CS W4701
Artificial Intelligence

Fall 2013

Chapter 3 Part 4:

Informed Search

Jonathan Voris

(based on slides by Sal Stolfo)

Announcements

- Midterm: Thursday **October 24** 2:40-3:55 PM in **Pupin 301**
- Final: Thursday **December 5th** 2:40-3:55 PM in **Pupin 301**

Summary of Uninformed Search Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Outline

- Best-first search
  - Greedy best-first search
  - A^* search
- Heuristics

Recap: Tree Search

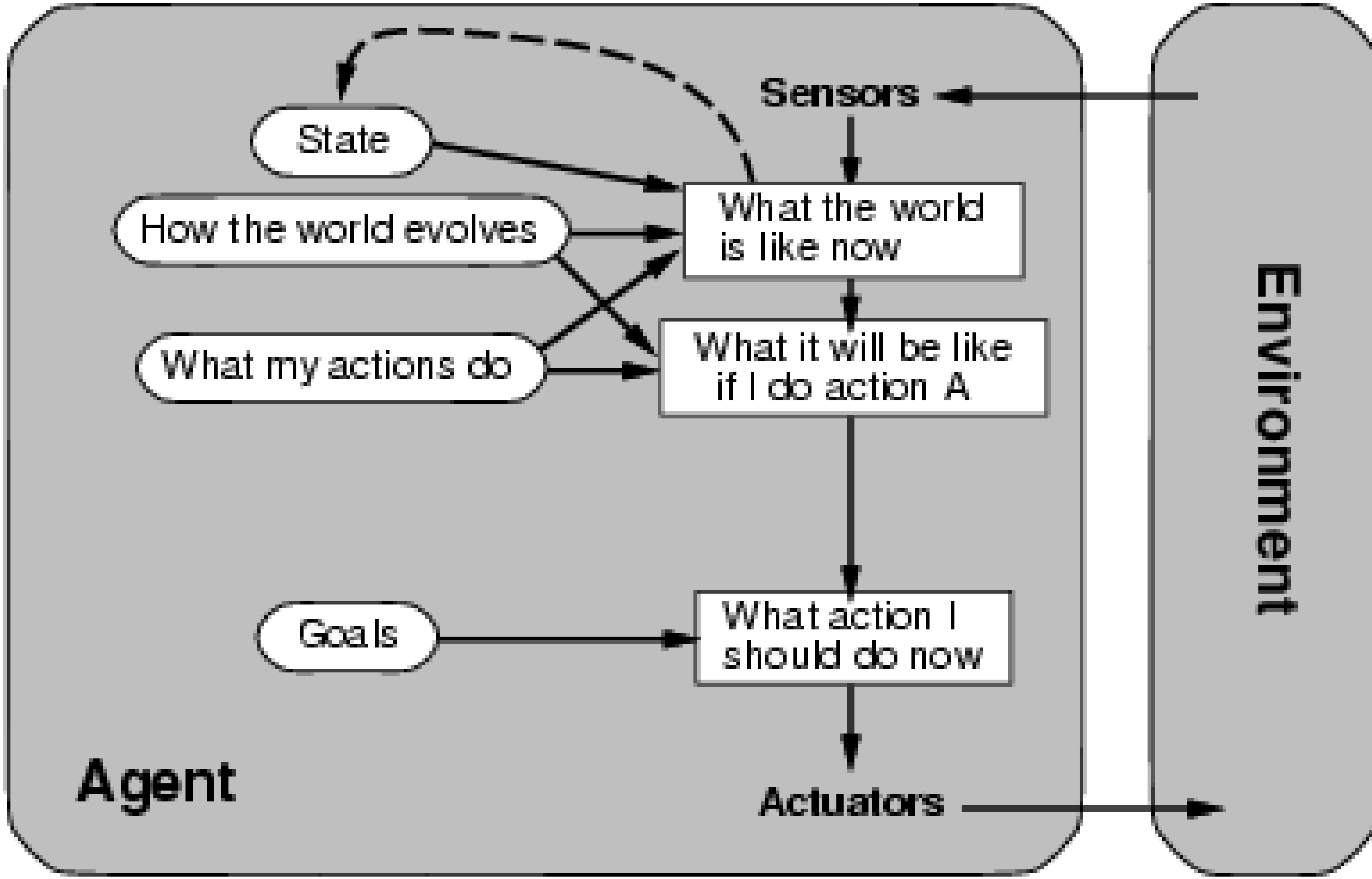
- Core concept:
  - Exploration of state space by generating successors of already-explored states (a.k.a. **expanding** states)

