CS W4701
Artificial Intelligence

Fall 2013

Chapter 7:

Logical Agents

Jonathan Voris

(based on slides by Sal Stolfo)

The Big Idea

- Humans know stuff

- We use the stuff we know to help us do things

The Big Idea

- Do our agents know stuff?

The Big Idea

- Do our agents know stuff?
  - Well, kind of…

The Big Idea

- Do our agents know stuff?
  - Well, kind of…
- Knowledge encoded in agent functions:

The Big Idea

- Do our agents know stuff?
  - Well, kind of…
- Knowledge encoded in agent functions:
  - Successor functions
  - Heuristics
  - Performance measures
  - Goal tests

The Big Idea

- Think about n-puzzle agent
  - Can it predict outcomes of future actions?
  - Can it conclude that a state is unreachable?
  - Can it prove that certain states are always unreachable from others?
- How to represent an environment that is
  - Atomic
  - Partially observable

The Big Idea

- Agents we’ve designed so far possess very inflexible knowledge
- What if we could teach our agents how to **reason**?
  - Combine information
  - Adapt to new tasks
  - Learn about environment
  - Update in response to environmental changes
- Agent will need a way to keep track of and apply stuff it knows

Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution

Outline

- **Knowledge-based agents**
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution

Knowledge Bases

- Knowledge base = set of sentences in a formal language
- Declarative approach to building an agent (or other system):
  - `Tell` it what it needs to know
- Then it can `Ask` itself what to do - answers should follow from the KB
- Agents can be viewed at the knowledge level
  i.e., what they know, regardless of how implemented
- Or at the implementation level
  - i.e., data structures in KB and algorithms that manipulate them

Knowledge Based Agents

- When knowledge-based agent runs it:
  - Tells KB about its latest perception
    - MAKE-PERCEPT-SENTENCE
  - Asks the KB what to do next
    - MAKE-ACTION-QUERY
  - Executes action and tells the KB so
    - MAKE-ACTION-SENTENCE

A Simple Knowledge Based Agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

- The agent must be able to:

A Simple Knowledge Based Agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

- The agent must be able to:
  - Represent states, actions, etc.

A Simple Knowledge Based Agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts

A Simple Knowledge Based Agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world

A Simple Knowledge Based Agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world

A Simple Knowledge Based Agent

```
function KB-AGENT(percept) returns an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
    action ← ASK(KB, MAKE-ACTION-QUERY(t))
    TELL(KB, MAKE-ACTION-SENTENCE(action, t))
    t ← t + 1
    return action
```

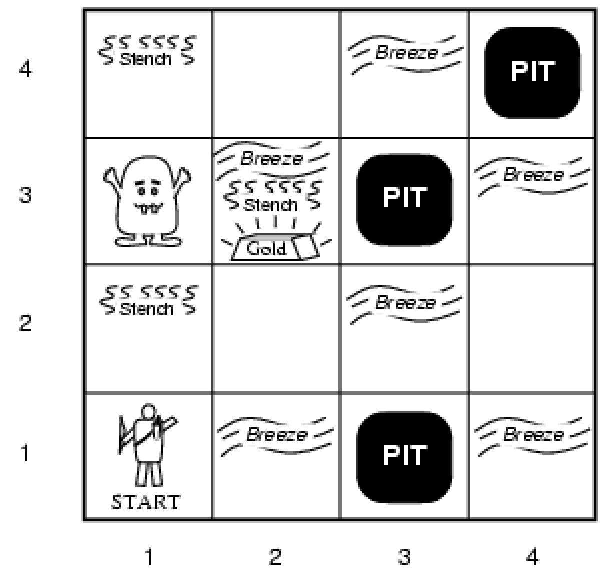
- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions

Outline

- Knowledge-based agents
- **Wumpus world**
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution

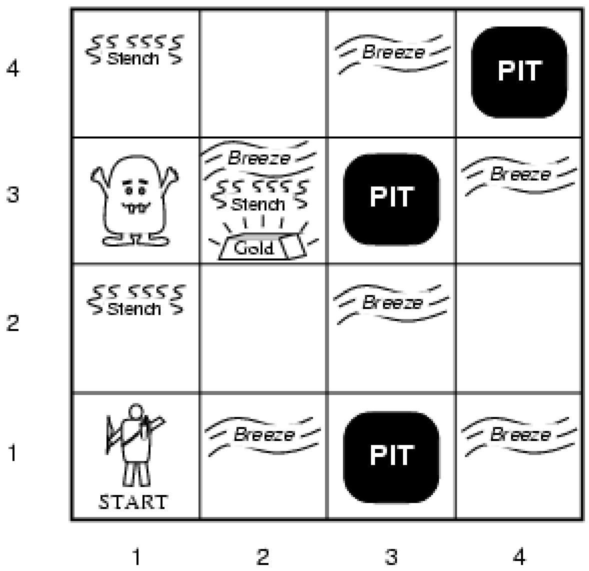
Wumpus World PEAS Description

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow



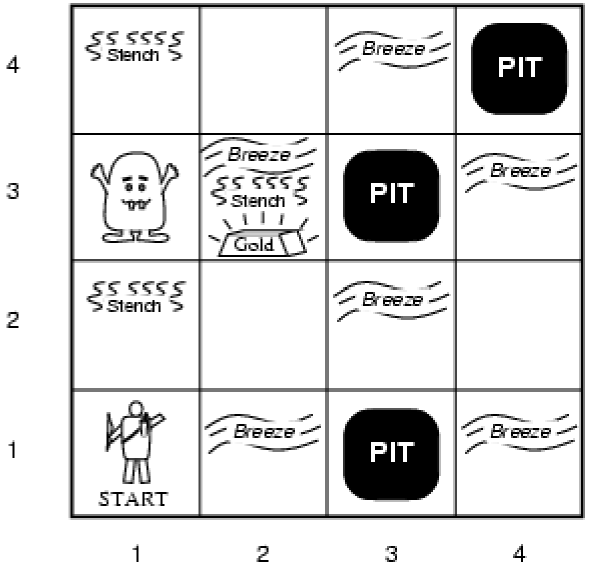
Wumpus World PEAS Description

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square



Wumpus World PEAS Description

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square

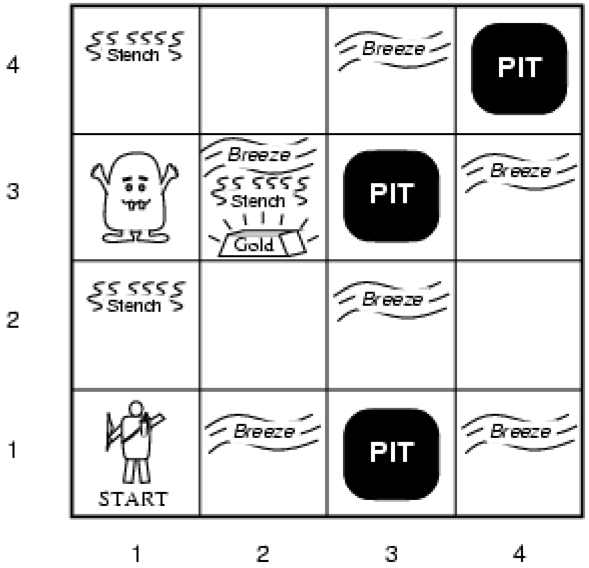


- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Wumpus World PEAS Description

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow

- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square



- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot
- Sensors: Stench, Breeze, Glitter, Bump, Scream

Wumpus World Characterization

- Fully Observable

Wumpus World Characterization

- Fully Observable No – only local perception

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete

Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes

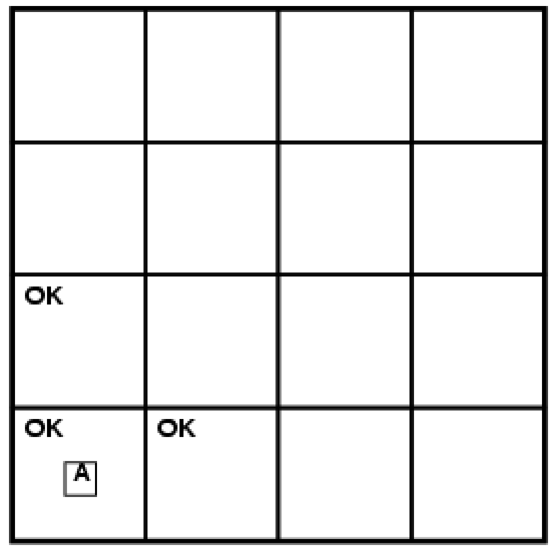
Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent?

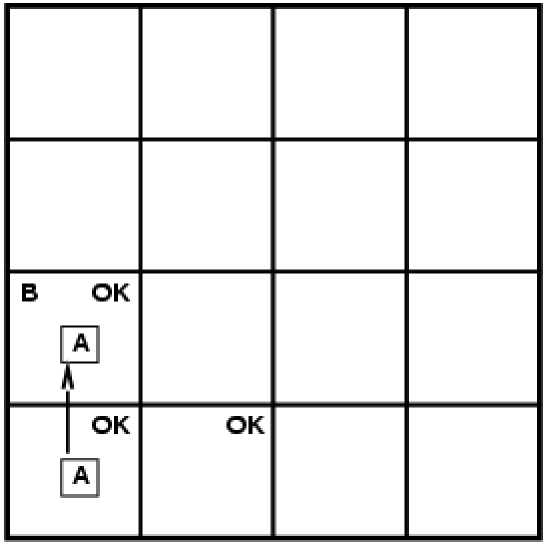
Wumpus World Characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent? Yes – Wumpus is essentially a natural feature

