

# View Reviews

## Paper ID

41

## Paper Title

On the proliferation of support vectors in high dimensions

### Reviewer #1

---

#### Questions

**1. Summary and contributions: Briefly summarize the paper and its contributions.**

The paper deals with the issue of support vector proliferation in high dimensional problems.

**2. Strengths: Describe the strengths of the work. Typical criteria include: soundness of the claims (theoretical grounding, empirical evaluation), significance and novelty of the contribution, and relevance to the AISTATS community.**

This addresses an interesting and important problem in SVM for high dimensional data.

**3. Weaknesses: Explain the limitations of this work along the same axes as above.**

Empirical results are lacking. I understand it is a theoretical work but would be good to see this verified.

**4. Correctness: Are the claims and method correct? Is the empirical methodology correct?**

I think it is correct, but I have not verified every part of the proof especially in the supplemental material.

**5. Clarity: Is the paper well written?**

Very clear.

**6. Relation to prior work: Is it clearly discussed how this work differs from previous contributions?**

Good introduction on the problem is given and relevant literature is cited.

**7. Reproducibility: Are there enough details to reproduce the major results of this work?**

3: Yes, most aspects are reproducible

**9. Please provide an "overall score" for this submission.**

9: Top 15% of accepted AISTATS papers. An excellent submission; a strong accept.

**10. Please provide a "confidence score" for your assessment of this submission.**

3: You are fairly confident in your assessment.

**13. While performing my duties as a reviewer (including writing reviews and participating in discussions), I have and will continue to abide by the AISTATS'21 code of conduct. (The AISTATS'21 code of conduct can be found here: <https://tinyurl.com/y32gdz7d> )**

Agreement accepted

### Reviewer #2

---

#### Questions

**1. Summary and contributions: Briefly summarize the paper and its contributions.**

This paper obtains lower bounds on the probability that every training example is a support vector.

**2. Strengths: Describe the strengths of the work. Typical criteria include: soundness of the claims (theoretical grounding, empirical evaluation), significance and novelty of the contribution, and relevance to**

**the AISTATS community.**

It looks a deep topic.

**3. Weaknesses: Explain the limitations of this work along the same axes as above.**

Hard to say, as I am really not an expert in the topic.

**4. Correctness: Are the claims and method correct? Is the empirical methodology correct?**

Seems true.

**5. Clarity: Is the paper well written?**

Yes, it is well written.

**6. Relation to prior work: Is it clearly discussed how this work differs from previous contributions?**

I am really not an expert in the topic.

**7. Reproducibility: Are there enough details to reproduce the major results of this work?**

3: Yes, most aspects are reproducible

**9. Please provide an "overall score" for this submission.**

6: Marginally above the acceptance threshold.

**10. Please provide a "confidence score" for your assessment of this submission.**

1: Your assessment is an educated guess.

**13. While performing my duties as a reviewer (including writing reviews and participating in discussions), I have and will continue to abide by the AISTATS'21 code of conduct. (The AISTATS'21 code of conduct can be found here: <https://tinyurl.com/y32gdz7d> )**

Agreement accepted

**Reviewer #3**

---

## Questions

**1. Summary and contributions: Briefly summarize the paper and its contributions.**

This paper studies in theory under which situation all the training data become the support vectors in hard-margin support vector machines. The major contribution of this work is to explore an equivalent condition when all the training data become the support vectors (condition 3 in Lemma 1) and use to improve the theoretical bound in Muthukumar et al (2020).

**2. Strengths: Describe the strengths of the work. Typical criteria include: soundness of the claims (theoretical grounding, empirical evaluation), significance and novelty of the contribution, and relevance to the AISTATS community.**

This is a theoretical paper based on the random matrix theory. The paper is related to the least-square interpolation, which is currently an active research topic. The paper is relevant to the AISTAT community.

**3. Weaknesses: Explain the limitations of this work along the same axes as above.**

(1) Novelty: all the main ideas in this paper are not new and have been published in Muthukumar et al (2020), whose concentration bounds are looser than this work.