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

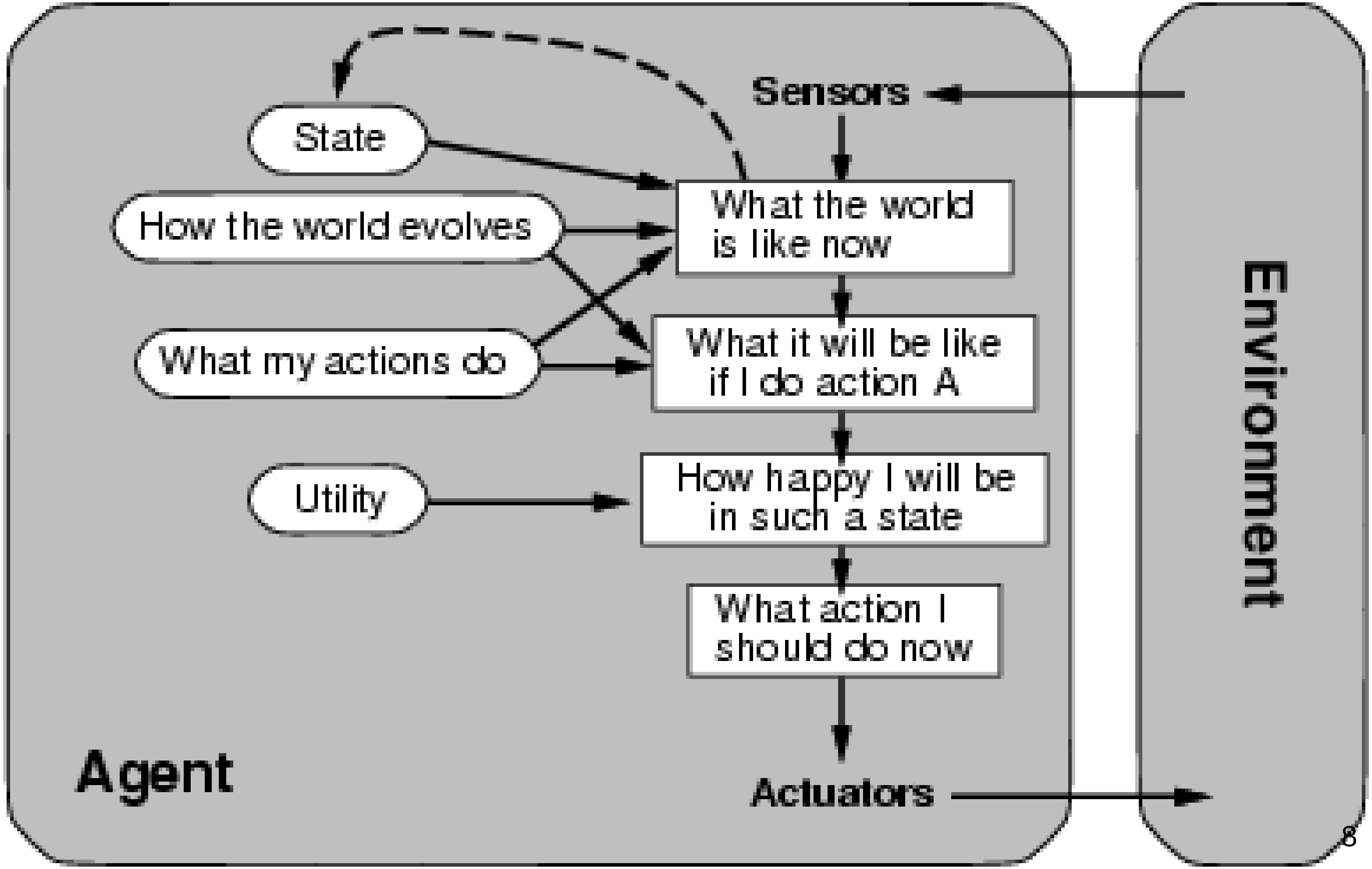
Smarter Search

- Uninformed search parameter selection: crude application of domain knowledge
  - Node ordering
  - Search strategy
- But still limited to:
  - Expand successors
  - Reached goal?
- What if we had a way to assess relative state quality?

Goal-based Agents



Utility-based Agents



Best-first Search

- Recall uniform-cost search
  - Expand node with lowest **path cost** function value g(n)
- New idea: use an **evaluation function** *f(n)* for each node
  - Estimate of "desirability“
  - Expand most desirable unexpanded node
- Implementation: Order the nodes in fringe in decreasing order of desirability
- Special cases:
  - Greedy best-first search
  - A^* search

