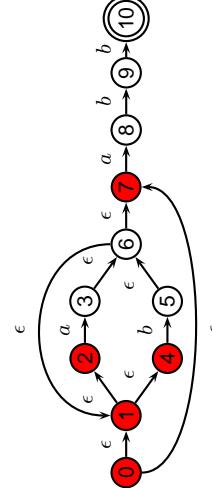
Problem: you must follow the “right” arcs to show that a string is accepted. How do you know which arc is right?

Solution: follow them all and sort it out later.

“Two-stack” NFA simulation algorithm:

1. Initial states: the ϵ -closure of the start state
2. For each character c ,
 - New states: follow all transitions labeled c
 - Form the ϵ -closure of the current states
3. Accept if any final state is accepting

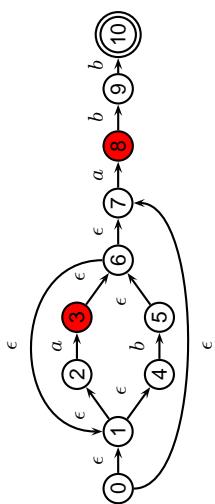
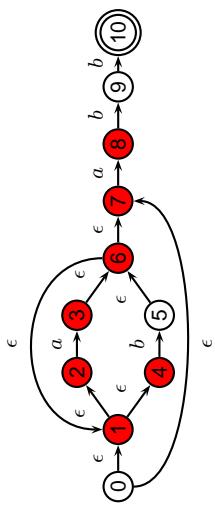
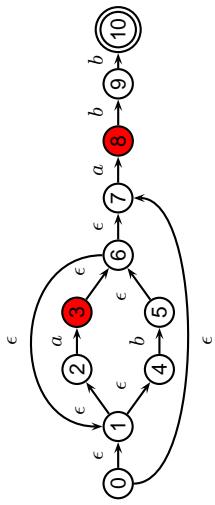
Simulating an NFA: ·aabbb, Start



Simulating an NFA: $a \cdot abb$

Simulating an NFA: $a \cdot abb, \epsilon$ -closure

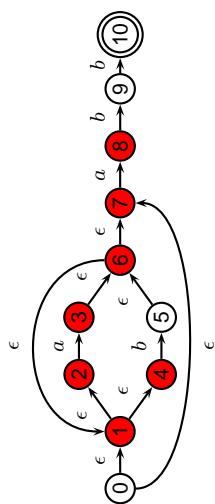
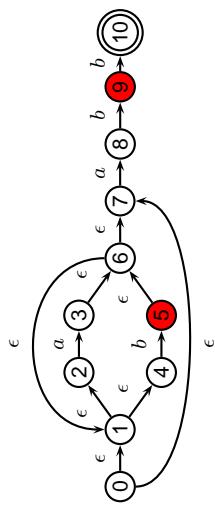
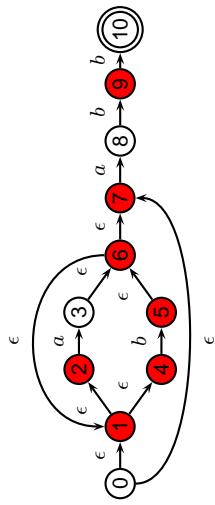
Simulating an NFA: $aa \cdot bb$



Simulating an NFA: $aa \cdot bb$, ϵ -closure

Simulating an NFA: $ab \cdot b$

Simulating an NFA: $aab \cdot b, \epsilon$ -closure



Simulating an NFA: $aabb$.

Simulating an NFA: $aabb$, Done

Deterministic Finite Automata

Restricted form of NFAs:

- No state has a transition on ϵ
 - For each state s and symbol a , there is at most one edge labeled a leaving s .