(2) Significance: there are two major limitations of the practical impact. First, this paper relies on hard-margin SVM; however the hard-margin SVM is rarely used in practice, as opposed to the soft-margin SVM. Second, the paper relies on heavy distributional assumptions of training data. However, the success of SVM is mainly because of its distribution-free feature.

**4. Correctness: Are the claims and method correct? Is the empirical methodology correct?**

The theory is based on random matrix theory like Hanson-Wright inequality. The claims in this paper are correct.

**5. Clarity: Is the paper well written?**

The paper is clearly well written.

**6. Relation to prior work: Is it clearly discussed how this work differs from previous contributions?**

This work has done a good job to review and discuss the difference from previous work.

**7. Reproducibility: Are there enough details to reproduce the major results of this work?**

3: Yes, most aspects are reproducible

**9. Please provide an "overall score" for this submission.**

5: Marginally below the acceptance threshold.

**10. Please provide a "confidence score" for your assessment of this submission.**

4: You are confident in your assessment, but not absolutely certain.

**13. While performing my duties as a reviewer (including writing reviews and participating in discussions), I have and will continue to abide by the AISTATS'21 code of conduct. (The AISTATS'21 code of conduct can be found here: <https://tinyurl.com/y32gdz7d> )**

Agreement accepted

**Reviewer #6**

---

**Questions**

**1. Summary and contributions: Briefly summarize the paper and its contributions.**

The paper derives some concentration results for the phenomenon of support vectors proliferation in high-dimensional linear classification settings of two types of designs: sub-Gaussian features and Harr features. A weak converse result coupled with its implication for generalization are also presented.

**2. Strengths: Describe the strengths of the work. Typical criteria include: soundness of the claims (theoretical grounding, empirical evaluation), significance and novelty of the contribution, and relevance to the AISTATS community.**

\* The motivation for theoretical analysis of the paper is sounded: their analysis is done in non-asymptotic regime, which differs from classical work. Their bounds on sufficient conditions on proliferation of support vector (i.e. probability of every training example is a support vector) is also an improvement on a very recent work of Muthukumar et al [2020]. This implies overall novelty of the work. Note that however I have not checked the proofs in Supplementary Materials.

\* The empirical result is a a simple simulation scheme demonstrates the support vector proliferation phenomenon, which agrees with

**3. Weaknesses: Explain the limitations of this work along the same axes as above.**

\* A minor concern is that the settings for empirical result seems a bit too simplistic.

**4. Correctness: Are the claims and method correct? Is the empirical methodology correct?**

\* For main theoretical analysis, the key result lies in Lemma 1 with its proof in section 4 that seems to be correct. The authors only make assumption that the kernel matrix of the dual SVM problem is non-singular, which is ok for the settings of the paper. Again note that I have not checked the proofs in Supplementary Material.

**5. Clarity: Is the paper well written?**

\* Yes, the paper is very well written and easy to pass through.

**6. Relation to prior work: Is it clearly discussed how this work differs from previous contributions?**

\* Yes, the connection and improvement on previous works is clearly discussed, most particularly in the latter half of Introduction section.

**7. Reproducibility: Are there enough details to reproduce the major results of this work?**

2: Some aspects of the work are reproducible

**8. Additional feedback, comments, suggestions for improvement and questions for the authors:**

\* Minor comment: some references is wrongly placed in parentheses, e.g. second last paragraph of the Introduction, or second last paragraph of section 3.2.

**9. Please provide an "overall score" for this submission.**

7: A good submission; an accept.

**10. Please provide a "confidence score" for your assessment of this submission.**

3: You are fairly confident in your assessment.

**13. While performing my duties as a reviewer (including writing reviews and participating in discussions), I have and will continue to abide by the AISTATS'21 code of conduct. (The AISTATS'21 code of conduct can be found here: <https://tinyurl.com/y32gdz7d> )**

Agreement accepted