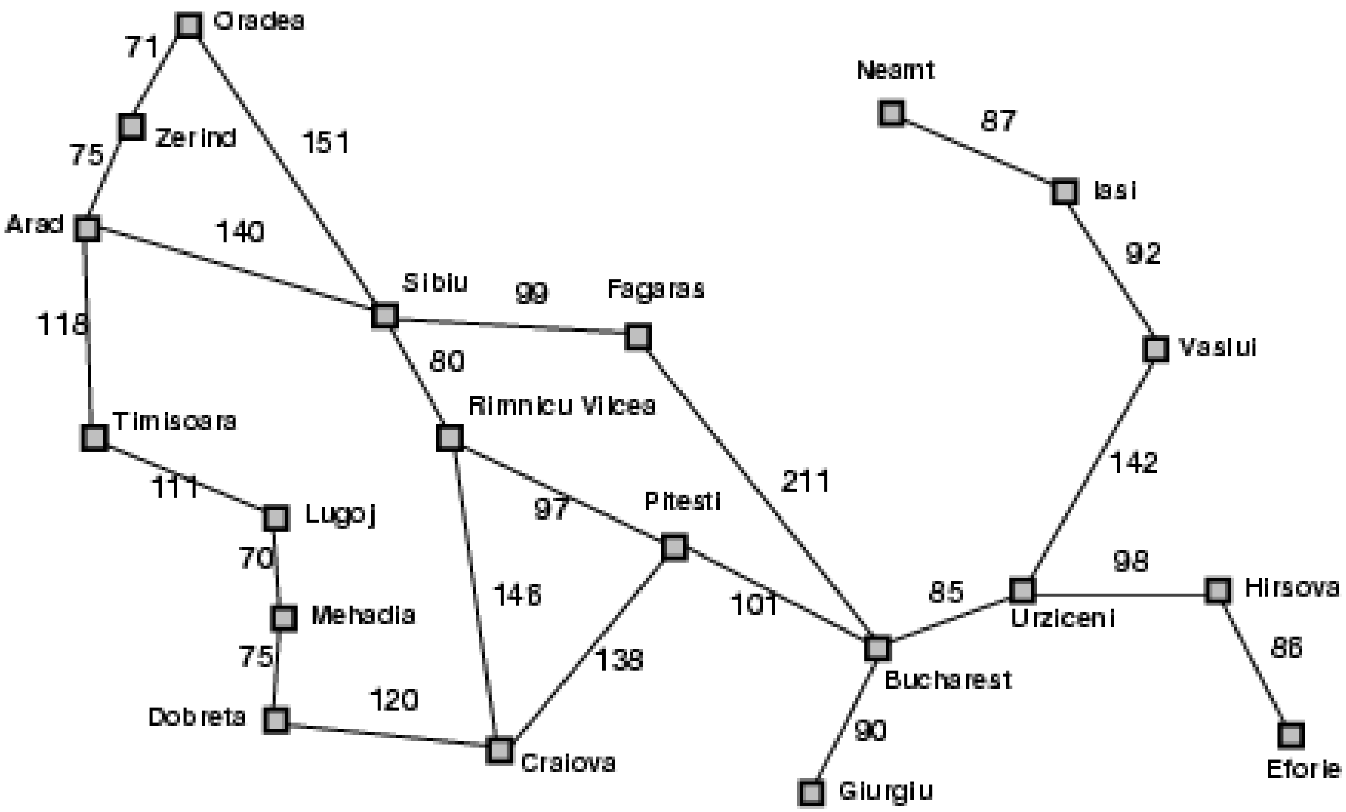
Heuristics

- f(n) presents a chicken and egg problem
  - Need to know which state is closest to goal
  - If we know that, what is the point of the agent?
- Instead, utilize domain specific knowledge to **estimate** preferable states
- Known as a **heuristic**
  - Greek word *heuriskein*: “To discover”
  - Learning aid
  - Feedback that facilitates self learning
- h(goal) = 0 always

Greedy Best-first Search

- Evaluation function $f(n) = h(n)$ (heuristic)
  - Estimate of cost from n to *goal*
- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest
- Greedy best-first search expands the node that **appears** to be closest to goal

Romania with Step Costs in km



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
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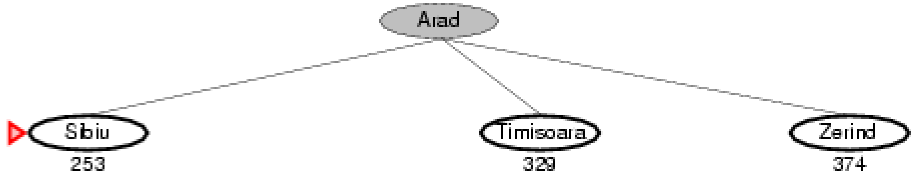
Greedy Best-first Search Example

Greedy Best-first Search Example



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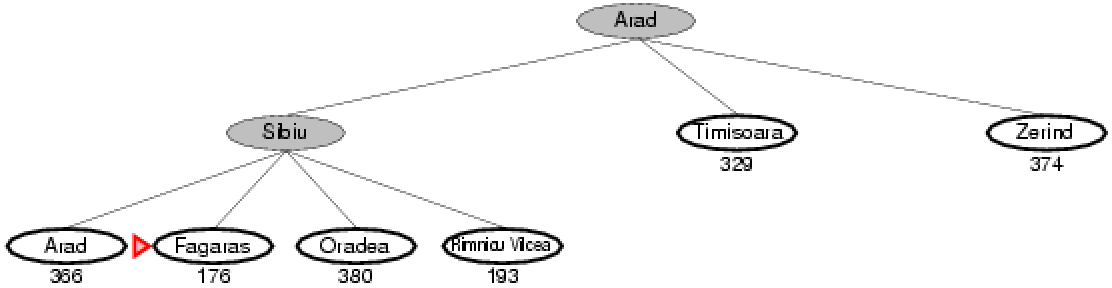

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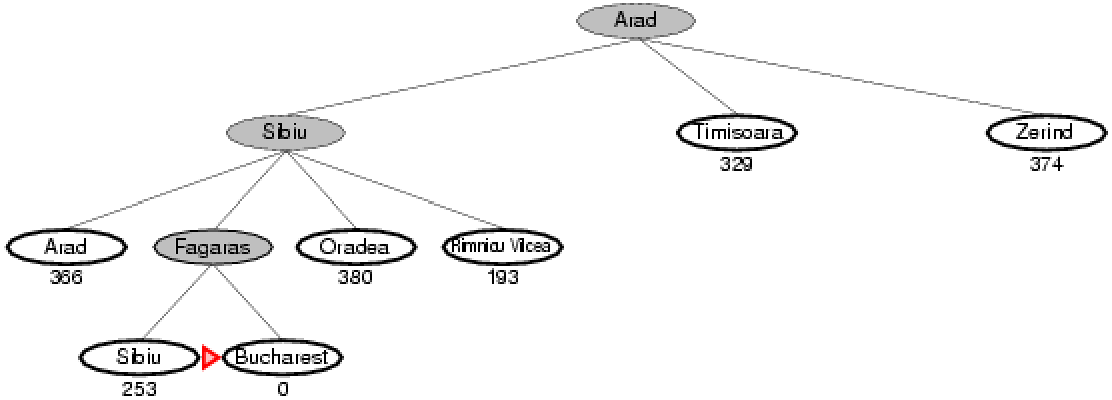