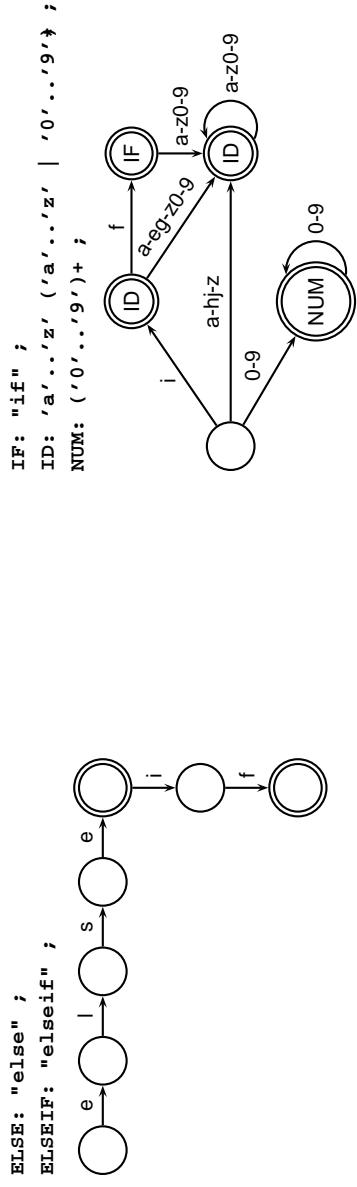
Differs subtly from the definition used in COMS W3261
 (Sipser, *Introduction to the Theory of Computation*)

Very easy to check acceptance: simulate by maintaining current state. Accept if you end up on an accepting state. Reject if you end on a non-accepting state or if there is no transition from the current state for the next symbol.

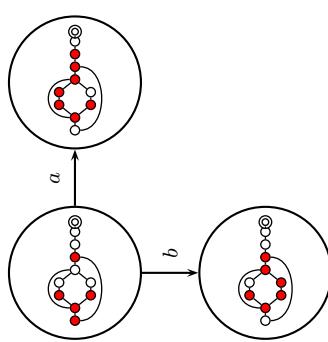
Deterministic Finite Automata

Deterministic Finite Automata

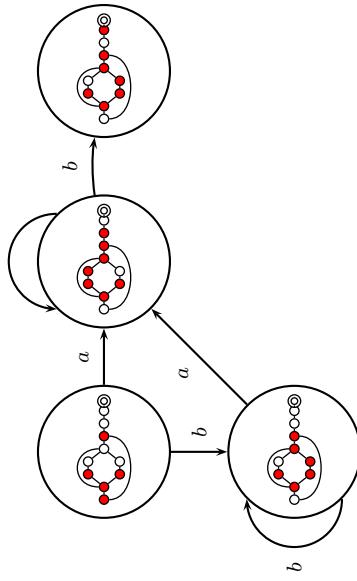
Building a DFA from an NFA



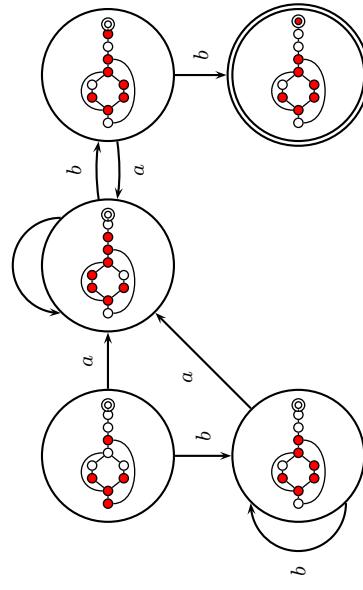
Subset construction for $(a|b)^*abb$ (1)



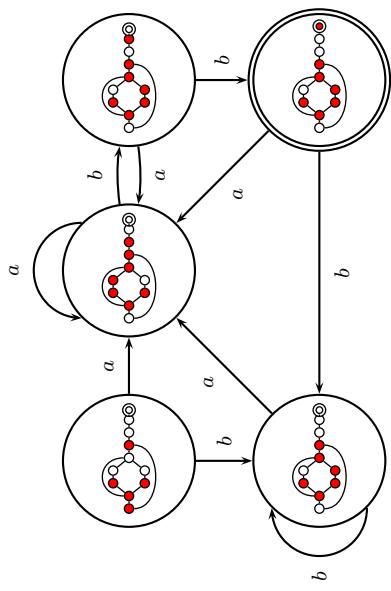
Subset construction for $(a|b)^*abb$ (2)



Subset construction for $(a|b)^*abb$ (3)



Subset construction for $(a|b)^*abb$ (4)



Subset Construction

An DFA can be exponentially larger than the corresponding NFA.
 n states versus 2^n
 Tools often try to strike a balance between the two representations.
 ANTLR uses a different technique.

