CS W4701
Artificial Intelligence

Fall 2013

Chapter 6:

Constraint Satisfaction Problems

Jonathan Voris

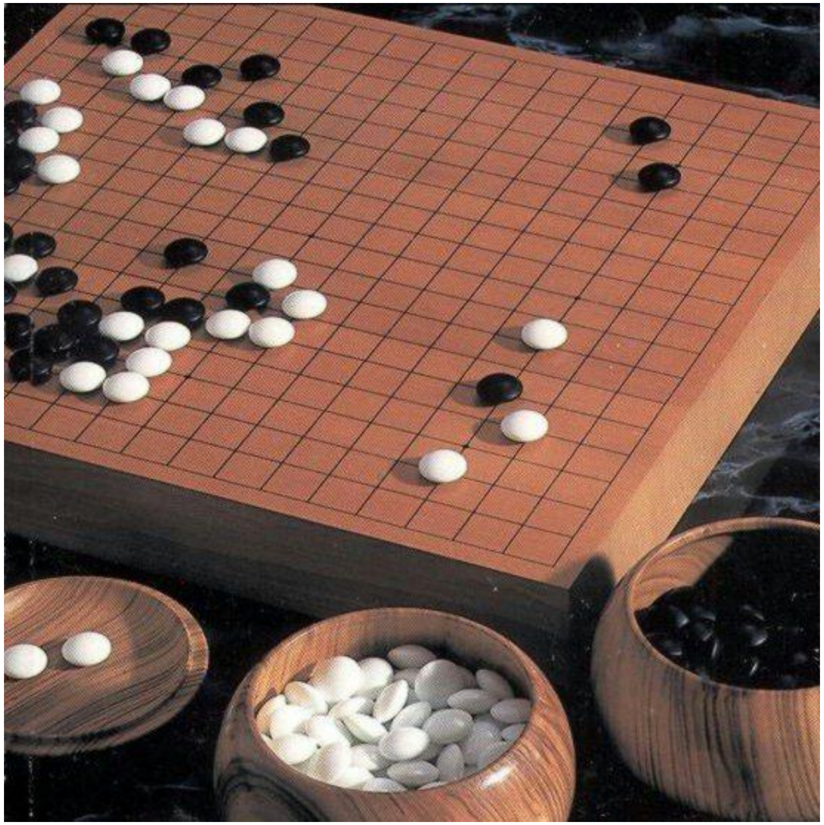
(based on slides by Sal Stolfo)

Assignment 3

- Go
  - “Encircling Game”
- Ancient Chinese game
  - Dates back
    - At least to the 4^{th} century B.C.
    - Probably to 2300 B.C.
  - Abstraction of war, or princely distraction?
- Spread to Japan by 1000 A.D.
- Immigrants brought to America in the 1800s
- German mathematician Otto Korschelt began analyzing it in the early 20^{th} century

Assignment 3

- Go gameplay
  - 19 x 19 board
  - Players take turns placing black and white stones
  - Stones are removed if surrounded by the other player's stones
- No set end condition
  - Game ends when both players pass
  - Winner has the most stones and controlled territory



Assignment 3

- Your objective is to develop a Go player agent

Assignment 3

- Your objective is to develop a Go player agent
- Any questions?

Assignment 3

- ~~Your objective is to develop a Go player agent~~
- I'M KIDDING

Assignment 3

- Go is a rare example of a game that is harder for computers than humans
  - Only recently a computer beat a human with a 9 move handicap!
- Tons of possible moves
- Extremely sequential
  - Impact of moves on future states potentially limitless
- Tricky to evaluate
- For more see
  - http://www.nytimes.com/1997/07/29/science/to-test-a-powerful-computer-play-an-ancient-game.html

Assignment 3

- Instead, you'll be working with Gomoku
  - Aka Gobang
  - Aka Five in a Row
- Same board and stones as Go
- Win condition: Precisely 5 in a row

Assignment 3

- Assignment overview:
  - Implement Gomoku playing agent using Minimax & Alpha-Beta Pruning
  - Input:
    - Board size
    - Winning chain length
    - Move time limit
- 3 game modes:
  - Play human
  - Play random
  - Play self

Assignment 3

- Due in 2.5 weeks
  - Tuesday November 19th @ 11:59:59 PM EST
- Please follow submission instructions
  - https://www.cs.columbia.edu/~jvoris/AI/notes/Assignment%20submission%20guideline-Spring11.pdf
- Submit:
  - Code File
  - Documentation File
- Submissions should run on GNU/Linux CLIC machines
  - https://www.cs.columbia.edu/~jvoris/AI/notes/simple%20Clic%20tutorial.pdf

Outline

- Constraint Satisfaction Problems (CSP)
- Backtracking search for CSPs
- Local search for CSPs