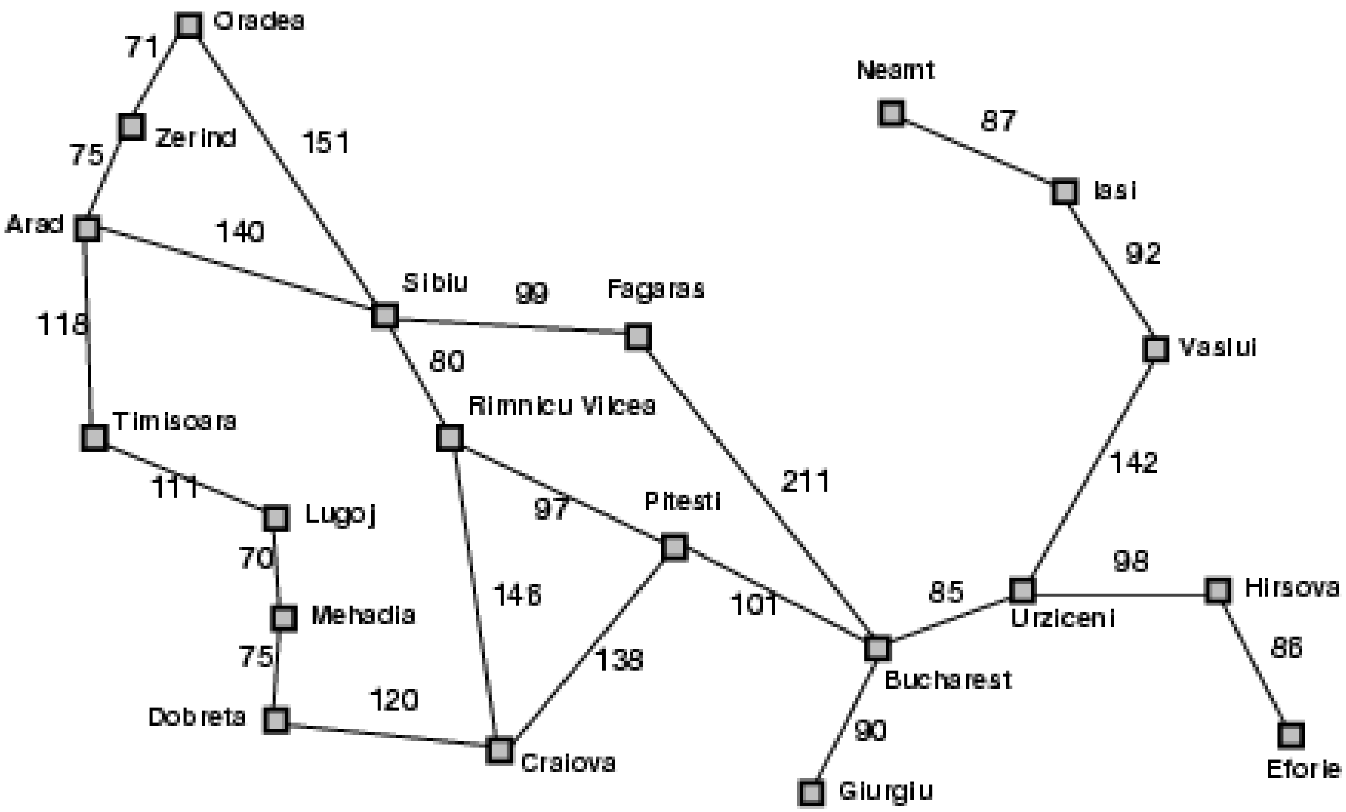
Properties of Greedy Best-first Search

- <u>Complete?</u>
  - No – Can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →…
- <u>Time?</u>
  - $O(b^m)$, but a good heuristic can give dramatic improvement
- <u>Space?</u>
  - $O(b^m)$ - keeps all nodes in memory
- <u>Optimal?</u>
  - No

A^* Search

- Idea: Avoid expanding paths that are already expensive
- Evaluation function
  - f(n) = estimated cost of cheapest path through n
  - $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal

Romania with Step Costs in km



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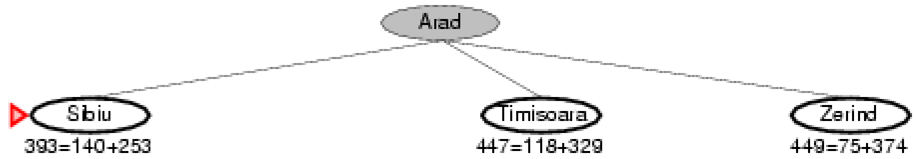
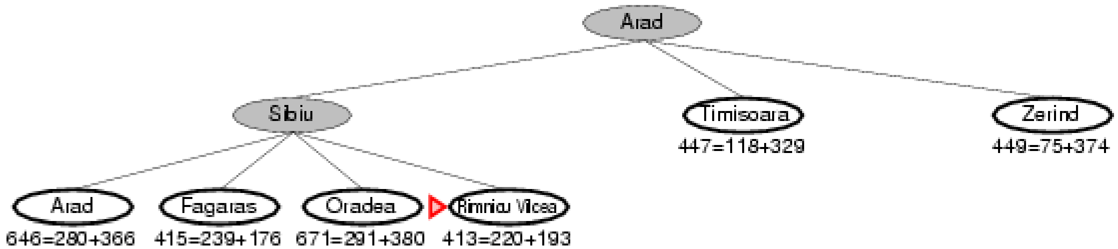
A^* Search Example