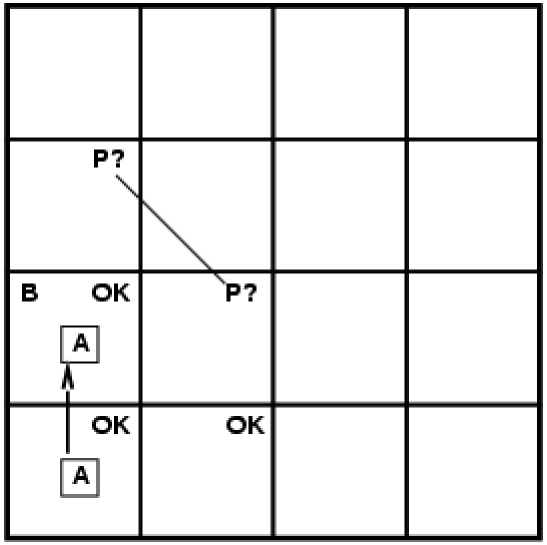
Exploring a Wumpus World



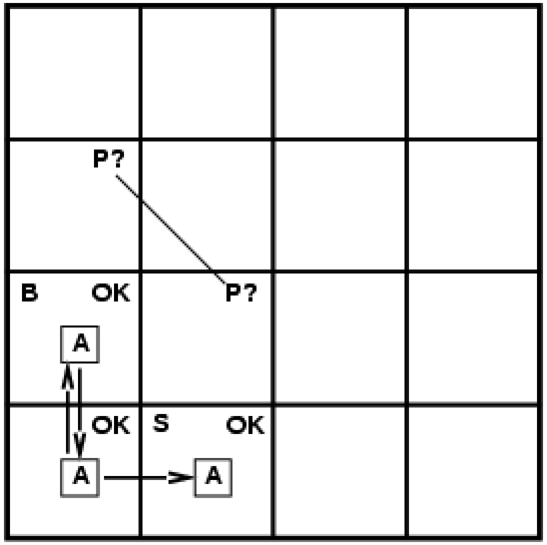
Exploring a Wumpus World



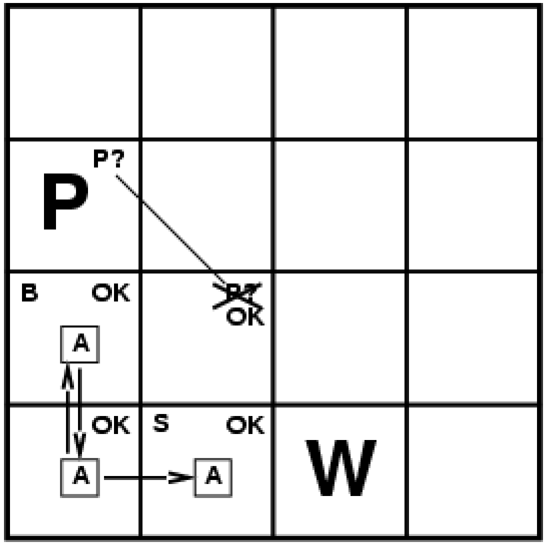
Exploring a Wumpus World



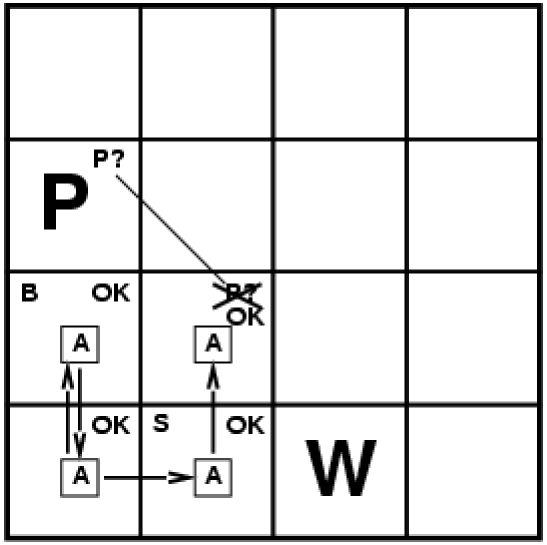
Exploring a Wumpus World



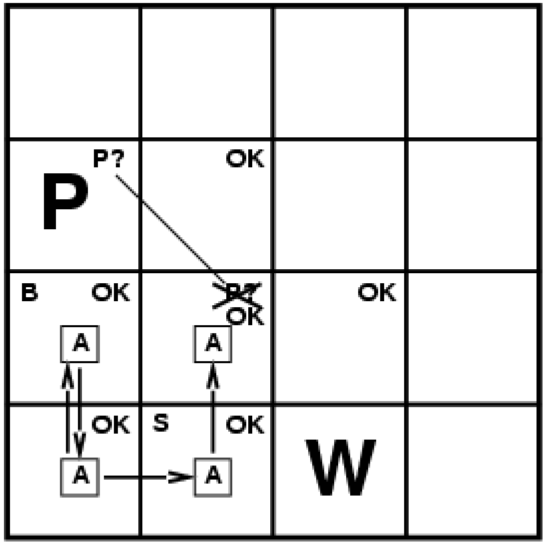
Exploring a Wumpus World



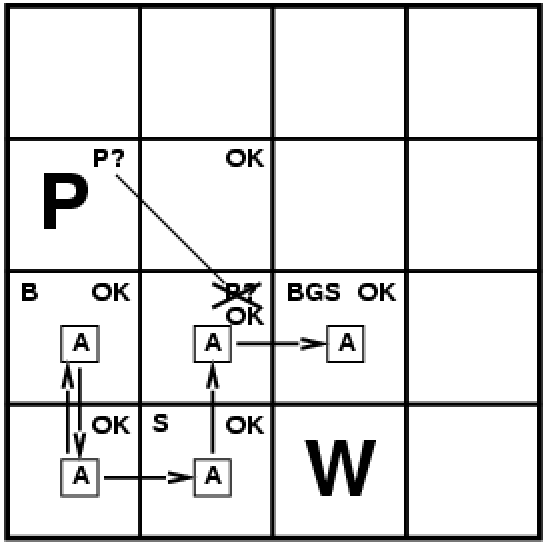
Exploring a Wumpus World



Exploring a Wumpus World



Exploring a Wumpus World



Outline

- Knowledge-based agents
- Wumpus world
- **Logic in general - models and entailment**
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution

Logic in General

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences;
  - i.e., define truth of a sentence in a world

Logic in General

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences;
  - i.e., define truth of a sentence in a world

- E.g., the language of arithmetic
  - $x+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence
  - $x+2 \geq y$ is true iff the number $x+2$ is no less than the number y
  - $x+2 \geq y$ is true in a world where $x = 7$, $y = 1$
  - $x+2 \geq y$ is false in a world where $x = 0$, $y = 6$
  - True in all worlds?
  - False in all worlds?

Entailment

- Entailment means that one thing follows from another

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
  - E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”
  - E.g., x+y = 4 entails 4 = x+y

Entailment

- Entailment means that one thing follows from another

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
  - E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”
  - E.g., x+y = 4 entails 4 = x+y
- Which part is syntax and which part is semantics?

Entailment

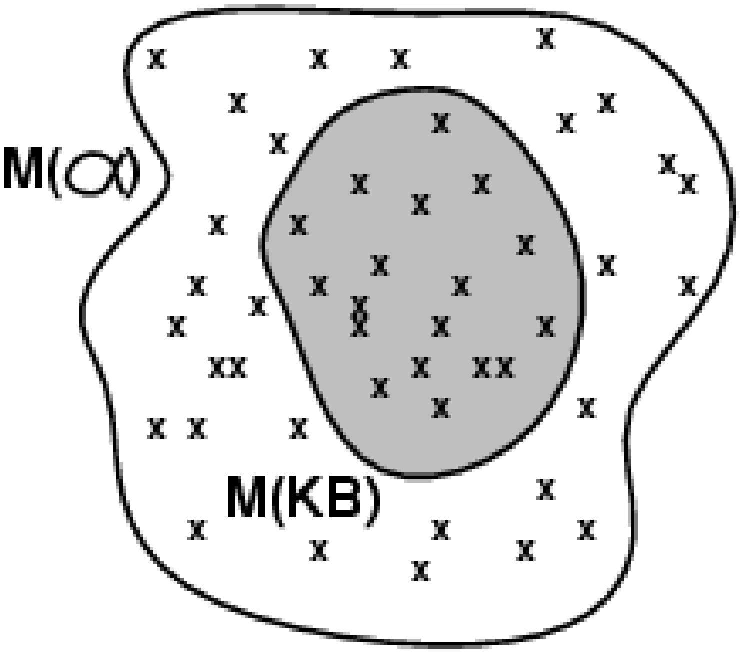
- Entailment means that one thing follows from another

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
  - E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”
  - E.g., x+y = 4 entails 4 = x+y
- Which part is syntax and which part is semantics?
  - Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

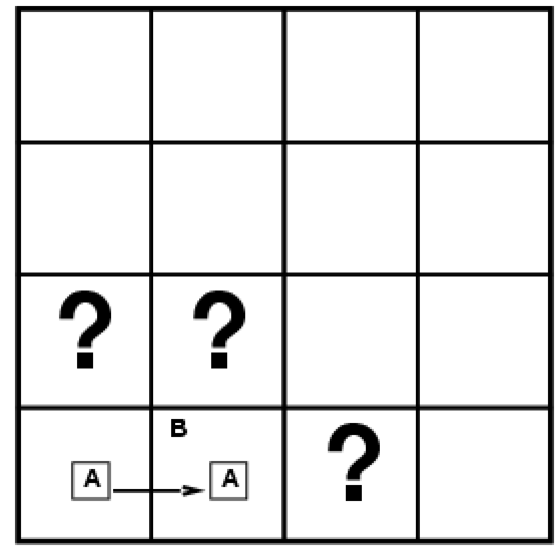
Models

- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
  - Alternative phrasing: m satisfies a
- *M(α)* is the set of all models of α
  - All models where α is true
- Then KB $\models$ α iff *M(KB)* $\subseteq$ *M*(α)
  - E.g. *KB* = Giants won and Reds won α = Giants won



Entailment in the Wumpus World

Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

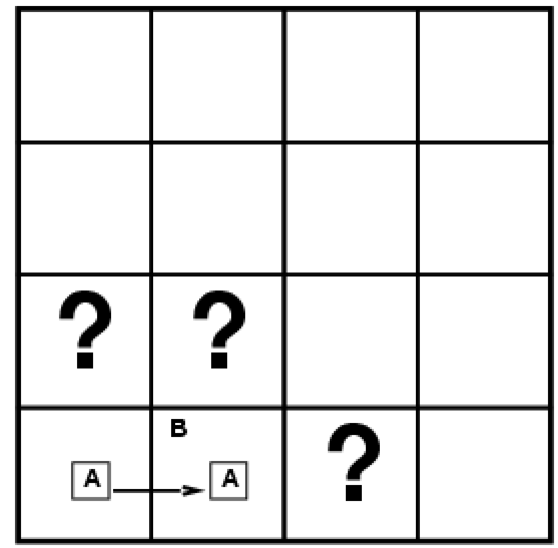


Consider possible models for *KB* assuming only pits

? Boolean choices

Entailment in the Wumpus World

Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

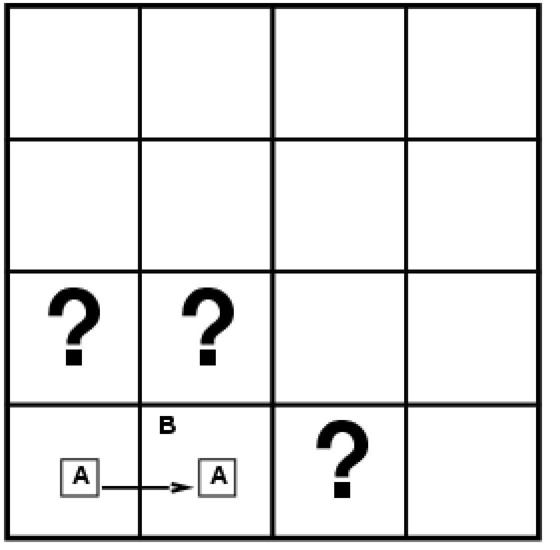


Consider possible models for *KB* assuming only pits

3 Boolean choices $\Rightarrow$? possible models

Entailment in the Wumpus World

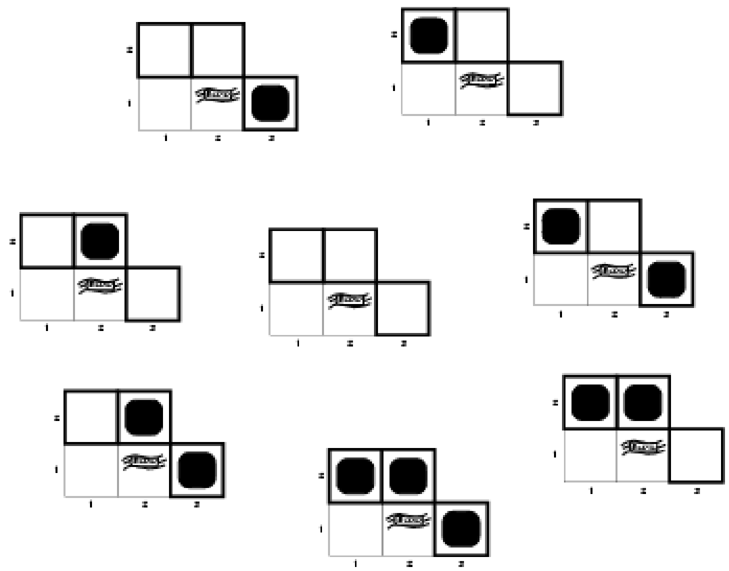
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]