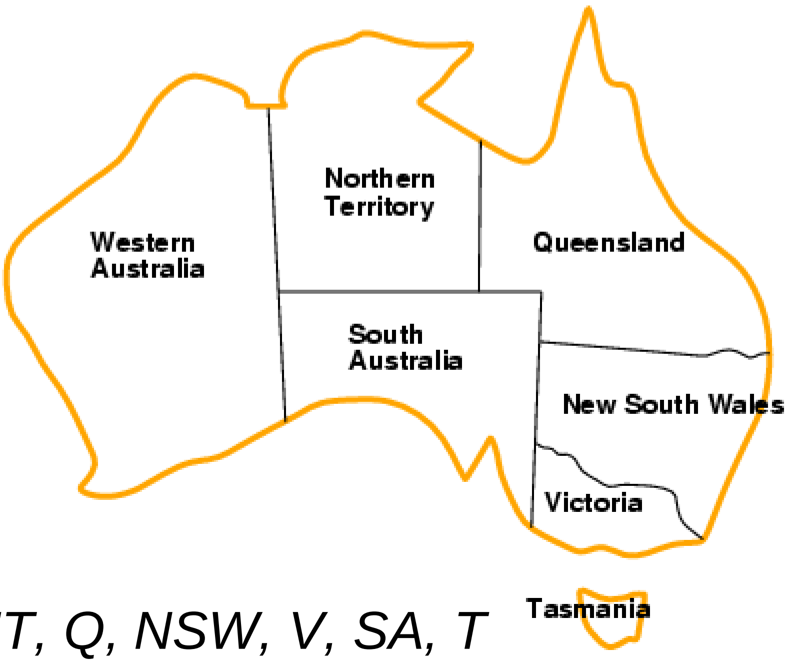
Constraint Satisfaction Problems (CSPs)

- Standard search problem:
  - **Atomic** state representation
  - **state** is a "black box" – any data structure that supports successor function, heuristic function, and goal test
- CSP:
  - **Factored** state representation
  - **state** is defined by *variables* X_i with *values* from *domain* D_i
  - **goal test** is a set of *constraints* specifying allowable combinations of values for subsets of variables
- Simple example of a *formal representation language*
- Allows useful general purpose algorithms with more power than standard search algorithms

Constraint Satisfaction Problems (CSPs)

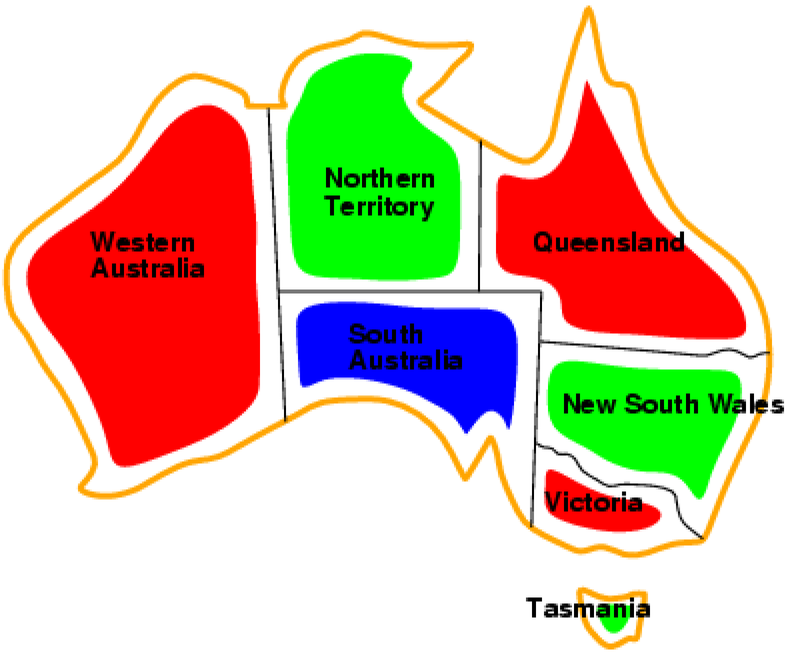
- Why CSPs?
- Natural way to formulate many problems
- Easier to apply existing CSP solver
- More efficient
  - Can greatly reduce size of search space

Example: Map Coloring



- Variables: *WA, NT, Q, NSW, V, SA, T*
- Domains: D_i = {red,green,blue}
- Constraints: adjacent regions must have different colors
  e.g., WA ≠ NT, or (WA,NT) in {(red,green),(red,blue),(green,red),(green,blue),(blue,red),(blue,green)}

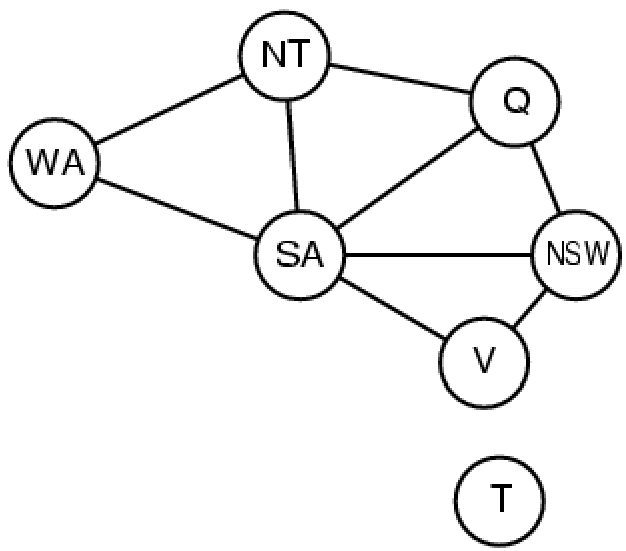
Example: Map Coloring



- **Solutions** are *complete* and *consistent* assignments, e.g., WA = red, NT = green,Q = red,NSW = green,V = red,SA = blue,T = green

Constraint Graph

- Binary CSP: each constraint relates two variables
- Constraint graph: nodes are variables, arcs are constraints