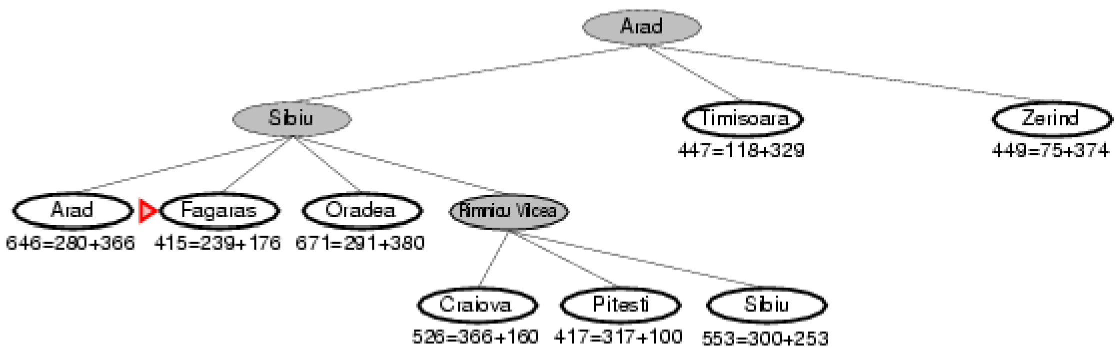
A* Search Example



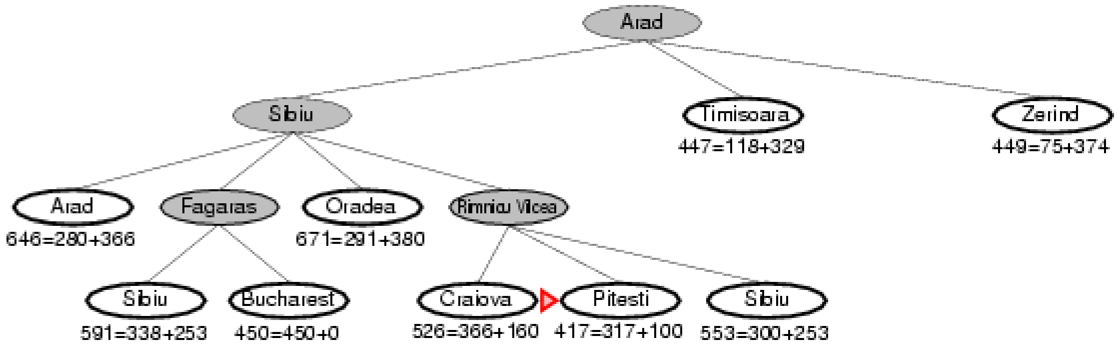
A* Search Example



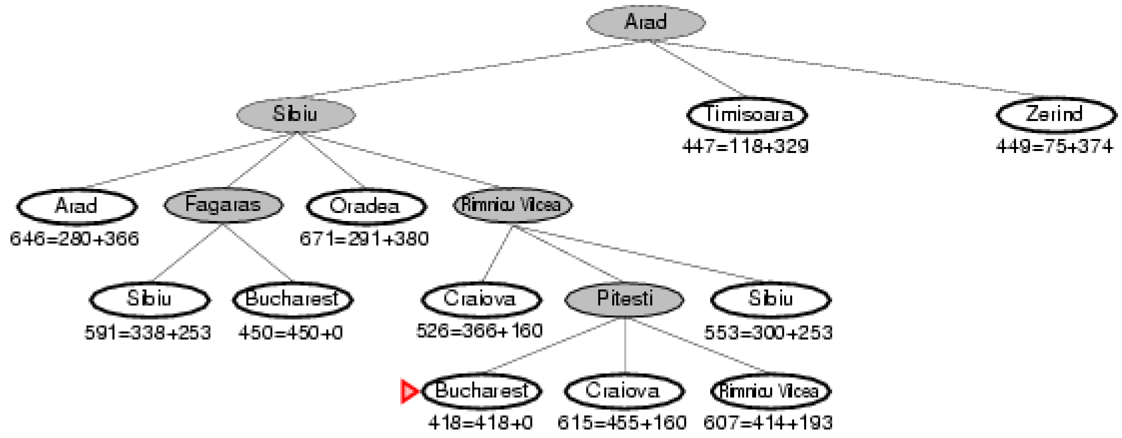
A^* Search Example



A* Search Example



A* Search Example

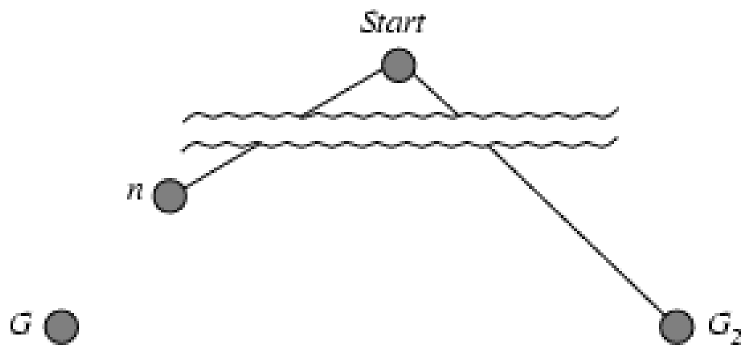


Admissible Heuristics

- A heuristic $h(n)$ is **admissible** if for every node *n*, $h(n) \leq h^*(n)$, where $h^*(n)$ is the **true** cost to reach the goal state from *n*.
- An admissible heuristic **never overestimates** the cost to reach the goal, i.e., it is **optimistic**
  - f(n) won't overestimate then either
- Example: $h_{SLD}(n)$
  - Never overestimates the actual road distance