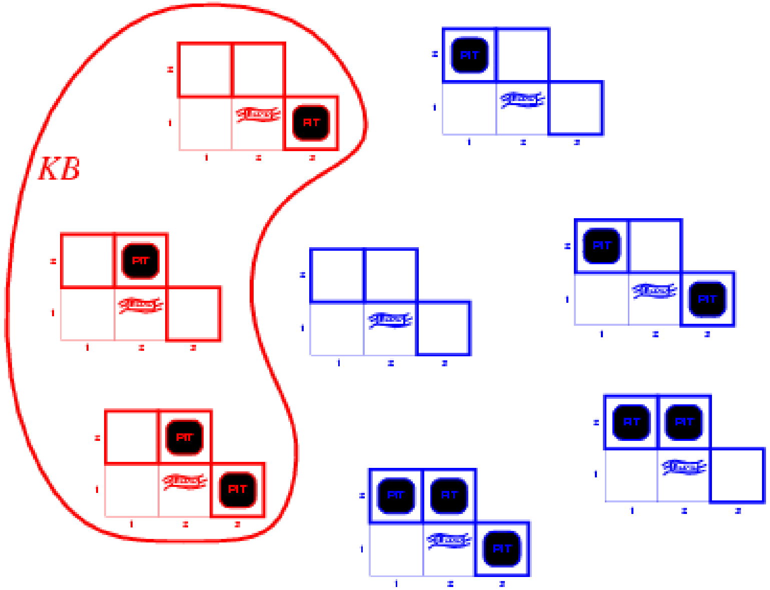
Consider possible models for *KB* assuming only pits

3 Boolean choices $\Rightarrow$ 8 possible models

Wumpus Models

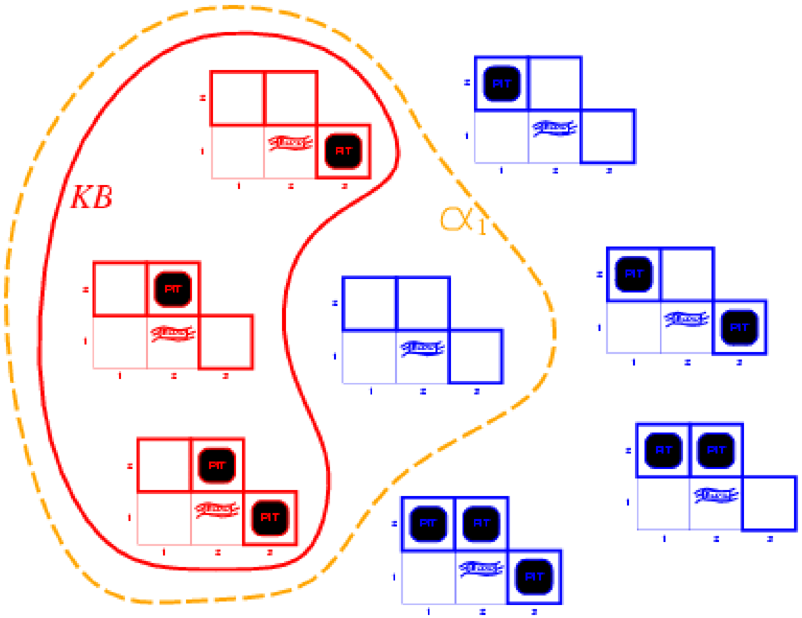


Wumpus Models



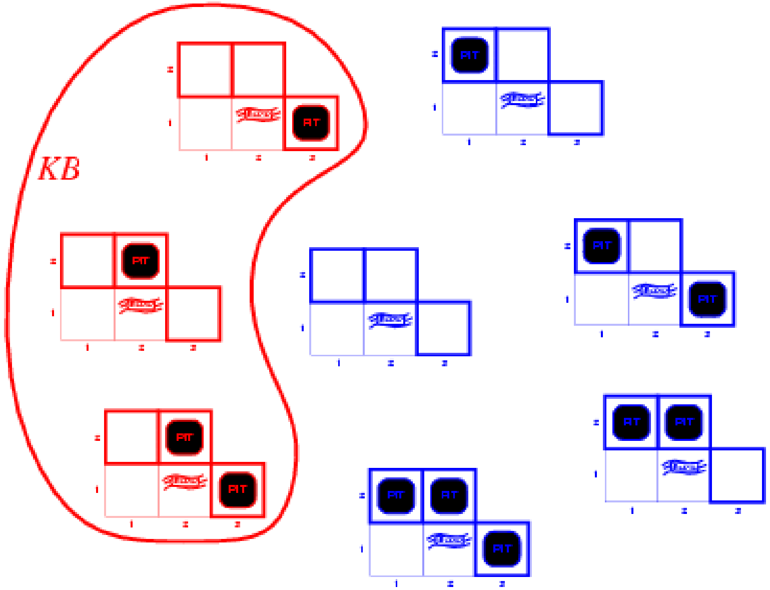
- *KB* = wumpus-world rules + observations

Wumpus Models



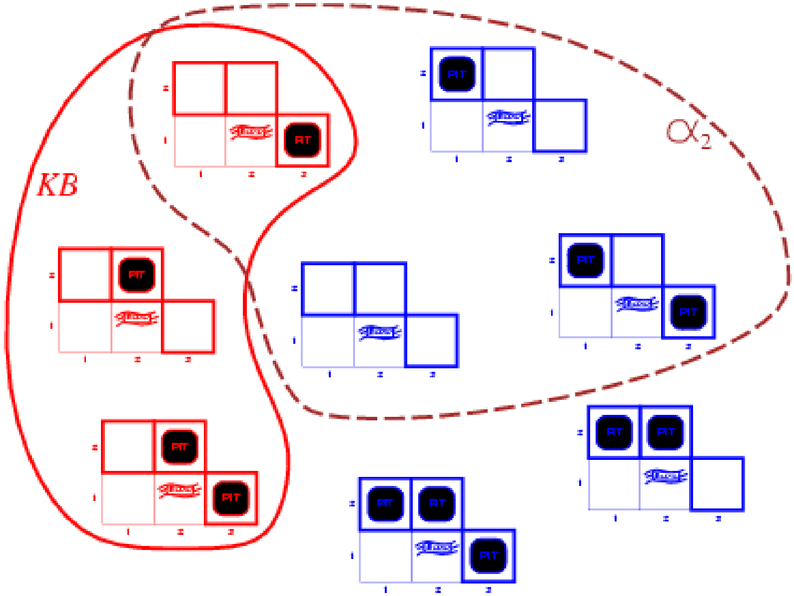
- *KB* = wumpus-world rules + observations
- α_1 = "[1,2] is safe", *KB* $\models \alpha_1$, proved by model checking

Wumpus Models



- *KB* = wumpus-world rules + observations

Wumpus Models



- *KB* = wumpus-world rules + observations
- α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Inference

- KB is your agent's haystack
  - Pile containing all possible conclusions
- Specific conclusion α is the needle agent is looking for
- Entailment: Is the needle in the haystack?
- Inference: Can you find the needle?

Inference

- *KB* $\vdash_i$ α = sentence α can be derived from *KB* by procedure *I*
  - α is derived from KB by I
  - i derives α from KB
- Soundness: *i* is sound if whenever *KB* $\vdash_i$ α, it is also true that *KB* $\models$ α
  - a.k.a. truth-preserving

Inference

- *KB* $\vdash_i \alpha$ = sentence α can be derived from *KB* by procedure *I*
  - α is derived from KB by I
  - i derives α from KB
- Soundness: *i* is sound if whenever *KB* $\vdash_i \alpha$, it is also true that *KB* $\models \alpha$
  - a.k.a. truth-preserving
- Is model checking sound?

Inference

- *KB* $\vdash_i \alpha$ = sentence α can be derived from *KB* by procedure *I*
  - α is derived from KB by I
  - i derives α from KB
- Soundness: *i* is sound if whenever *KB* $\vdash_i \alpha$, it is also true that *KB* $\models \alpha$
  - a.k.a. truth-preserving
- Is model checking sound? **Yes**

Inference

- $KB \vdash_i \alpha$ = sentence α can be derived from *KB* by procedure *I*
  - α is derived from KB by I
  - i derives α from KB
- Soundness: *i* is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
  - a.k.a. truth-preserving
- Is model checking sound? **Yes**
- What would it mean for an inference algorithm to *not* be sound?

Inference

- $KB \vdash_i \alpha$ = sentence α can be derived from *KB* by procedure *I*
  - α is derived from KB by I
  - i derives α from KB
- Soundness: *i* is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
  - a.k.a. truth-preserving
- Completeness: *i* is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the *KB*.

Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- **Propositional (Boolean) logic**
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution

Propositional Logic: Syntax

- Propositional logic is the simplest logic
  - Illustrates basic ideas
- The proposition symbols S_1, S_2 etc are sentences
- Atomic sentences: Single proposition
  - Special fixed meaning symbols: True and False
- Complex sentences:
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
  - If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
  - If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$ is true iff

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$ is true iff S is false

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
E.g.	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false
$S_1 \wedge S_2$	is true iff	

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
E.g.	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff		

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff		

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
E.g.	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff		

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	$S_1$is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff		

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and	$S_2 \Rightarrow S_1$ is true

Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
	false	true	false

With these symbols, 8 possible models, can be enumerated automatically.
Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \textit{true} \wedge (\textit{true} \vee \textit{false}) = \textit{true} \wedge \textit{true} = \textit{true}$$

Truth Tables for Connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

Wumpus World Sentences

Let $P_{i,j}$ be true if there is a pit in [i, j].

Let $B_{i,j}$ be true if there is a breeze in [i, j].