Varieties of CSPs

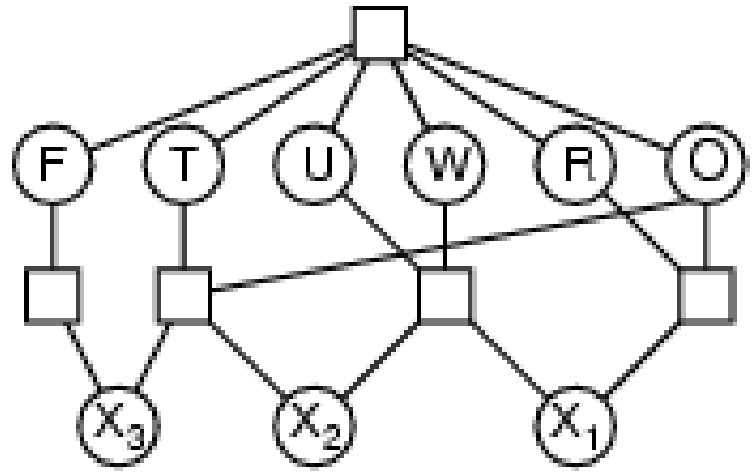
- Discrete variables
  - finite domains:
    - *n* variables, domain size *d* → $O(d^n)$ complete assignments
    - e.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - infinite domains:
    - integers, strings, etc.
    - e.g., job scheduling, variables are start/end days for each job
    - need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$
- Continuous variables
  - e.g., start/end times for Hubble Space Telescope observations
  - linear constraints solvable in polynomial time by linear programming

Varieties of Constraints

- **Unary** constraints involve a single variable
  - e.g., SA ≠ green
- **Binary** constraints involve pairs of variables
  - e.g., SA ≠ WA
- **Higher-order** constraints involve 3 or more variables
  - Aka global constraints
  - Alldiff
  - Sudoku
  - Cryptarithmetic column constraints

Example: Cryptarithmetic

```
    T W O
+   T W O
---------
  F O U R
```



- Variables: *F T U W R O* X_1 X_2 X_3
- Domains: {*0,1,2,3,4,5,6,7,8,9*}
- Constraints: *Alldiff (F,T,U,W,R,O)*
  - $O + O = R + 10 \cdot X_1$
  - $X_1 + W + W = U + 10 \cdot X_2$
  - $X_2 + T + T = O + 10 \cdot X_3$
  - $X_3 = F$, $T \neq 0$, $F \neq 0$

Real World CSPs

- Assignment problems
  - e.g., Who teaches what class?
- Timetabling problems
  - e.g., Which class is offered when and where?
- Transportation scheduling
- Factory scheduling
- Notice that many real-world problems involve real-valued variables

Standard Search Formulation (Incremental)

Let's start with the straightforward approach, then fix it

States are defined by the values assigned so far

- Initial state: the empty assignment { }
- Successor function: assign a value to an unassigned variable that does not conflict with current assignment
  → fail if no legal assignments
- Goal test: the current assignment is complete

1. This is the same for all CSPs
2. Every solution appears at depth *n* with *n* variables
   → use *depth-limited search*
3. Path is irrelevant, so can also use complete-state formulation
4. $b = (n - \ell)d$ at depth ℓ, hence $n! \cdot d^n$ leaves

Standard Search Formulation (Incremental)

- n variables
- domain size d

Standard Search Formulation (Incremental)

- n variables
- domain size d
- Branching factor:

Standard Search Formulation (Incremental)

- n variables
- domain size d
- Branching factor:
  - (n-1)d

Standard Search Formulation (Incremental)

- n variables
- domain size d
- Branching factor:
  - (n-1)d
- So number of leaves:

Standard Search Formulation (Incremental)

- n variables
- domain size d
- Branching factor:
  - (n-1)d
- So number of leaves:
  - $n!*d^n$

Standard Search Formulation (Incremental)

- n variables
- domain size d
- Branching factor:
  - (n-1)d
- So number of leaves:
  - $n!*d^n$
- But there are only d^n complete assignments!
  - What went wrong?

Backtracking Search

- Variable assignments are *commutative*, i.e.,

[WA = red then NT = green] same as [NT = green then WA = red]

  Assignment order is irrelevant

- Only need to consider assignments to a single variable at each node
  → b = d and there are d^n leaves

- Depth-first search for CSPs with single-variable assignments is called *backtracking* search

- Backtracking search is the basic uninformed algorithm for CSPs

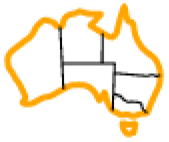
- Can solve *n*-queens for $n \approx 25$

Backtracking Search

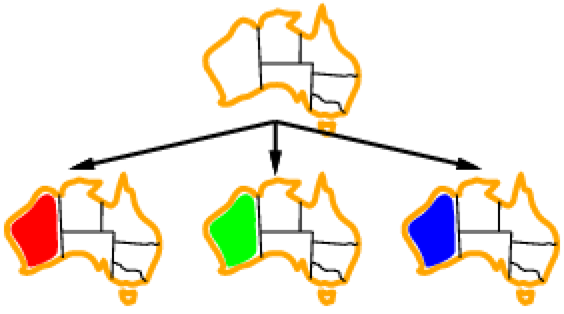
```
function BACKTRACKING-SEARCH( csp) returns a solution, or failure
   return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING( assignment, csp) returns a solution, or
failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE( Variables[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
      if value is consistent with assignment according to Constraints[csp] then
         add { var = value } to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failue then return result
         remove { var = value } from assignment
   return failure
```