The ANTLR Compiler Generator

Language and compiler for writing compilers
 Running ANTLR on an ANTLR file produces Java source files that can be compiled and run.
 ANTLR can generate

- Scanners (lexical analyzers)
- Parsers
- Tree walkers

ANTLR 2.0 vs. 3.0

As usual, software marches on. ANTLR 3.0 is only a “small” change from 2.0, but of course, it breaks every one of my examples.

The biggest change was in the tree walker behavior.

All the nice tricks I'll show you for writing an interpreter no longer work.

Use ANTLR 2.0

```
class CalcLexer extends Lexer;
{
    LPAREN : '(' ;          // Rules for punctuation
    RPAREN : ')' ;          ;
    STAR : '*' ;           ;
    PLUS : '+' ;           ;
    SEMI : ';' ;           ;
    protected             // Can only be used as a sub-rule
    DIGIT : '0'.. '9' ;     // Any character between 0 and 9
    INT : (DIGIT)+ ;       // One or more digits
    ws : (' ' | '\t' | '\n' | '\r') //Whitespace
        { $setType(Token.SKIP); } //Action: ignore
}
```

An ANTLR File for a Simple Scanner

ANTLR Specifications for Scanners

Rules are names starting with a capital letter.
A character in single quotes matches that character.

```
LPAREN : '(' ;
RPAREN : ')' ;
STAR : '*' ;
PLUS : '+' ;
SEMI : ';' ;
protected           // Can only be used as a sub-rule
DIGIT : '0'.. '9' ; // Any character between 0 and 9
INT : (DIGIT)+ ;   // One or more digits
ws : (' ' | '\t' | '\n' | '\r') //Whitespace
{ $setType(Token.SKIP); } //Action: ignore
```

ANTLR Specifications

Question mark makes a clause optional.

```
PERSON : ("wo")? 'm' ('a' | 'e') 'n' ;
```

(Matches man, men, woman, and women.)

Double dots indicate a range of characters:

```
DIGIT : '0'.. '9' ;
```

Asterisk and plus match “zero or more,” “one or more.”

```
ID : LETTER (LETTER | DIGIT)* ;
```

```
NUMBER : (DIGIT)+ ;
```

Free-Format Languages

Typical style arising from scanner/parser division

Program text is a series of tokens possibly separated by whitespace and comments, which are both ignored.

- keywords (`if` `while`)
- punctuation (, (+) -)
- identifiers (`foo` `bar`)
- numbers (10 -3.14159e+32)
- strings ("A String")

Free-Format Languages

Java C C++ Algol Pascal

Some deviate a little (e.g., C and C++ have a separate preprocessor)

But not all languages are free-format.

FORTRAN 77

FORTRAN 77 is not free-format. 72-character lines:

```
100 IF(IN .EQ. 'Y' .OR. IN .EQ. 'Y' .OR.
      $ IN .EQ. 'T' .OR. IN .EQ. 'T') THEN
      
      Statement label Continuation Normal
```

When column 6 is not a space, line is considered part of the previous.

Fixed-length line works well with a one-line buffer.

Makes sense on punch cards.

Python

The Python scripting language groups with indentation

```
i = 0
while i < 10:
    i = i + 1
    print i # Prints 1, 2, ..., 10
```

```
print i # Just prints 10
```

This is succinct, but can be error-prone.

How do you wrap a conditional around instructions?

Syntax and Language Design

Does syntax matter? Yes and no

More important is a language's *semantics*—its meaning.

The syntax is aesthetic, but can be a religious issue.

But aesthetics matter to people, and can be critical.

Verbosity does matter: smaller is usually better.