- <u>Theorem</u>: If $h(n)$ is admissible, A^* using `TREE-SEARCH` is optimal

Proof: Optimality of A^*

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

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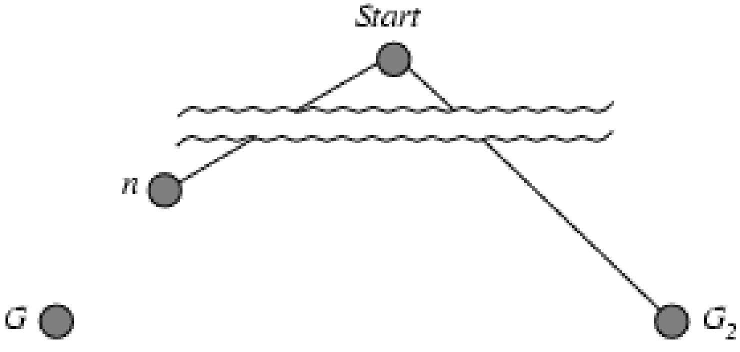


- $f(G_2) > f(G)$ from above
- $h(n) \leq h^*(n)$ since h is admissible
- $g(n) + h(n) \leq g(n) + h^*(n)$
- $f(n) \leq f(G)$

So $f(G_2) > f(G) \geq f(n)$ and hence $f(G_2) > f(n)$

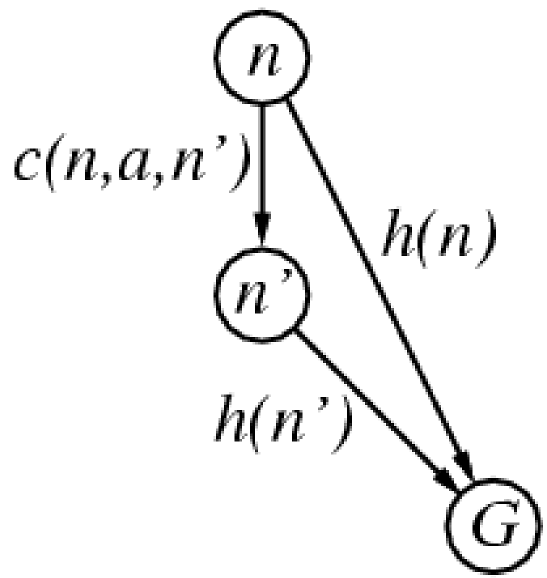
Therefore A* will never select G_2 for expansion.

Consistent Heuristics

- A heuristic is **consistent** if, for every node *n*, every successor *n'* of *n* generated by any action *a*:

  $h(n) \leq c(n,a,n') + h(n')$
- Consistency means f(n) should not

  decrease along path



Consistent Heuristics

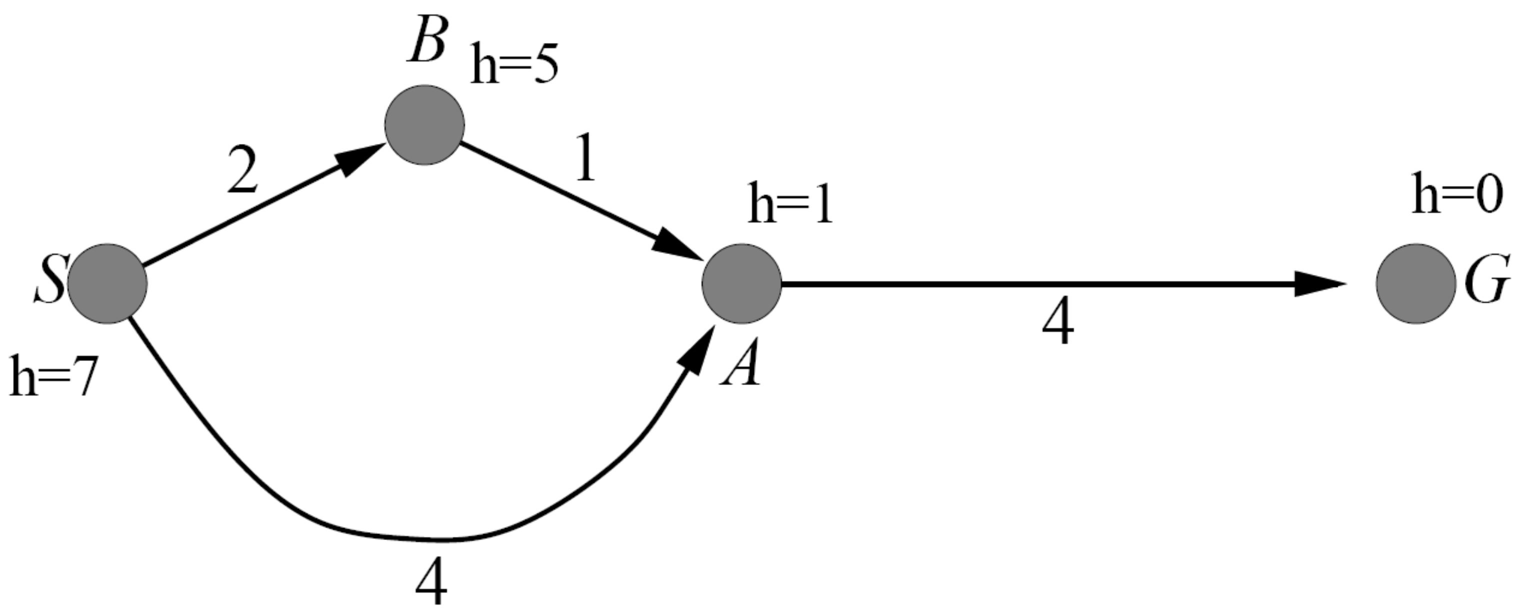
- **Theorem**: If *h(n)* is consistent, A* using `GRAPH-SEARCH` is optimal
- Assume h(n) is consistent
- f(n) along path is non-decreasing
- Mathematically speaking:

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\geq g(n) + h(n) \\
&\geq f(n)
\end{aligned}
$$

- Whenever n is expanded, we've found the best path to n
  - Otherwise we would've followed the better path first
- Thus all nodes expanded in non-decreasing order of f(n)
- f(goal) = g(goal)
  - Because f(goal) = g(goal) + h(goal) and h(goal) = 0
- First goal node expanded must be therefore be least expensive goal

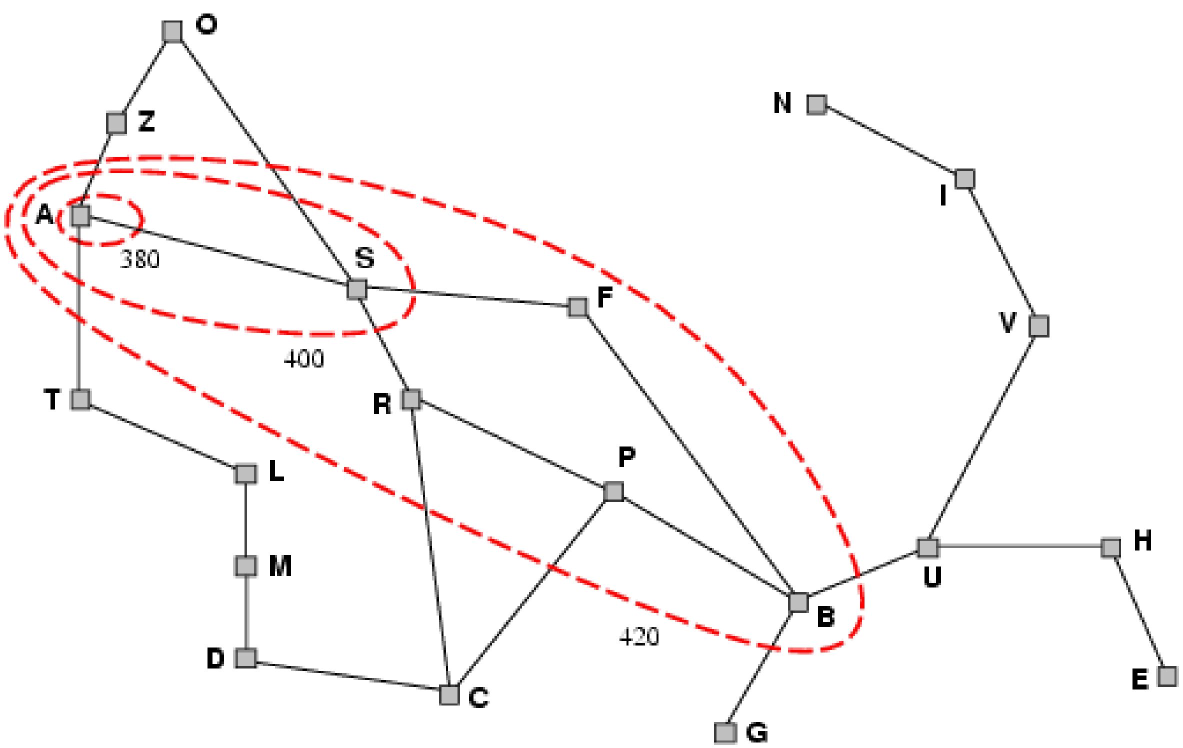
Admissibility and Consistency

- Consistency is stricter than admissibility
  - All consistent heuristics are admissible
  - Not all admissible heuristics are consistent