$\neg P_{1,1}$

$\neg B_{1,1}$

$B_{2,1}$

- "Pits cause breezes in adjacent squares"

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

Truth Tables for Inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
$false$	$false$	$false$	$false$	$false$	$false$	$false$	$false$	$true$
$false$	$false$	$false$	$false$	$false$	$false$	$true$	$false$	$true$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$false$	$true$	$false$	$false$	$false$	$false$	$false$	$false$	$true$
$false$	$true$	$false$	$false$	$false$	$false$	$true$	$\underline{true}$	$\underline{true}$
$false$	$true$	$false$	$false$	$false$	$true$	$false$	$\underline{true}$	$\underline{true}$
$false$	$true$	$false$	$false$	$false$	$true$	$true$	$\underline{true}$	$\underline{true}$
$false$	$true$	$false$	$false$	$true$	$false$	$false$	$false$	$true$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$true$	$true$	$true$	$true$	$true$	$true$	$true$	$false$	$false$

Inference by Enumeration

```
function TT-ENTAILS?(KB, α) returns true or false
    symbols ← a list of the proposition symbols in KB and α
    return TT-CHECK-ALL(KB, α, symbols, [])
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else do
        P ← FIRST(symbols); rest ← REST(symbols)
        return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
                TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

- For *n* symbols, time complexity is $O(2^n)$, space complexity is $O(n)$

Inference by Enumeration

- Sound?

Inference by Enumeration

- Sound? Yes

Inference by Enumeration

- Sound? Yes
  - Entailment is used directly!

Inference by Enumeration

- Sound? Yes
  - Entailment is used directly!
- Complete?

Inference by Enumeration

- Sound? Yes
  - Entailment is used directly!
- Complete? Yes

Inference by Enumeration

- Sound? Yes
  - Entailment is used directly!
- Complete? Yes
  - Works for all KB and a
  - Always stops

Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- **Equivalence, validity, satisfiability**
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution

Logical Equivalence

- Two sentences are logically equivalent iff true in same models:
- α ≡ ß iff α ╞ β and β ╞ α

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Validity and Satisfiability

- A sentence is valid if it is true in all models,
  - e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$
  - a.k.a. Tautologies
- Validity is connected to inference via the Deduction Theorem:
  - $KB \models \alpha$ if and only if ($KB \Rightarrow \alpha$) is valid
  - Every valid implication sentence describes a legitimate inference
- A sentence is satisfiable if it is true in some model
  - e.g., $A \vee B$
- A sentence is unsatisfiable if it is true in no models
  - e.g., $A \wedge \neg A$
- Satisfiability is connected to inference via the following:
  - $KB \models \alpha$ if and only if ($KB \wedge \neg \alpha$) is unsatisfiable
  - Proof by contradiction
  - Assume α is false and show this causes a contradiction in KB

Proof Methods

- Proof methods divide into (roughly) two kinds:
  - Natural Deduction: Application of inference rules
    - Legitimate (sound) generation of new sentences from old
    - Proof = a sequence of inference rule applications
      Can use inference rules as operators in a standard search algorithm
    - Typically require transformation of sentences into a normal form
  - Model checking
    - truth table enumeration (always exponential in *n*)
    - improved backtracking, e.g., Davis--Putnam-Logemann-Loveland (DPLL)
    - heuristic search in model space (sound but incomplete)
      e.g., min-conflicts like hill-climbing algorithms

Resolution

Conjunctive Normal Form (CNF)

conjunction of disjunctions of literals

clauses

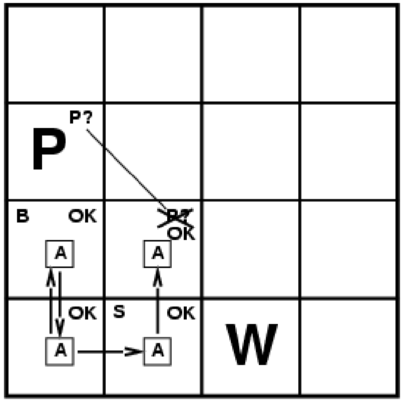
E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

- Resolution inference rule (for CNF):

$$\frac{\ell_i \vee \dots \vee \ell_k, \qquad\qquad m_1 \vee \dots \vee m_n}{\ell_i \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

where ℓ_i and m_j are complementary literals.

E.g., $$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution

Conjunctive Normal Form (CNF)

conjunction of disjunctions of literals

clauses

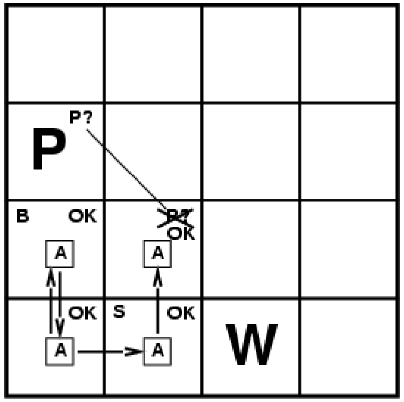
E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

- Resolution inference rule (for CNF):

$$\frac{\ell_i \lor \ldots \lor \ell_k, \qquad\qquad m_1 \lor \ldots \lor m_n}{\ell_i \lor \ldots \lor \ell_{i-1} \lor \ell_{i+1} \lor \ldots \lor \ell_k \lor m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n}$$

where ℓ_i and m_j are complementary literals.

E.g., $\dfrac{P_{1,3} \lor P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$



- Resolution is sound and complete for propositional logic

Resolution

Soundness of resolution inference rule:

$$\frac{\begin{array}{r}\neg(\ell_i \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k) \Rightarrow \ell_i \\ \neg m_j \Rightarrow (m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)\end{array}}{\neg(\ell_i \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k) \Rightarrow (m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}$$

Resolution

- Assume l_i is true
  - Then m_j is false
  - We were given $m_1 \vee \ldots \vee m_n$
  - Thus $m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n$ is true
- Assume l_i is false
  - Then m_j is true
  - We were given $l_i \vee \ldots \vee l_k$
  - Thus $l_i \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k$ is true

Conversion to CNF

- Resolution rule can derive any conclusion entailed by any propositonal knowledge base
- But only works for disjunctions of literals!
- We want to work with KBs that have statements in other forms
- What to do?

Conversion to CNF

- Fortunately, every propositional logic sentence is equivalent to a **conjunction** of **disjunctive** literals
  - Known as **Conjunctive Normal Form** (**CNF**)

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\wedge$ over $\vee$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

Resolution Algorithm

- Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable
  - Recall $KB \models \alpha$ if and only if ($KB \wedge \neg\alpha$) is unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false

    clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
    new ← { }
    loop do
        for each Ci, Cj in clauses do
            resolvents ← PL-RESOLVE(Ci, Cj)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false
        clauses ← clauses ∪ new
```

Resolution Algorithm

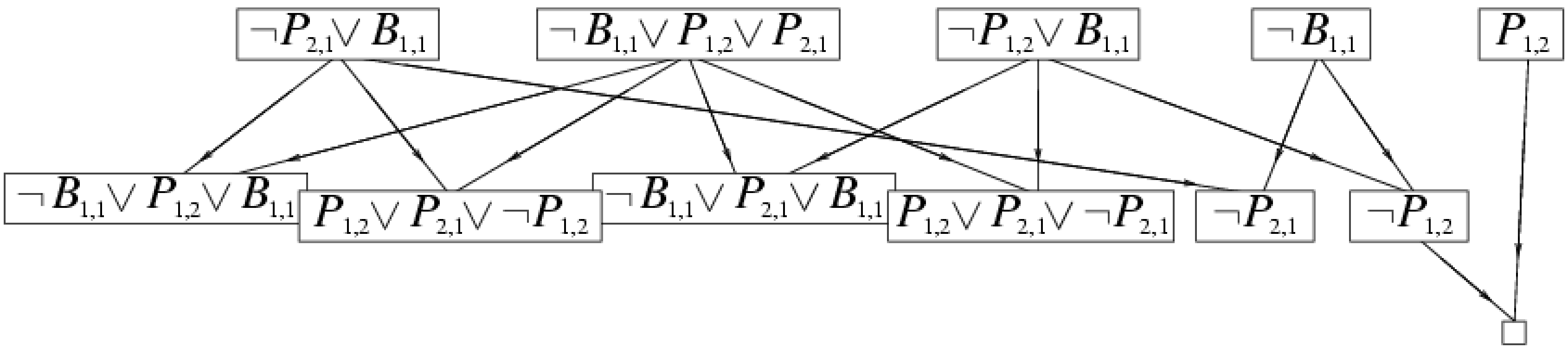
- Why is empty clause equivalent to False?

Resolution Algorithm

- Why is empty clause equivalent to False?
- Disjunction only true if one value is false
- Also, only happens if include P and not P

Resolution Example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $\alpha = \neg P_{1,2}$



Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - **Forward chaining**
  - Backward chaining
  - Resolution

Forward and Backward Chaining

- Often don't need full power of resolution because data is in Horn Form
  - KB = conjunction of Horn clauses
  - Horn clause =
    - Disjunction with **at most one** positive literal, or
    - proposition symbol, or
    - (conjunction of symbols) $\Rightarrow$ symbol
  - E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$
- Modus Ponens (for Horn Form): complete for Horn KBs

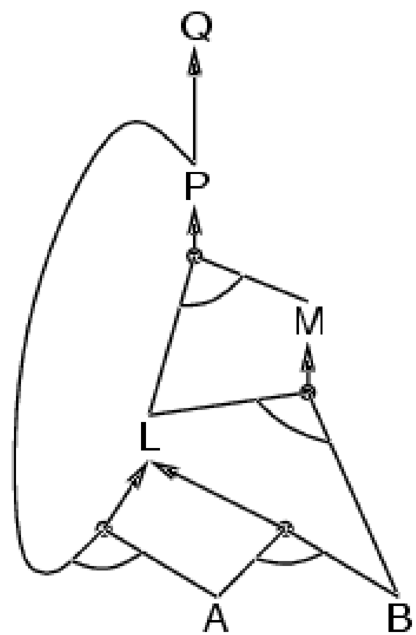
$$\frac{\alpha_1, \ldots ,\alpha_n, \qquad\qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with forward chaining or backward chaining.
- These algorithms are very natural and run in linear time

Forward Chaining

- Idea: fire any rule whose premises are satisfied in the *KB*,
  - Add its conclusion to the *KB*, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