Too small can be a problem: API is a compact, cryptic language with its own character set (!)

```
E→ A TEST B ; L
L→ 0 . 5
E→ ((A×A)+B×B)* L
```

Syntax and Language Design

Some syntax is error-prone. Classic FORTRAN example:

```
DO 5 I = 1,25 ! Loop header (for i = 1 to 25)
DO 5 I = 1.25 ! Assignment to variable DO5I
```

Trying too hard to reuse existing syntax in C++:

```
vector< vector<int> > foo; // Syntax error
```

C distinguishes `>` and `>>` as different operators.

Parsing

Parsing

Objective: build an abstract syntax tree (AST) for the token sequence from the scanner.

$$2 * 3 + 4 \Rightarrow \begin{array}{c} + \\ / \quad \backslash \\ 2 \quad 3 \end{array}$$

Goal: discard irrelevant information to make it easier for the next stage.

Parentheses and most other forms of punctuation removed.

Grammars

Languages

Regular languages (t is a terminal):

$$A \rightarrow t_1 \dots t_n B$$

$$A \rightarrow t_1 \dots t_n$$

Context-free languages (P is terminal or a variable):

$$A \rightarrow P_1 \dots P_n$$

Context-sensitive languages:

$$\alpha_1 A \alpha_2 \rightarrow \alpha_1 B \alpha_2$$

" $B \rightarrow A$ only in the 'context' of $\alpha_1 \dots \alpha_2$ "

Issues

Ambiguous grammars

Precedence of operators

Left- versus right-recursive

Top-down vs. bottom-up parsers

Parse Tree vs. Abstract Syntax Tree

Ambiguous Grammars

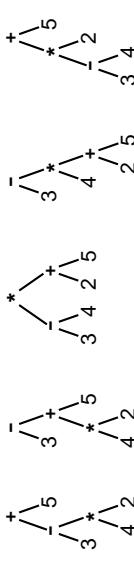
A grammar can easily be ambiguous. Consider parsing

$$3 - 4 * 2 + 5$$

with the grammar

$$e \rightarrow e + e \mid e - e \mid e * e \mid e / e \mid N$$

$e \rightarrow e + e \mid e - e \mid e * e \mid e / e \mid N$



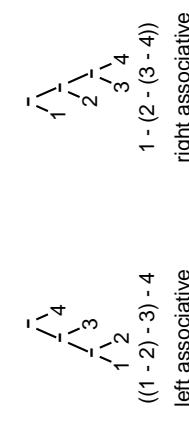
Associativity

Fixing Ambiguous Grammars

Whether to evaluate left-to-right or right-to-left

Most operators are left-associative

$$1 - 2 - 3 = 4$$



left associative

right associative

Original ANTLR grammar specification

```
expr : expr '+' expr
      | expr '-' expr
      | expr '*' expr
      | expr '/' expr
      | NUMBER
      ;
term : term '*' term
      | term '/' term
      | atom ;
atom : NUMBER ;
```

Ambiguous: no precedence or associativity.

Still ambiguous: associativity not defined

Assigning Precedence Levels

```
expr : expr '+' expr
      | expr '-' expr
      | expr '*' expr
      | expr '/' expr
      | term ;
```

```
term : term '*' term
      | term '/' term
      | atom ;
```

Ambiguous: no precedence or associativity.

Still ambiguous: associativity not defined

Parsing Context-Free Grammars

Parsing LL(k) Grammars

There are $O(n^3)$ algorithms for parsing arbitrary CFGs, but most compilers demand $O(n)$ algorithms.

Fortunately, the LL and LR subclasses of CFGs have $O(n)$ parsing algorithms. People use these in practice.

```
term : term '*' atom
      | term '/' atom
      | atom ;
```

```
atom : NUMBER ;
```

LL: Left-to-right, Left-most derivation
k: number of tokens to look ahead
Parsed by top-down, predictive, recursive parsers
Basic idea: look at the next token to predict which production to use
ANTLR builds recursive LL(k) parsers
Almost a direct translation from the grammar.