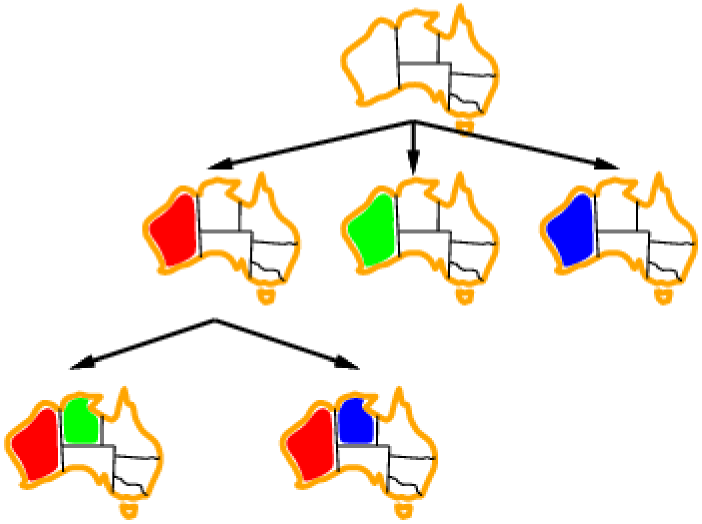
Backtracking Example



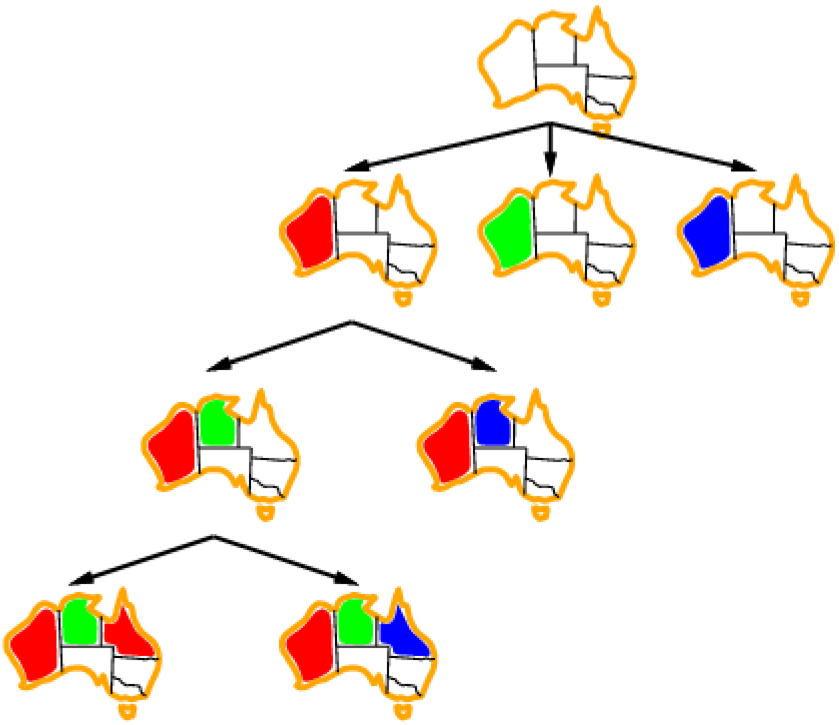
Backtracking Example



Backtracking Example



Backtracking Example



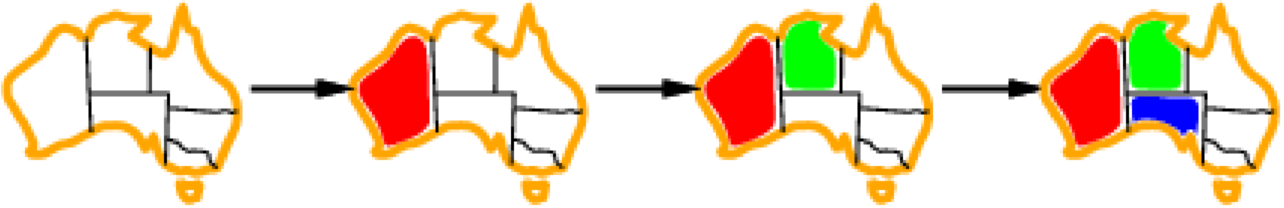
Improving Backtracking Efficiency

- **General-purpose** heuristic methods can give huge gains in speed:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can inferences be made along the way?
  - Can we detect inevitable failure early?

Most Constrained Variable

- Most constrained variable:

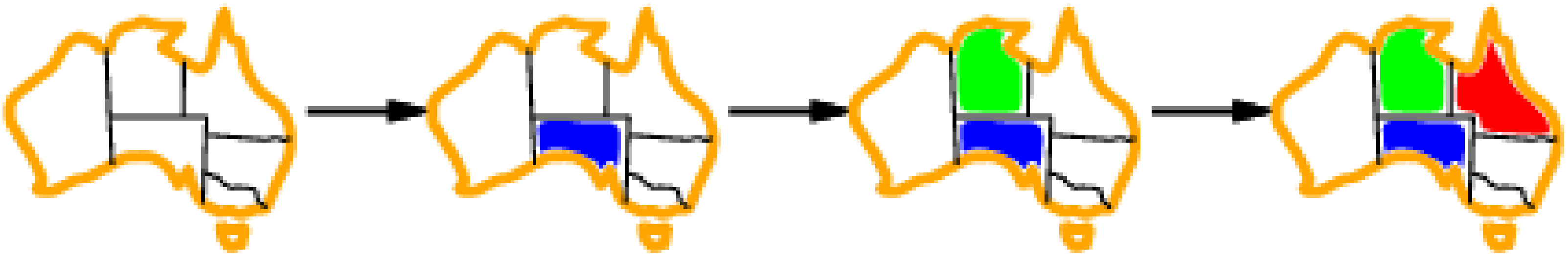
  choose the variable with the fewest legal values