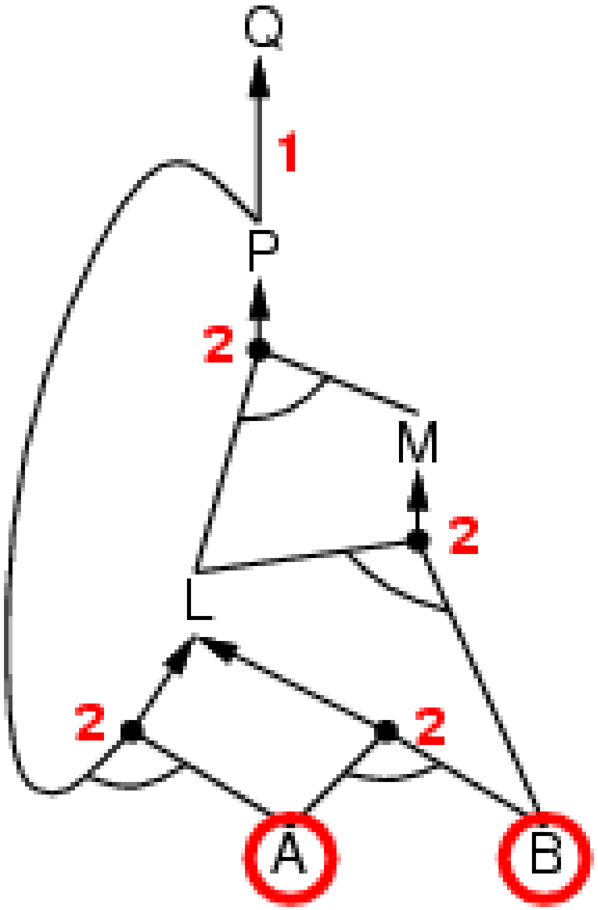


Forward Chaining Algorithm

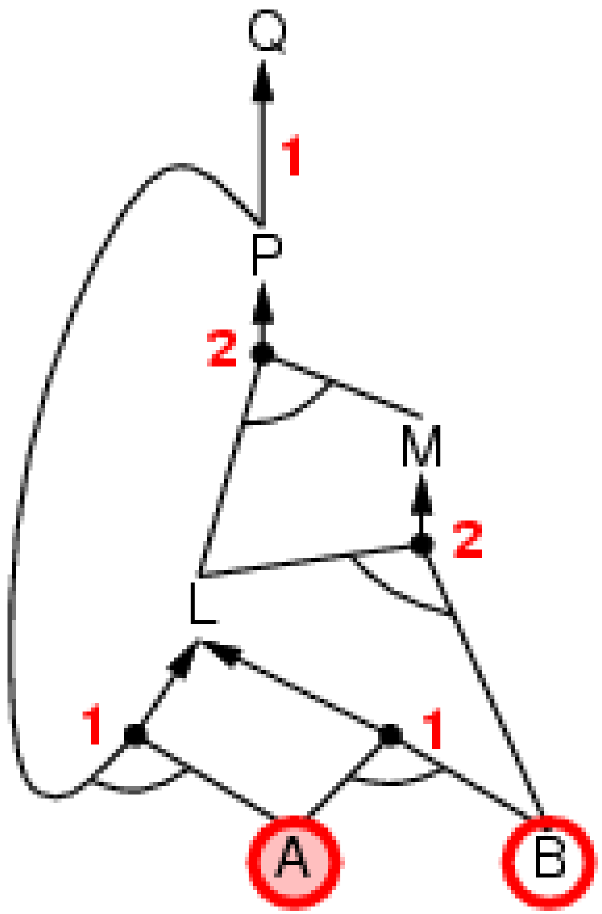
```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if HEAD[c] = q then return true
                  PUSH(HEAD[c], agenda)
  return false
```