Optimality of A*

- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

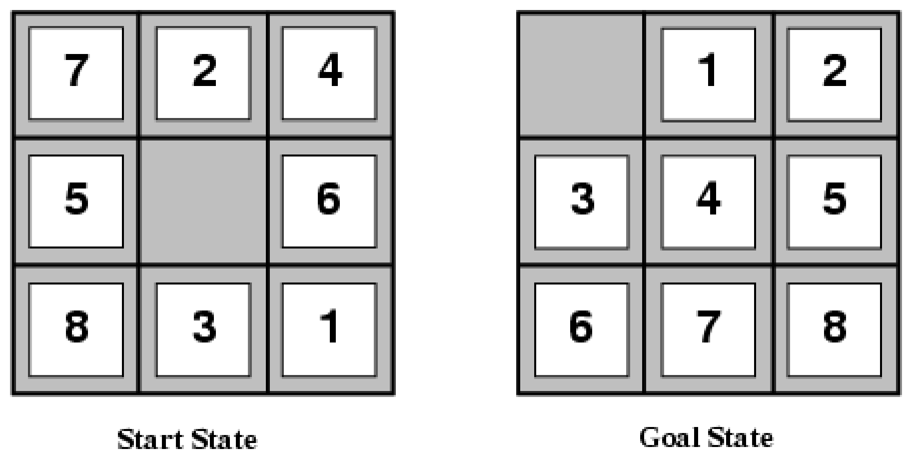
- Complete?
  - Yes (unless there are infinitely many nodes with f(n) ≤ optimal solution cost C*)
- Time?
  - Exponential
  - $O(b^{(h^*-h)})$
- Space?
  - Keeps all nodes in memory
- Optimal?
  - Yes
  - Also optimally efficient
    - Expanding fewer nodes may miss optimal solution

Admissible Heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible Heuristics

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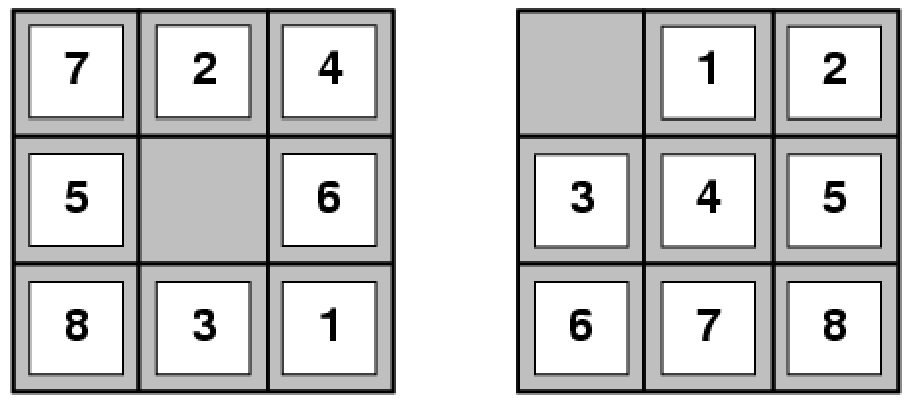

Start State Goal State

- $h_1(S) = 8$
- $h_2(S) = ?$

Admissible Heuristics

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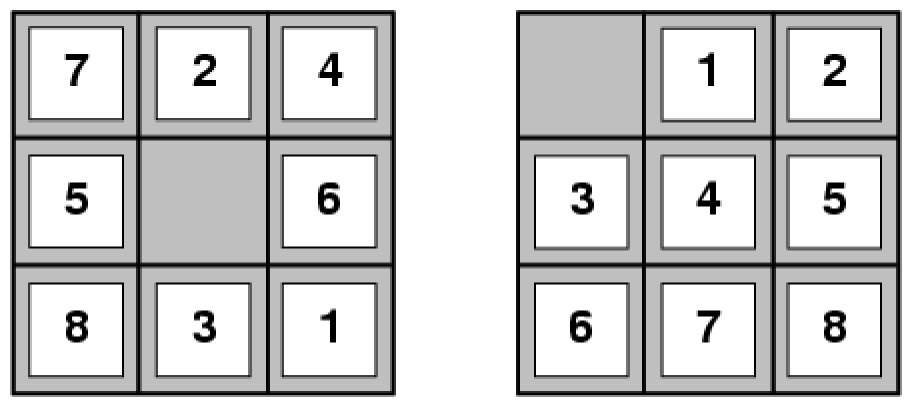

Start State Goal State

- $h_1(S) = 8$
- $h_2(S) = 3+1+2+2+2+3+3+2 = 18$

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 **dominates** h_1
- h_2 is better for search
- Typical search costs (average number of nodes expanded) over 100 8-puzzle instances:
- $d=12$
  - IDS = 364,404 nodes
  - $A^*(h_1)$ = 227 nodes
  - $A^*(h_2)$ = 73 nodes
- $d=24$
  - IDS = too many nodes
  - $A^*(h_1)$ = 39,135 nodes
  - $A^*(h_2)$ = 1,641 nodes

Relaxed Problems

- A problem with fewer restrictions on the actions is called a **relaxed problem**
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Summary

- Heuristic functions estimate costs of shortest paths
- Good heuristics can dramatically reduce search cost
- Greedy best-first search expands lowest ***h***
  - incomplete and not always optimal
- A* search expands lowest ***g + h***
  - Complete
  - Optimal
  - Also optimally efficient (up to tie-breaks, for forward search)
    - Can’t explore fewer nodes due to risk of missing optimal solution
- Admissible heuristics can be derived from exact solution of relaxed problems