- a.k.a. minimum remaining values (MRV) heuristic
- "Fail first"
  - Picking the variable most likely to cause a conflict

Most Constraining Variable

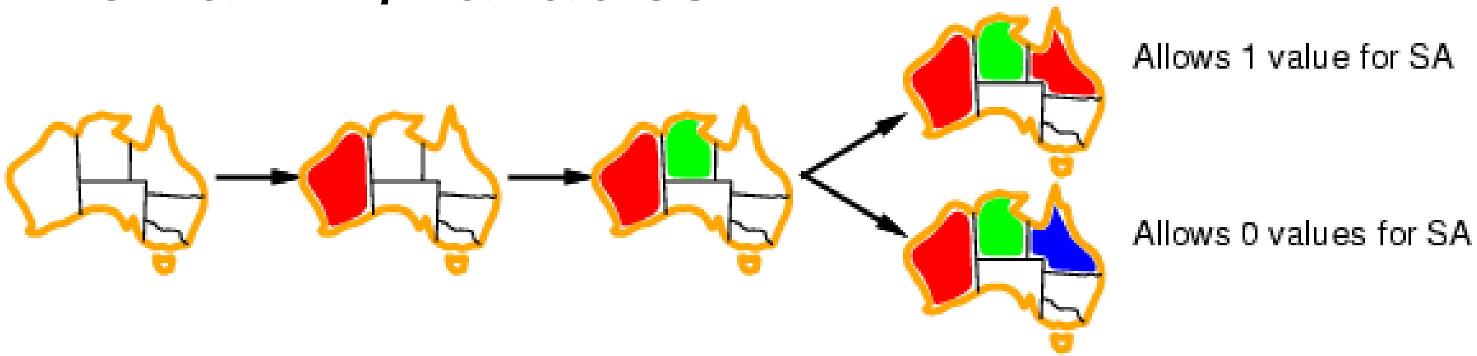
- Tie-breaker among most constrained variables
- Most constraining variable:
  - choose the variable with the most constraints on remaining variables



- aka degree heuristic

Least Constraining Value

- Given a variable, choose the least constraining value:
  - the one that rules out the fewest values in the remaining variables



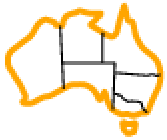
- "Fail last"
  - Selecting value least likely to cause future conflicts

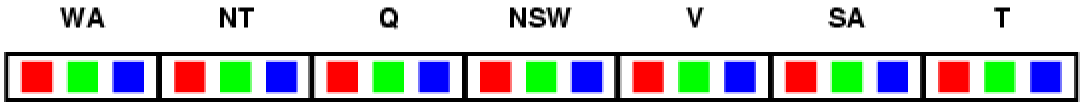
Improving Backtracking Efficiency

- Why fail first when selecting variables?
  - Prunes large portions of tree early on
- Why fail last when selecting values?
  - Only need one solution, so examine probable values first

Forward Checking

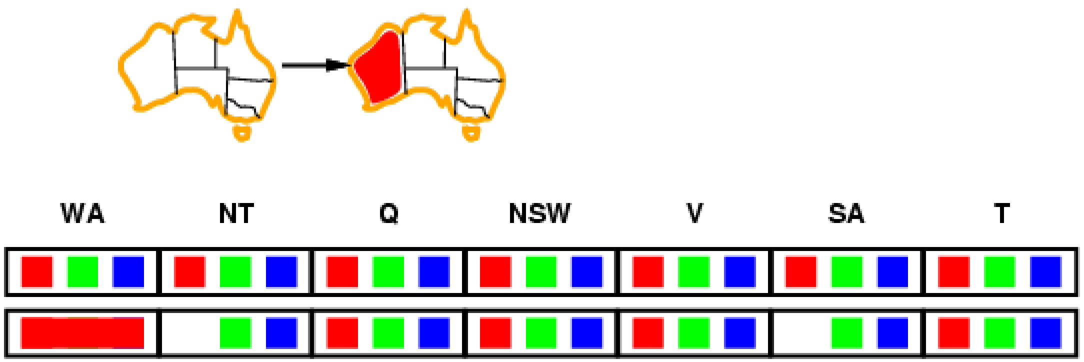
- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values





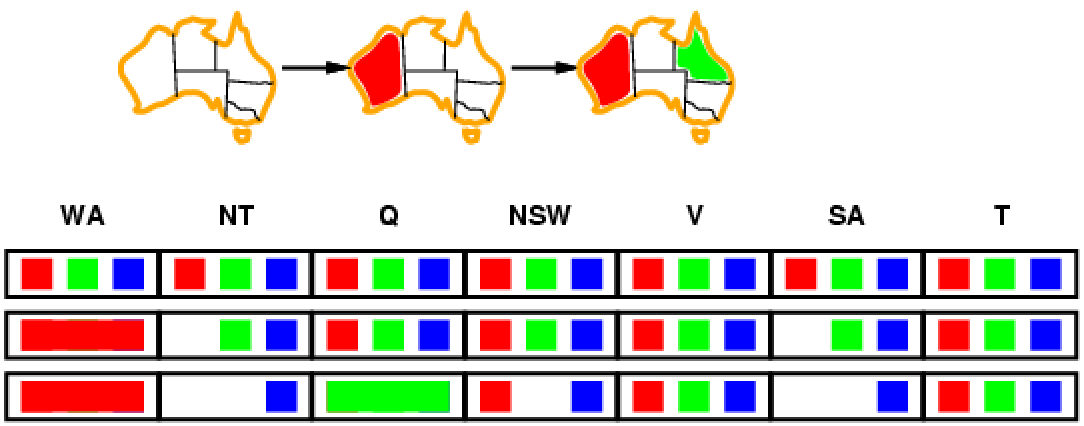
Forward Checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values