- Forward chaining is sound and complete for Horn KB

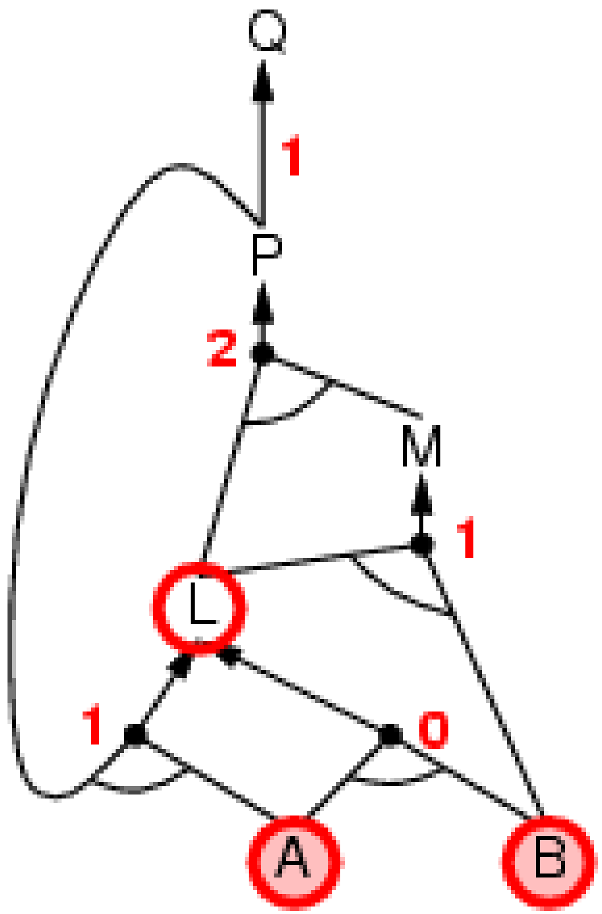
Forward Chaining Example



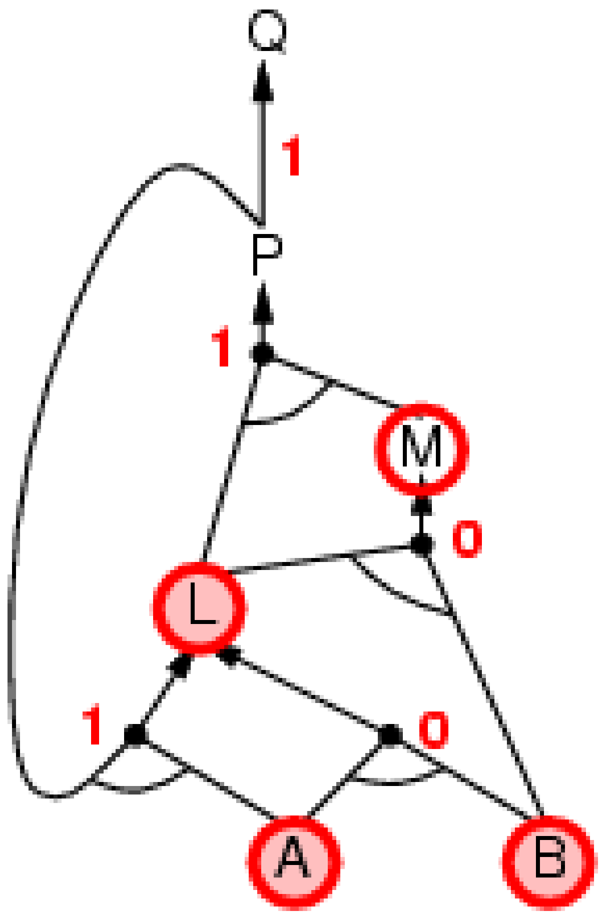
Forward Chaining Example



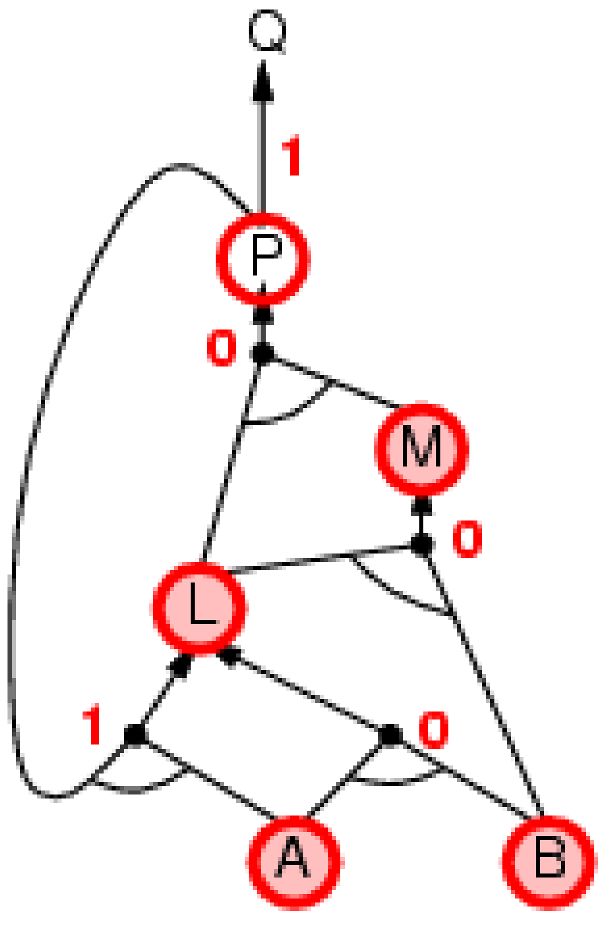
Forward Chaining Example



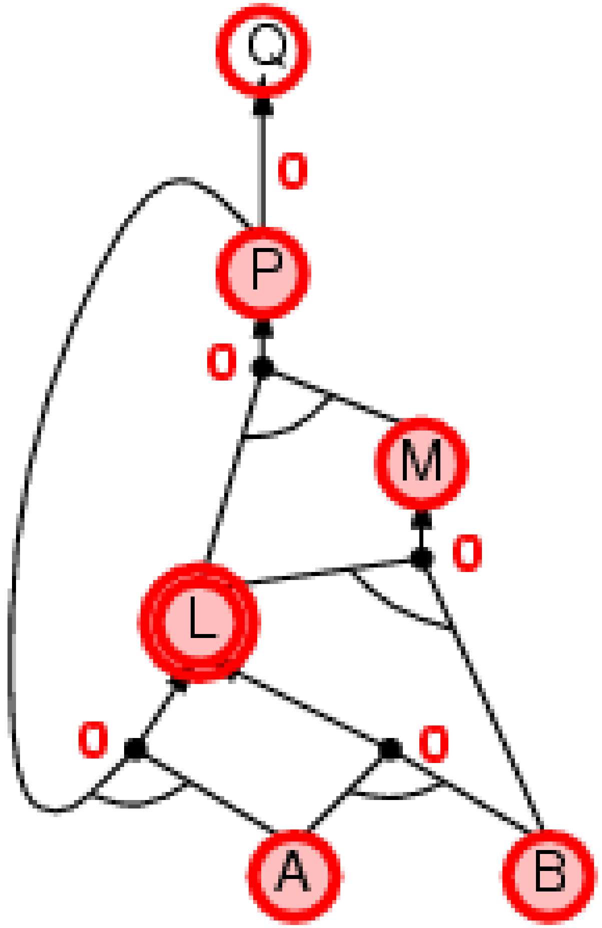
Forward Chaining Example



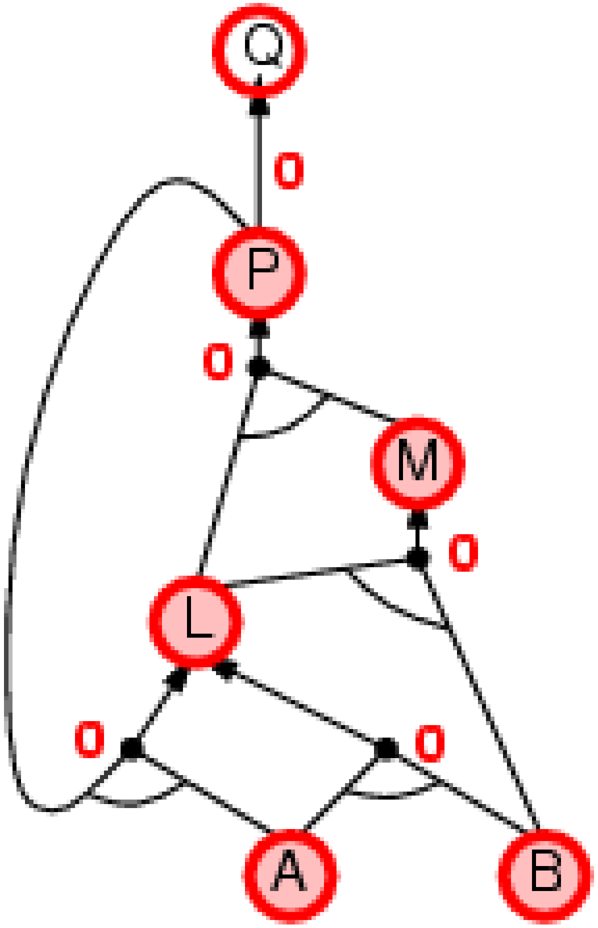
Forward Chaining Example



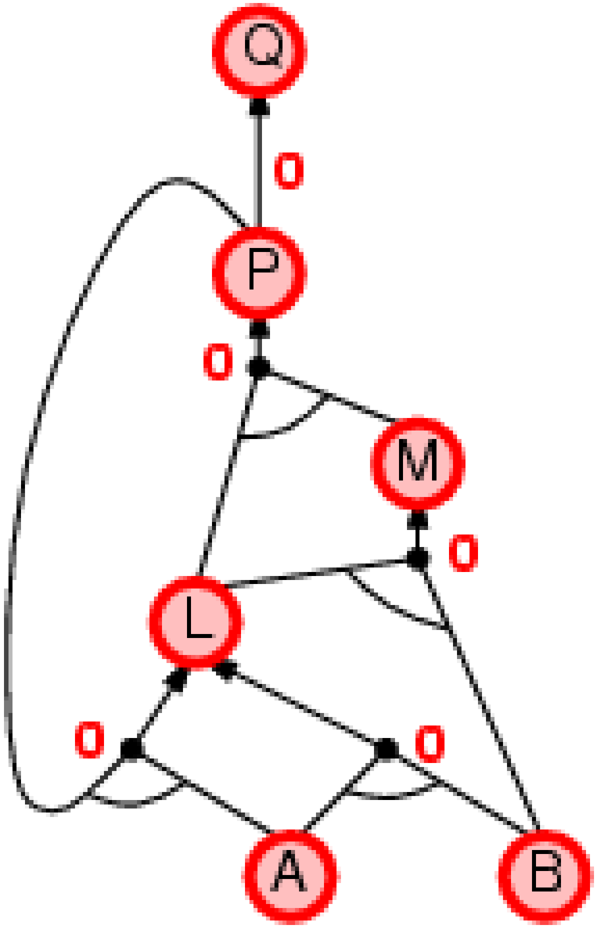
Forward Chaining Example



Forward Chaining Example



Forward Chaining Example



Proof of Completeness

- FC derives every atomic sentence that is entailed by *KB*

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model m, assigning true/false to symbols
3. Every clause in the original KB is true in m
4. Assume some $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ *is false*

   Then $a_1 \wedge \ldots \wedge a_k$ is true and is b false; contradicting 1)
5. Hence m is a model of KB
6. If $KB \models q$, q is true in every model of KB, including m

Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - **Backward chaining**
  - Resolution

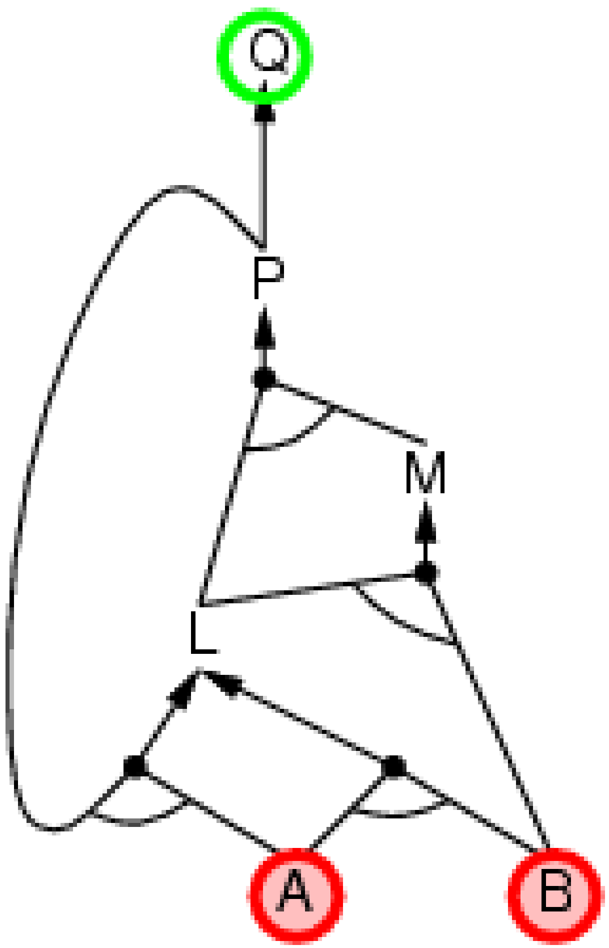
Backward Chaining

- Idea: work backwards from the query *q*
  - To prove *q* by BC:
    - check if *q* is known already, or
    - prove by BC all premises of some rule concluding *q*

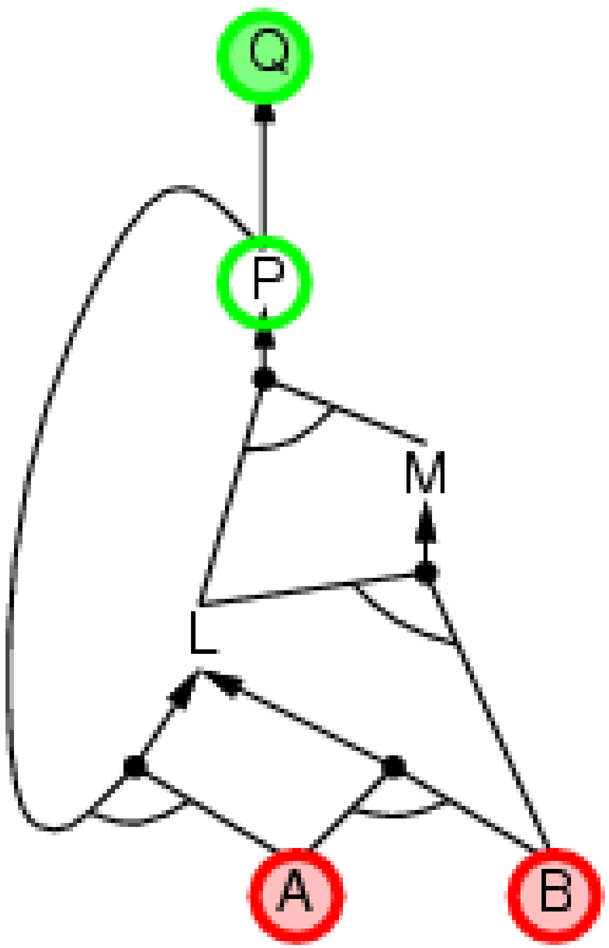
- Avoid loops: check if new subgoal is already on the goal stack

- Avoid repeated work: check if new subgoal
  - has already been proved true, or
  - has already failed

