CS Theory (Spring '25) Assigned: January 28, 2025

Homework 1

Instructor: *Josh Alman* Due: February 6, 2025 at noon

Collaboration on homework is allowed. However, you must write up solutions by yourself and understand everything that you hand in. You may also consult other reference materials, but you may not seek out answers from other sources or use AI tools. See the course webpage for more details.

Write the solution to each part of each problem on a separate page. Please do not write your name on any page of the submission; we are using anonymous grading in GradeScope.

There are 100 total possible points, and also bonus questions worth an additional 20 points.

0 Exercises

Here are some recommended exercises from the Sipser textbook. You should not turn in your solutions for these, and we will not grade them. Exercises 1.1, 1.2, 1.3, 1.6 (parts b,c,i,k,n).

1 Constructing DFAs (30 points; each part 10 points)

For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, give a DFA that recognizes the language, and **explain why it's correct**. Try to use as few states as possible; you may lose points if you use many more states than necessary.

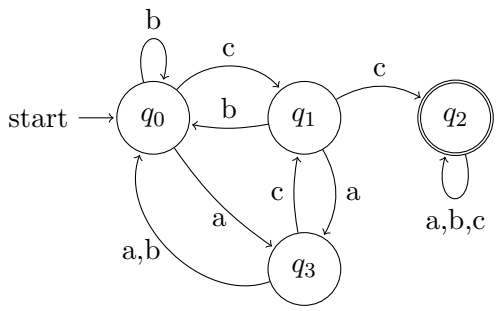
(a) $\{w \in \Sigma^* \mid w$ has an even number of 0s$\}$. For instance, 0000 has four 0s and is in the language, and 11100111 has two 0s and is in the language, while 011001010 has five 0s and is not in the language.

(b) $\{w \in \Sigma^* \mid w$ starts and ends with different characters$\}$. For instance, 100100 starts with 1 and ends with 0 and is in the language, but 001000 starts and ends with 0 and is not in the language.

(c) $\{w \in \Sigma^* \mid w$ starts and ends with different characters and also has an even number of 0s$\}$.

2 A DFA with Four States (20 points; each part 10 points)

Let D denote the following DFA over the alphabet $\Sigma = \{a, b, c\}$:



(a) Write out the formal definition of D (as a 5-tuple). Describe the transition function in a table.

(b) What is the language recognized by D? Give a short English description, then explain in a paragraph why your description is correct. Please give an explanation that is as short and simple as possible!

(c) (BONUS, 10 additional points) Prove or disprove: For all integers $n \geq 100$, there are at least 2.5^n strings in Σ^n which are rejected by D.

3 Shady Dealings (30 points; each part 10 points)

A string s over the alphabet $\Sigma = \{a, b\}$ is called *illicit* if it has length exactly 7 and it has more as than bs. For instance, *ababaaa* is illicit, but *aababa* is not (it doesn't have length 7) and *babbaba* is not (it has more bs than as). A language L over Σ is called *clandestine* if it only contains illicit strings. (A clandestine language may contain all the illicit strings, just some of them, or no strings at all!)

(a) How many illicit strings are there? Prove your answer.

(b) How many clandestine languages are there? Prove your answer.

(c) Prove that every clandestine language has a DFA that recognizes it.

(d) (BONUS, 10 additional points) Give, with proof, a clandestine language that is not recognized by any DFA with fewer than 11 states.

4 Day-to-day DFA (10 points)

Give an example of a device you encounter in your daily life whose behavior could be modeled using a DFA. Describe the DFA using a transition diagram or formal definition, and explain why it captures the behavior of the device. Please use an example that's not in the textbook. 1 extra BONUS point if you include a picture that you took of the device.

5 Collaborators, References, and Formatting (10 points)

Please list who you collaborated with on each problem, including any TAs or students you discussed the problems with in office hours. Also list any reference materials consulted other than the lectures and textbook for our class. Finally, please follow the formatting guidelines at the top of this document.