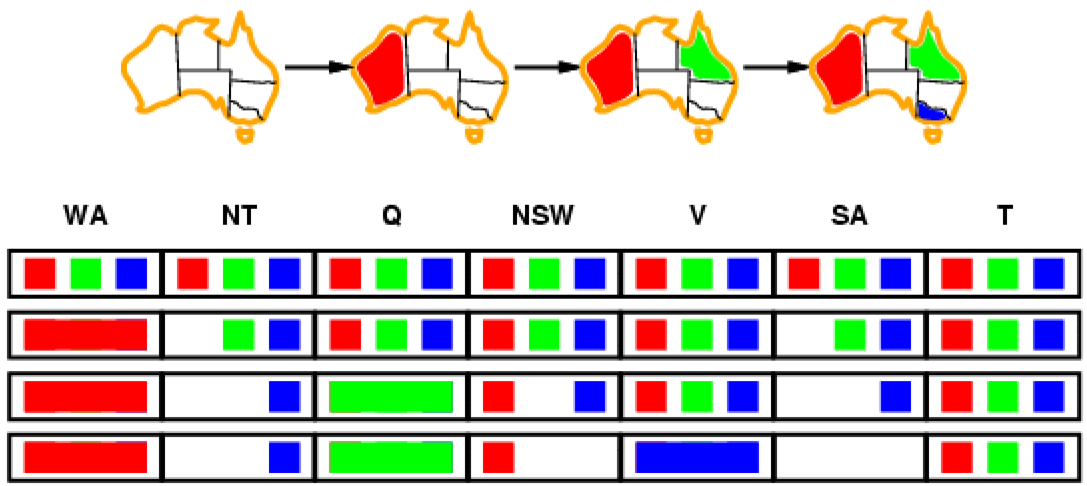
Forward Checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values



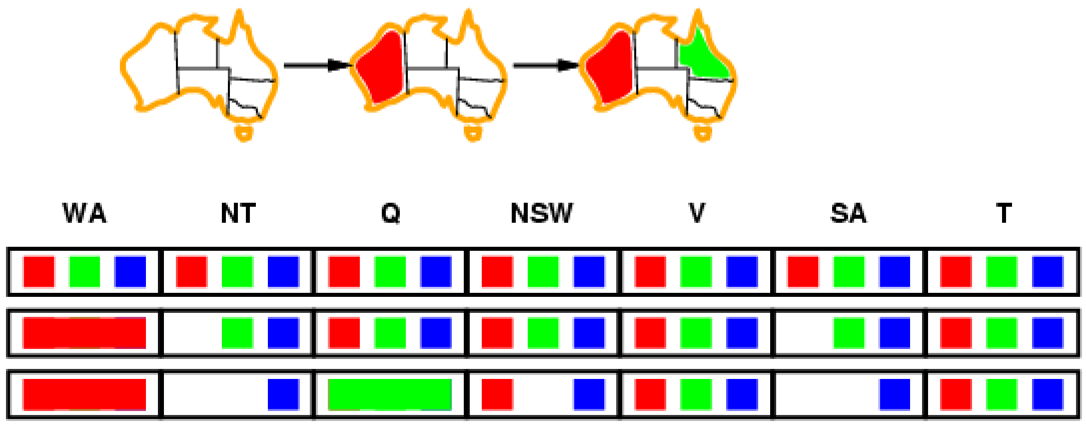
Forward Checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values



Constraint Propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

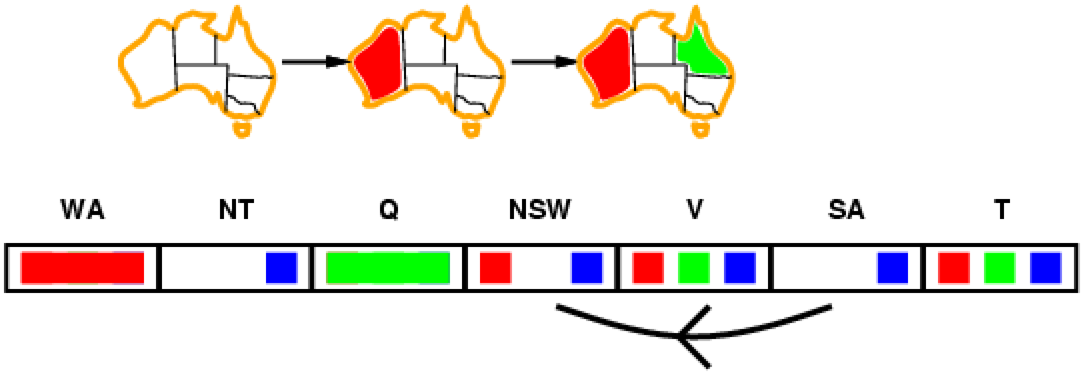


- NT and SA cannot both be blue!
- **Constraint propagation** repeatedly enforces constraints locally