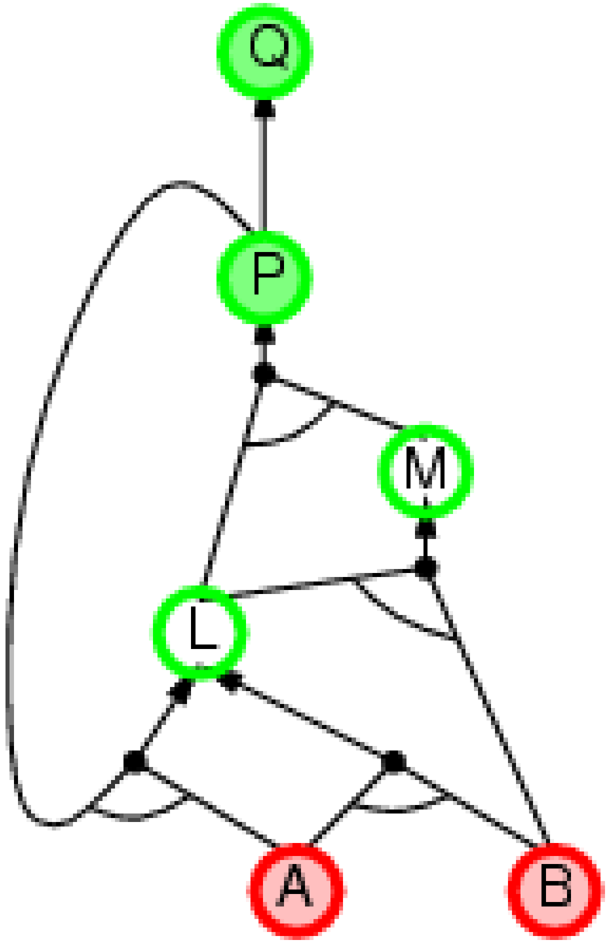
Backward Chaining Example



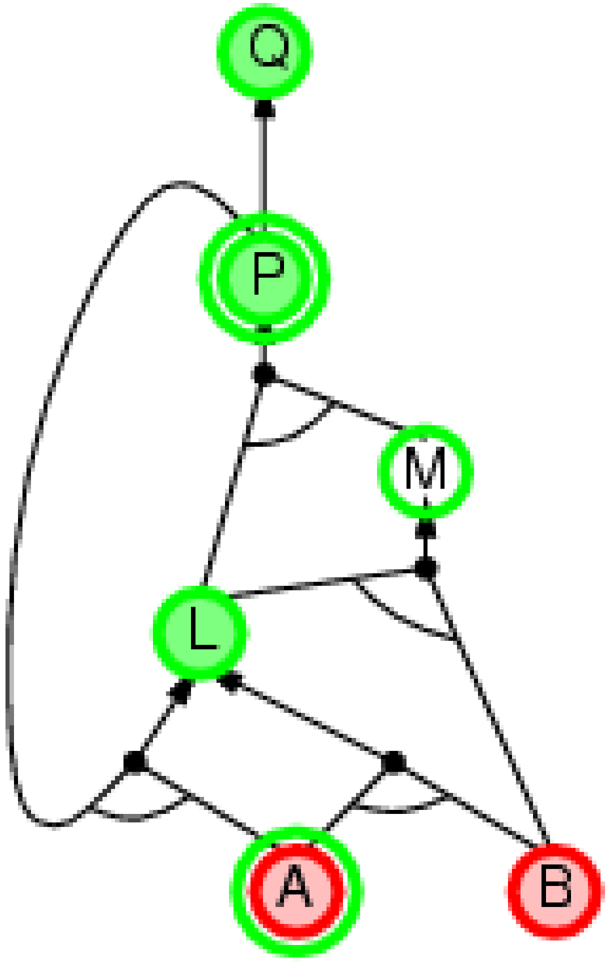
Backward Chaining Example



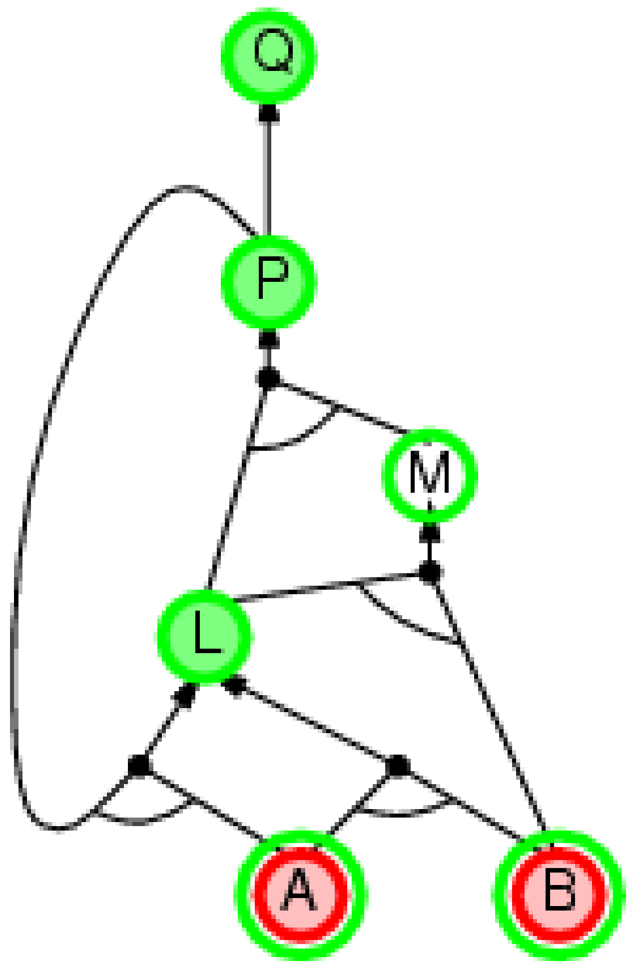
Backward Chaining Example



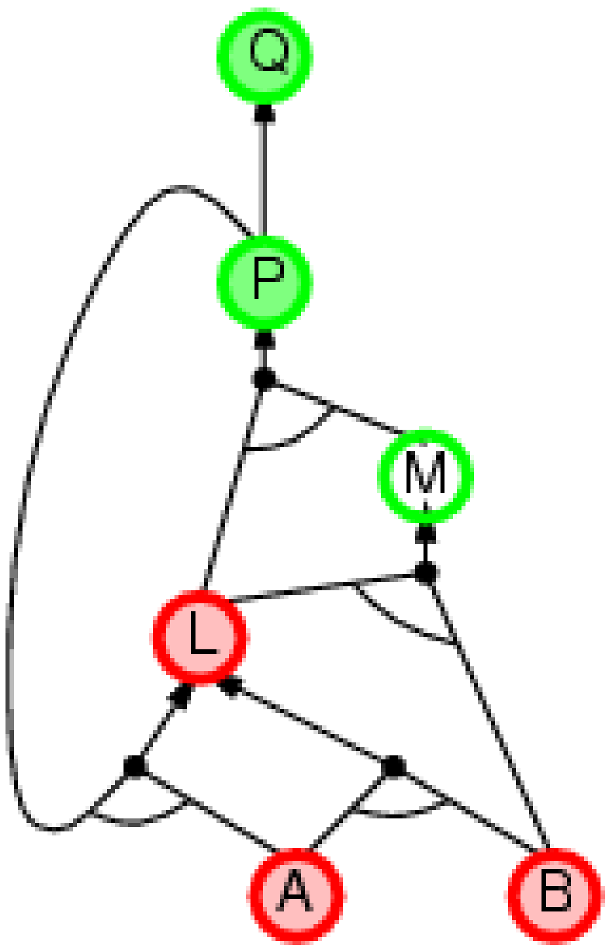
Backward Chaining Example



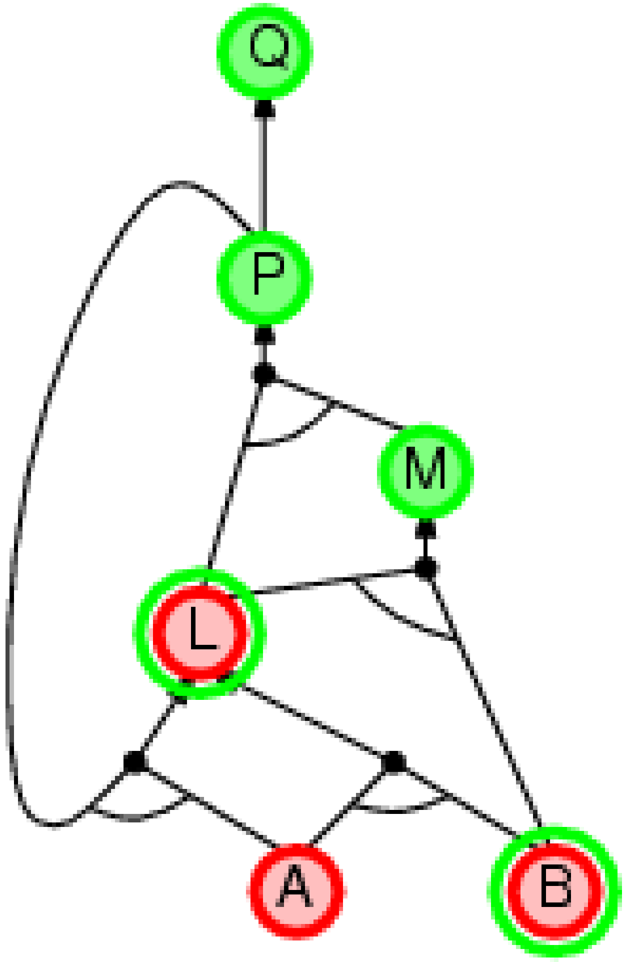
Backward Chaining Example



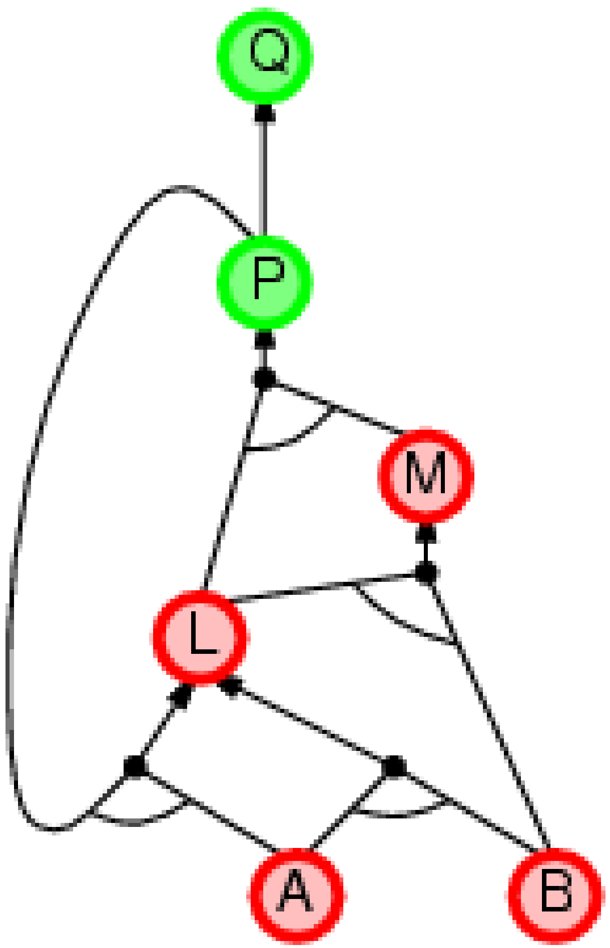
Backward Chaining Example



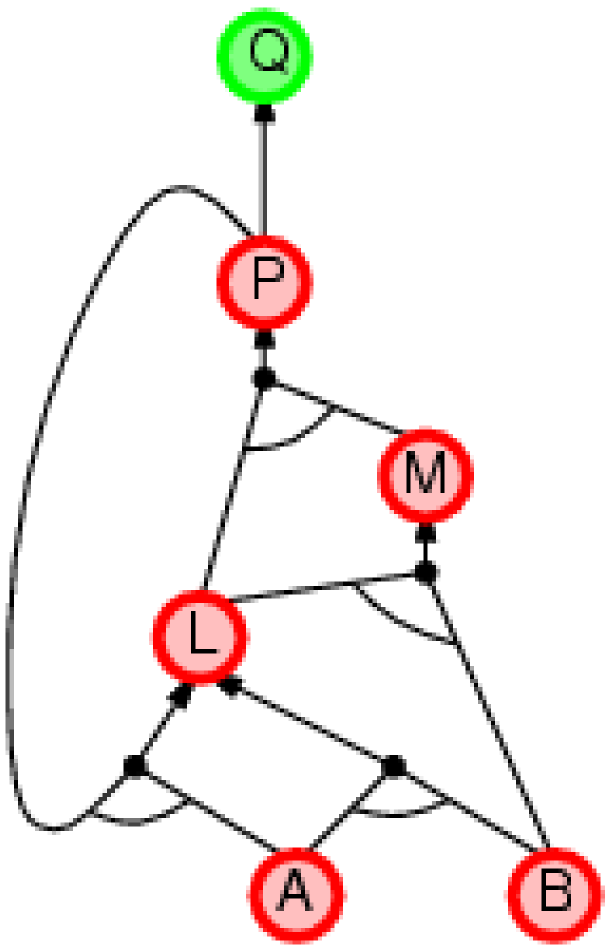
Backward Chaining Example



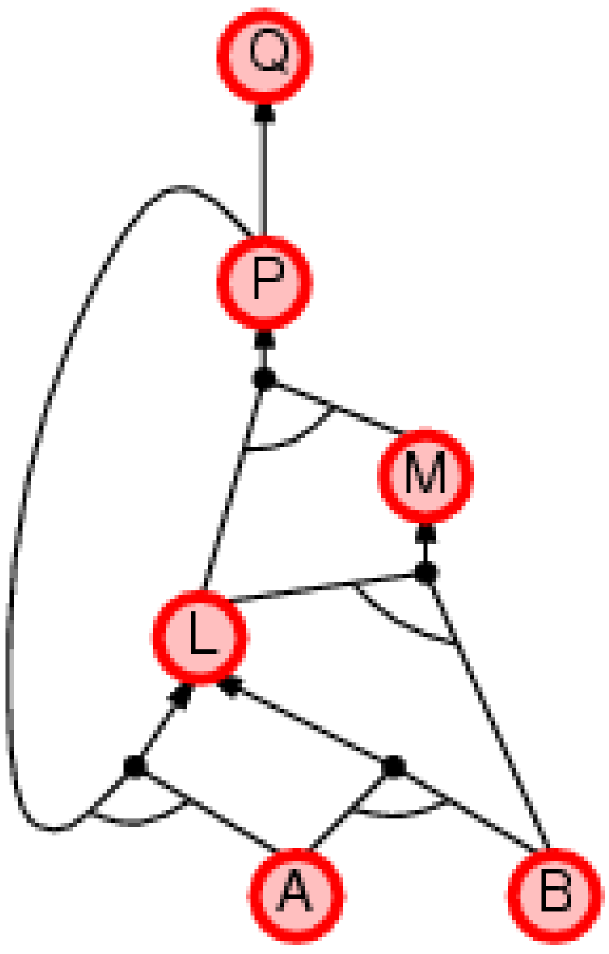
Backward Chaining Example



Backward Chaining Example



Backward Chaining Example



Forward vs. Backward Chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB
- Generally, agents share work between both
  - Limit forward reasoning to find facts that are relevant while backwards chaining

Efficient Propositional Inference

- Two families of efficient algorithms for propositional inference:
  - Complete backtracking search algorithms
    - DPLL algorithm (Davis, Putnam, Logemann, Loveland)
  - Incomplete local search algorithms
    - `WalkSAT` algorithm

The DPLL Algorithm

- Determine if an input propositional logic sentence (in CNF) is satisfiable.