Parsing Context-Free Grammars

Parsing LL(k) Grammars

Cannot have common prefixes

```
expr : ID '(' expr ')'
      | ID '=' expr
```

becomes

```
expr() {
    switch (next-token) {
        case ID: match(ID); match(LPAR); expr(); match(RPAR); break;
        case NUMBER: expr(); /* Infinite Recursion */
        case WHILE: match(WHILE); expr(); match(DO); expr();
                    match(WHILE); expr(); match(DO); expr();
        case NUMBER or LPAREN: expr();
        case EQUALS: expr();
    }
}
```

Writing LL(k) Grammars

Cannot have left-recursion

```
expr : expr '+' term
      | expr '-' term
      | expr '*' term
      | expr '/' term
      | atom ;
```

becomes

```
stmt() {
    switch (next-token) {
        case IF: match(IF); expr(); match(THEN); expr();
        case WHILE: match(WHILE); expr(); match(DO); expr();
                    match(WHILE); expr(); match(DO); expr();
        case NUMBER or LPAREN: expr();
        case EQUALS: expr();
    }
}
```

Writing LL(k) Grammars

Cannot have common prefixes

```
expr : ID '(' expr ')'
      | ID '=' expr
```

becomes

```
expr() {
    switch (next-token) {
        case ID: match(ID); match(LPAR); expr(); match(RPAR); break;
        case NUMBER: expr();
        case EQUALS: expr();
    }
}
```

Implementing a Top-Down Parser

Cannot have common prefixes

```
expr : expr '+' term
      | expr '-' term
      | expr '*' term
      | expr '/' term
      | atom ;
```

becomes

```
stmt() {
    switch (next-token) {
        case IF: match(IF); expr(); match(THEN); expr();
        case WHILE: match(WHILE); expr(); match(DO); expr();
                    match(WHILE); expr(); match(DO); expr();
        case NUMBER or LPAREN: expr();
        case EQUALS: expr();
    }
}
```

Eliminating Common Prefixes

Consolidate common prefixes:

```
expr
: expr '+' term
| expr '-' term
| term
;
becomes
if (a < b) c(); else d();
    |
    / \   / \   / \
    a if   a if   d()
    | \   / \   |
    b c() b c()
```

Eliminating Left Recursion

Understand the recursion and add tail rules

```
expr
: expr ('+' term | '-' term )
| term
;
becomes
expr : term ('+' term | '-' term );
expr : term expr ;
expr : '+', term exprt
| ',', term exprt
| /* nothing */
;

```

Using ANTLR's EBNF

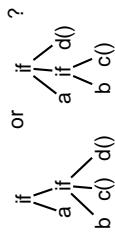
ANTLR makes this easier since it supports * and -:

```
expr : expr '+' term
      | expr '-' term
      | term ;
becomes
expr : term ('+' term | '-' term ) *
```

The Dangling Else Problem

Who owns the else?

```
if (a) if (b) c(); else d();
```



Grammars are usually ambiguous; manuals give disambiguating rules such as C's:

As usual the "else" is resolved by connecting an else with the last encountered elseif if.

The Dangling Else Problem

```
stmt : "if" expr "then" stmt iftail
      | other-statements ;
```

```
iftail
: "else" stmt
| /* nothing */
```

Problem comes when matching "iftail".

Normally, an empty choice is taken if the next token is in the "follow set" of the rule. But since "else" can follow an iftail, the decision is ambiguous.

The Dangling Else Problem

ANTLR can resolve this problem by insisting on "greedy." If a conditional is marked as greedy, it will take that option even if the "nothing" option would also match:

```
stmt : "if" expr "then" stmt
      ( options {greedy = true;} :
        "else" stmt
      )?
      | other-statements
;
```

The Dangling Else Problem

Some languages resolve this problem by insisting on nesting everything.

E.g., Algol 68:

```
if a < b then a else b fi;
```

"fi" is "if" spelled backwards. The language also uses do-od and case-esac.

Statement separators/terminators

C uses ; as a statement terminator.

```
if (a>b) printf("a less");
else {
  printf("b"); printf(" less");
}
```

Pascal uses ; as a statement separator.

```
if a < b then writeln('a less');
else begin
  write('a'); writeln(' less')
end
```

Pascal later made a final ; optional.

Bottom-up Parsing

Rightmost Derivation

Handles

LR Parsing

Constructing the SLR Parse Table

The states are places we could be in a reverse-rightmost derivation. Let's represent such a place with a dot.