Arc Consistency

- Simplest form of propagation makes each arc **consistent**
- *X* → *Y* is consistent iff

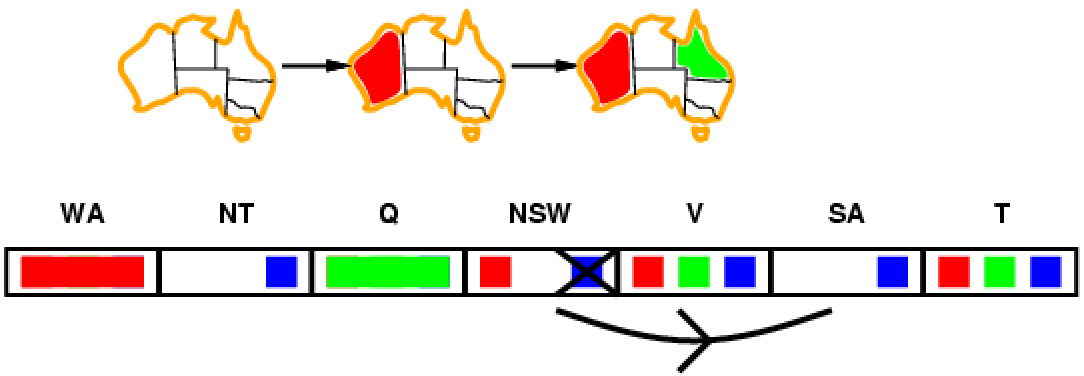
  for *every* value *x* of *X* there is *some* allowed *y*



Arc Consistency

- Simplest form of propagation makes each arc **consistent**
- *X* → *Y* is consistent iff

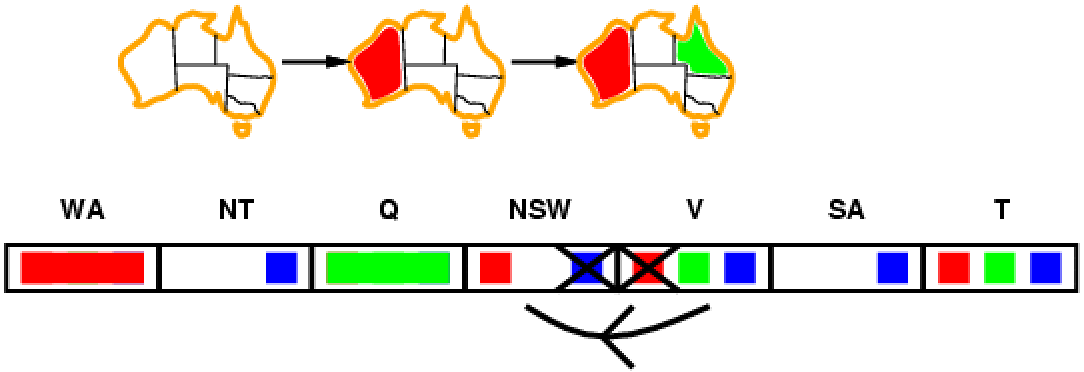
  for *every* value *x* of *X* there is *some* allowed *y*



Arc Consistency

- Simplest form of propagation makes each arc **consistent**
- *X* → *Y* is consistent iff

  for *every* value *x* of *X* there is *some* allowed *y*

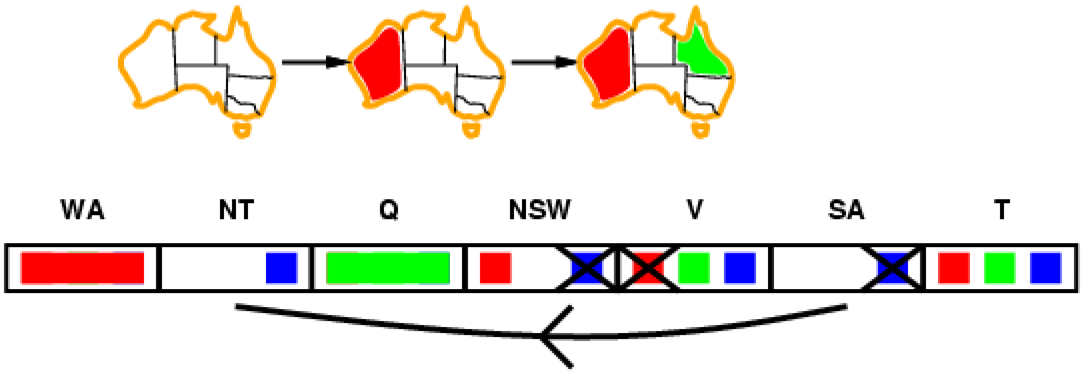


- If *X* loses a value, neighbors of *X* need to be rechecked

Arc Consistency

- Simplest form of propagation makes each arc **consistent**
- *X* → *Y* is consistent iff

  for *every* value *x* of *X* there is *some* allowed *y*



- If *X* loses a value, neighbors of *X* need to be rechecked
- Arc consistency detects failure earlier than forward checking
  - Like forward checking, but recursively applies constraints
- Can be run as a preprocessor or after each assignment

Arc Consistency Algorithm AC-3

function AC-3(csp) **returns** the CSP, possibly with reduced domains
  inputs: csp, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
  local variables: $queue$, a queue of arcs, initially all the arcs in csp

  while $queue$ is not empty **do**
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST($queue$)
    if RM-INCONSISTENT-VALUES(X_i, X_j) **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add (X_k, X_i) to $queue$