- Improvements over truth table enumeration:
  1. Early termination
     - A clause is true if any literal is true.
     - A sentence is false if any clause is false.

  2. Pure symbol heuristic
     - Pure symbol: always appears with the same "sign" in all clauses.
     - e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, A and B are pure, C is impure.
     - Make a pure symbol literal true.

  3. Unit clause heuristic
     - Unit clause: only one literal in the clause
     - The only literal in a unit clause must be true.

The DPLL Algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(*clauses*, *symbols*, [ ])

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

  if every clause in *clauses* is true in *model* **then return** *true*
  if some clause in *clauses* is false in *model* **then return** *false*
  P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
  if *P* is non-null **then return** DPLL(*clauses*, *symbols*–*P*, [*P* = *value*|*model*])
  P, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)
  if *P* is non-null **then return** DPLL(*clauses*, *symbols*–*P*, [*P* = *value*|*model*])
  P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses*, *rest*, [*P* = *true*|*model*]) **or**
      DPLL(*clauses*, *rest*, [*P* = *false*|*model*])

The `WalkSAT` Algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

The `WalkSAT` Algorithm

function WALKSAT(*clauses*, *p*, *max-flips*) **returns** a satisfying model or *failure*
  inputs: *clauses*, a set of clauses in propositional logic
    p, the probability of choosing to do a "random walk" move
    max-flips, number of flips allowed before giving up

  model ← a random assignment of *true*/*false* to the symbols in *clauses*
  for $i = 1$ **to** *max-flips* **do**
    if *model* satisfies *clauses* **then return** *model*
    clause ← a randomly selected clause from *clauses* that is false in *model*
    with probability *p* flip the value in *model* of a randomly selected symbol
      from *clause*
    else flip whichever symbol in *clause* maximizes the number of satisfied clauses
  return *failure*

Hard Satisfiability Problems

- Consider random 3-CNF sentences. e.g.,

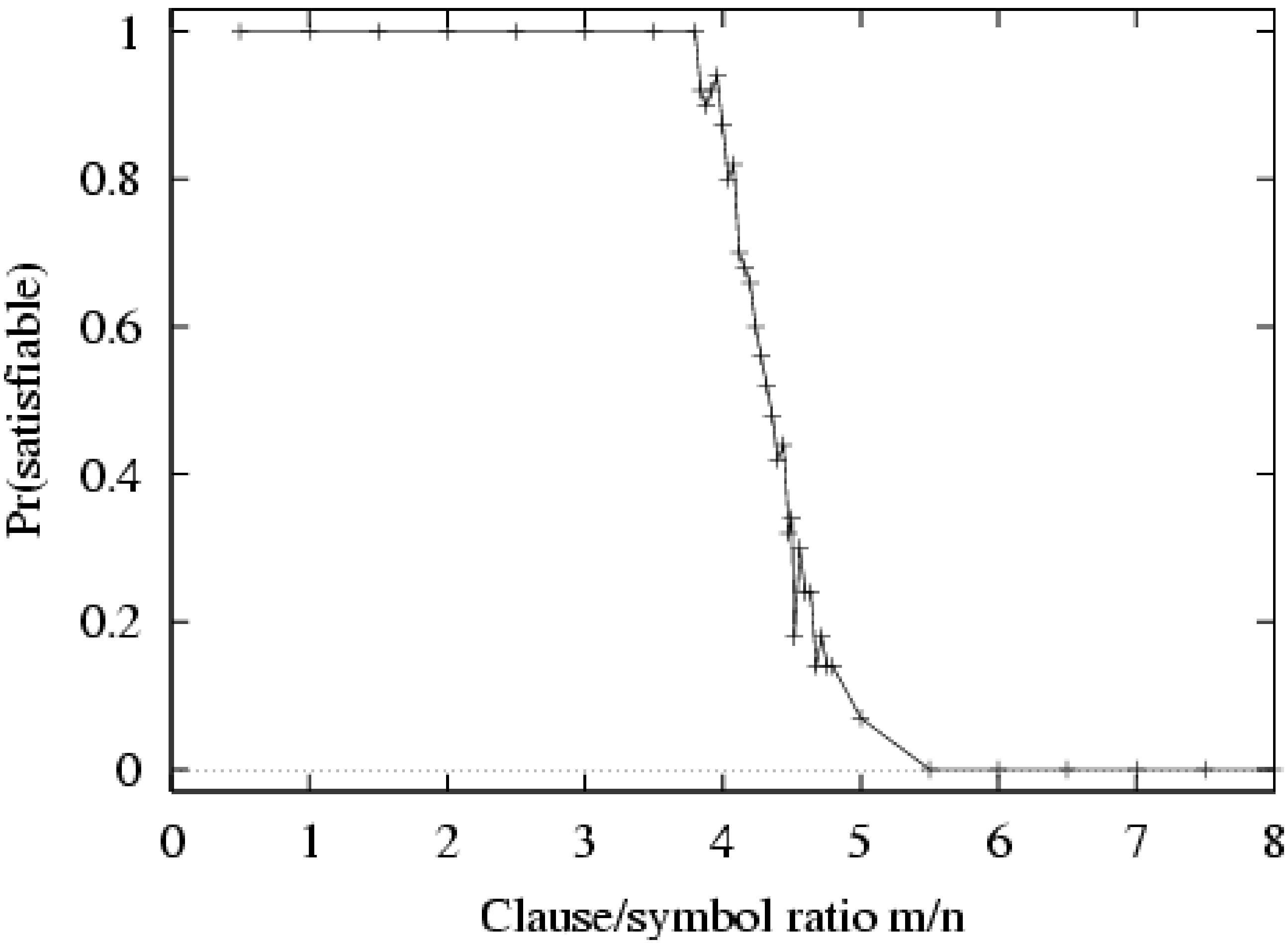
  $(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

  m = number of clauses

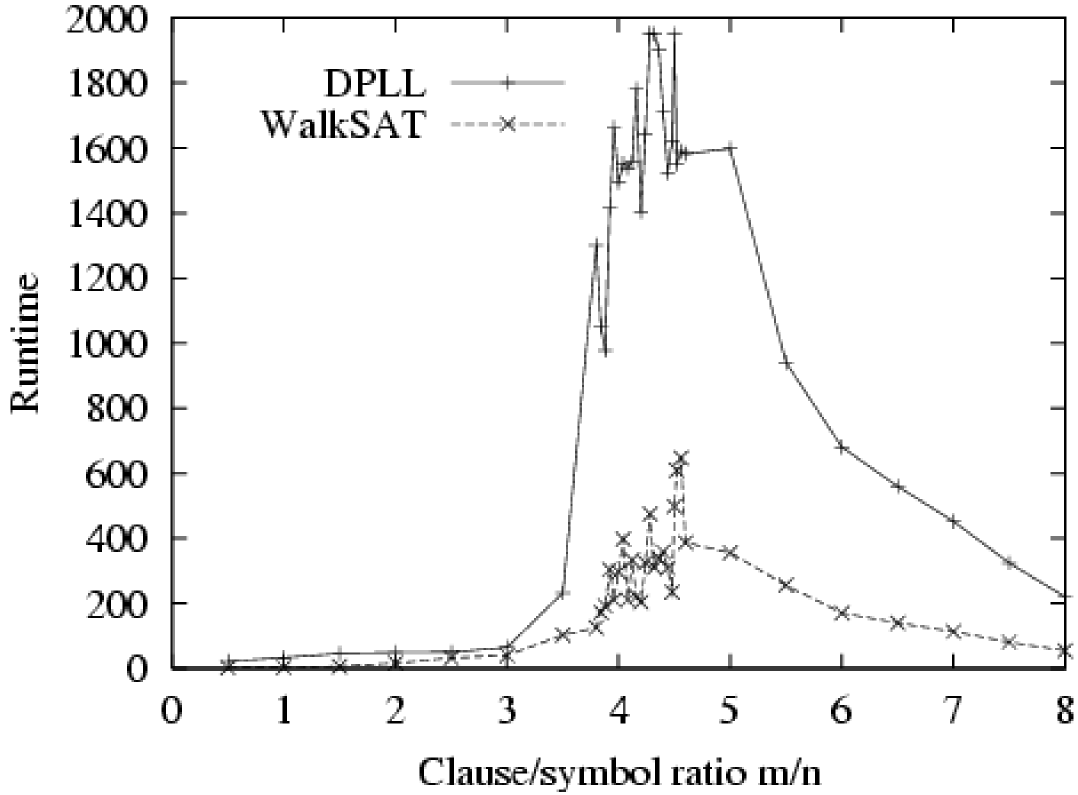
  n = number of symbols

  – Hard problems seem to cluster near $m/n = 4.3$ (critical point)

Hard Satisfiability Problems



Hard Satisfiability Problems



- Median runtime for 100 satisfiable random 3-CNF sentences, $n = 50$

Inference-based Agents in The Wumpus World

- A wumpus-world agent using propositional logic:

  $\neg P_{1,1}$
  $\neg W_{1,1}$
  $B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$
  $S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$
  $W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,4}$
  $\neg W_{1,1} \vee \neg W_{1,2}$
  $\neg W_{1,1} \vee \neg W_{1,3}$
  …
- On a 4x4 board:
  - 64 distinct proposition symbols
  - 155 sentences

function PL-WUMPUS-AGENT(*percept*) **returns** an *action*
  inputs: *percept*, a list, [*stench*,*breeze*,*glitter*]
  static: *KB*, initially containing the "physics" of the wumpus world
    x, *y*, *orientation*, the agent's position (init. [1,1]) and orient. (init. *right*)
    visited, an array indicating which squares have been visited, initially *false*
    action, the agent's most recent action, initially null
    plan, an action sequence, initially empty

  update *x*,*y*,*orientation*, *visited* based on *action*
  if *stench* **then** TELL(KB, $S_{x,y}$) **else** TELL(KB, $\neg S_{x,y}$)
  if *breeze* **then** TELL(KB, $B_{x,y}$) **else** TELL(KB, $\neg B_{x,y}$)
  if *glitter* **then** *action* ← *grab*
  else if *plan* is nonempty then *action* ← POP(*plan*)
  else if for some fringe square [*i*,*j*], ASK(KB, $(\neg P_{i,j} \wedge \neg W_{i,j})$) is *true* **or**
      for some fringe square [*i*,*j*], ASK(KB, $(P_{i,j} \vee W_{i,j})$) is *false* **then do**
    plan ← A*-GRAPH-SEARCH(ROUTE-PB([*x*,*y*], *orientation*, [*i*,*j*],*visited*))
    action ← POP(*plan*)
  else *action* ← a randomly chosen move
  return *action*

Expressiveness Limitation of Propositional Logic

- KB contains "physics" sentences for every single square

- For every time *t* and every location [*x,y*],

  $L_{x,y}^{t} \wedge FacingRight^{t} \wedge Forward^{t} \Rightarrow L_{x+1,y}^{t}$

- Rapid proliferation of clauses

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - Syntax: formal structure of sentences
  - Semantics: truth of sentences w.r.t. models
  - Entailment: necessary truth of one sentence given another
  - Inference: deriving sentences from other sentences
  - Soundness: derivations produce only entailed sentences
  - Completeness: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic
  - Forward, backward chaining are linear-time, complete for Horn clauses
- Propositional logic lacks expressive power to scale well