Say we were at the beginning ($\cdot e$). This corresponds to

1 :	$e \rightarrow t + e$	$e' \rightarrow \cdot e$	The first is a placeholder. The
2 :	$e \rightarrow t$	$e \rightarrow \cdot t + e$	second are the two possibilities
3 :	$t \rightarrow \mathbf{id} * t$	$e \rightarrow \cdot t$	when we're just before e . The last
4 :	$t \rightarrow \mathbf{id}$	$t \rightarrow \mathbf{id} * t$	two are the two possibilities when

$t \rightarrow \cdot \mathbf{id}$ we're just before t .

LR Parsing

	stack	input	action	1 : $e \rightarrow t + e$	input	stack	input
1 :	$e \rightarrow t + e$			$\text{Id} * \text{Id} + \text{Id} \$$	shift, goto 1	$\text{Id} * \text{Id} + \text{Id} \$$	sh
2 :	$e \rightarrow t$			$* \text{Id} + \text{Id} \$$	shift, goto 3	$* \text{Id} + \text{Id} \$$	sh
3 :	$t \rightarrow \text{Id} * t$			$\text{Id} + \text{Id} \$$	shift, goto 1	$\text{Id} + \text{Id} \$$	sh
4 :	$t \rightarrow \text{Id}$			$\text{Id} + \text{Id} \$$	reduce w/ 4	$\text{Id} + \text{Id} \$$	red
	action	goto				action	
	$\text{Id} + * \$$	$e \quad t$				$\text{Id} + * \$$	$e \quad t$
	0	s1		0	st	7	2
	1	r4 r4 s3 r4		1	r4 r4 s3 r4	2	
	2	r2 s4 r2 r2		2	r2 s4 r2 r2	0	
	3	s1		3	s1	5	
	4	s1		4	s1	6	
	5	r3 r3 r3		5	r3 r3 r3	2	
	6	r1 r1 r1 r1		6	r1 r1 r1 r1	0	
	7	acc		7	acc	0	
	stack	input	action				
	0	$\boxed{\text{q}} \quad \boxed{\text{q}}$	$\boxed{\frac{t}{e}}$	$+ \text{Id} \$$			

Action is reduce with rule 4 ($t \rightarrow \text{Id}$). The right side is removed from the stack to reveal state 3. The goto table in state 3 tells us to go to state 5 when we reduce a t .

LR Parsing

Constructing the SLR Parsing Table

Diagram illustrating the states and transitions of a parser:

- S7: $e' \rightarrow e.$** Start state.
- S0: $e' \rightarrow e \cdot$**
 - Transition $e' \rightarrow e \cdot$ leads to **S1: $t \rightarrow \text{Id} \cdot \text{Id}.$**
 - Transition $e' \rightarrow e \cdot t + e$ leads to **S2: $e \rightarrow t \cdot + e$** .
- S1: $t \rightarrow \text{Id} \cdot \text{Id}.$**
 - Transition Id leads to **S3: $t \rightarrow \text{Id} \cdot \text{Id} \cdot t$** .
 - Transition Id leads to **S4: $e \rightarrow t \cdot : \text{Id}$** .
- S2: $e \rightarrow t \cdot + e$**
 - Transition t leads to **S4: $e \rightarrow t \cdot : \text{Id}$** .
 - Transition $+$ leads to **S6: $e \rightarrow t + e.$**
- S4: $e \rightarrow t \cdot : \text{Id}$**
 - Transition e leads to **S6: $e \rightarrow t + e.$**
- S6: $e \rightarrow t + e.$** Final state.

The Punchline

This is a tricky, but mechanical procedure. The parser generators YACC, Bison, Cup, and others (but not ANTLR) use a modified version of this technique to generate fast bottom-up parsers.

Shift/reduce conflicts are caused by a state like	Reduce/reduce conflicts are caused by a state like
$t \rightarrow \text{Id} \cdot *t$	$t \rightarrow \text{Id} * t$.
$t \rightarrow \text{Id} * t$.	$e \rightarrow t + e$.