---

function RM-INCONSISTENT-VALUES(X_i, X_j) **returns** true iff remove a value
  $removed \leftarrow false$
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy constraint(X_i, X_j)
      then delete x from DOMAIN[X_i]; $removed \leftarrow true$
  return $removed$

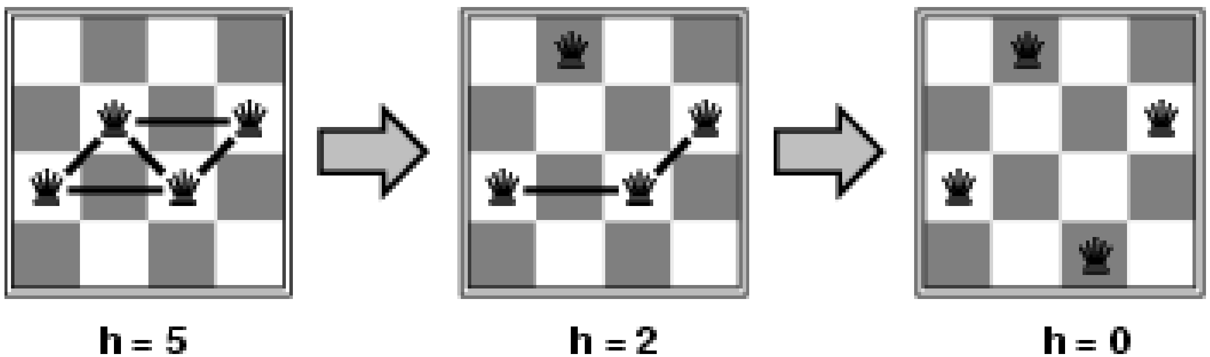
- Time complexity: $O(n^2d^3)$

Local Search for CSPs

- Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned
- To apply to CSPs:
  - Allow states with unsatisfied constraints
  - Operators reassign variable values
- Variable selection: randomly select any conflicted variable
- Value selection by **min-conflicts** heuristic:
  - Choose value that violates the fewest constraints
  - i.e., hill-climb with *h(n)* = total number of violated constraints

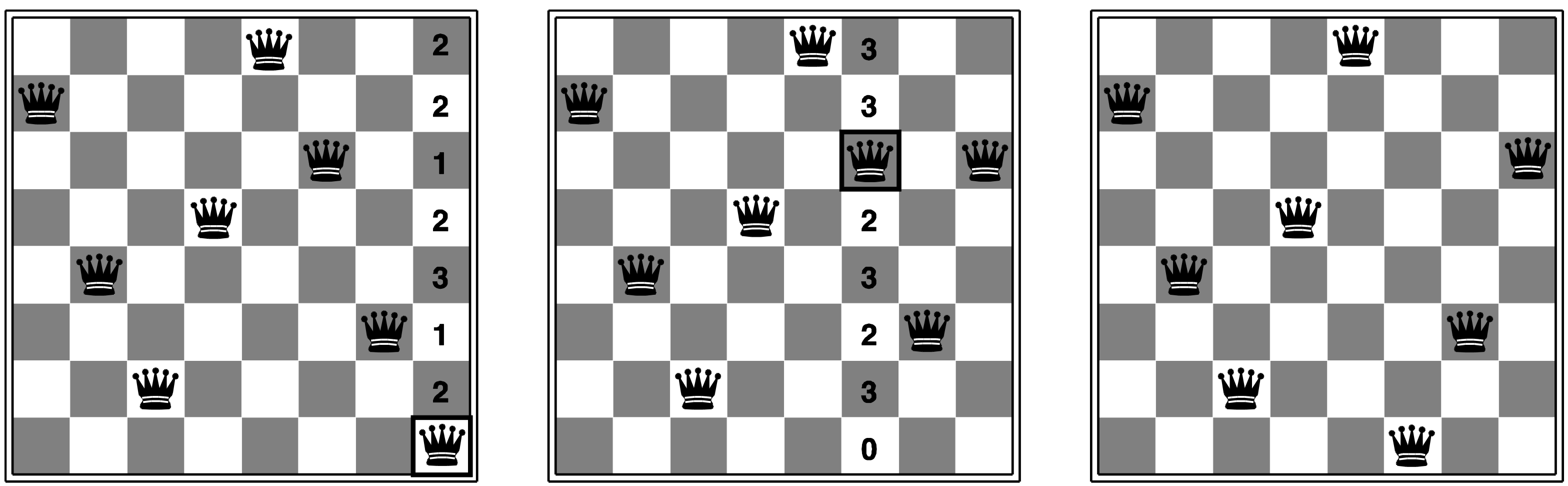
Example: 4-Queens

- <u>States</u>: 4 queens in 4 columns (4^4 = 256 states)
- <u>Actions</u>: move queen in column
- <u>Goal test</u>: no attacks
- <u>Evaluation</u>: *h(n)* = number of attacks



- Given random initial state, can solve *n*-queens in almost **constant** time for arbitrary *n* with high probability (e.g., *n* = 10,000,000)
  - 50 steps on average

Example: 8-queens



Summary

- CSPs are a special kind of problem:
  - States are factored; defined by values of a fixed set of variables
  - Goal test defined by constraints on variable values

- Backtracking
  - Depth-first search with one variable assigned per node

- Variable ordering and value selection heuristics help significantly

- Forward checking prevents assignments that guarantee later failure

- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies

- Iterative min-conflicts is usually effective in practice