Boosting

COMS 4721 Spring 2022
Daniel Hsu

AdaBoost

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Boosting: Convert a "weak learning algorithm" into a "strong learning algorithm"

AdaBoost (Adaptive Boosting)

Ensemble learning algorithm for binary classification data $(\vec{x}_1, y_1), \ldots, (\vec{x}_n, y_n) \in \mathbb{R}^d \times \{-1, 1\}$

AdaBoost

- For $t = 1, 2, \ldots, T$:
  - Construct probability distribution $(D_t(1), \ldots, D_t(n))$
    (importance weights on training examples)
  - Run "**weak learning algorithm**" on D_t-weighted training examples
    $\longrightarrow$ "**weak classifier**" $h_t \colon \mathbb{R}^d \to \{-1, 1\}$, with

$$D_t\text{-weighted training error rate} \quad \epsilon_t \;:=\; \sum_{i=1}^{n} D_t(i) \cdot \mathbb{1}\{h_t(\vec{x}_i) \neq y_i\}$$

- Return final (ensemble) classifier $f \colon \mathbb{R}^d \to \{-1, 1\}$ based on $h_1, \ldots, h_T$

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Questions:

- How to construct importance weights D_t?
- How to construct final (ensemble) classifier f?

Constructing the importance weights and final classifier

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- Final classifier:

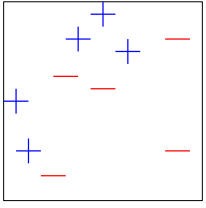
$$f(\vec{x}) \;:=\; \text{sign} \left(\sum_{t=1}^{T} \alpha_t \, h_t(\vec{x}) \right)$$

Example: AdaBoost with decision stumps

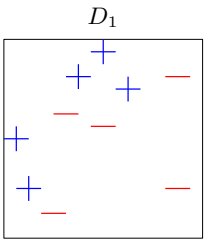
(Example from Figures 1.1 and 1.2 of Schapire & Freund text)

- Use decision stump learning algorithm as "weak learning algorithm"
- Each h_t has the form

$$h_t(\vec{x}) = \begin{cases} +1 & \text{if } x_{i_t} > \theta_t \\ -1 & \text{if } x_i \leq \theta \end{cases} \quad \text{or} \quad h_t(\vec{x}) = \begin{cases} -1 & \text{if } x_{i_t} > \theta_t \\ +1 & \text{if } x_i \leq \theta \end{cases}$$

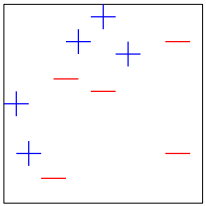


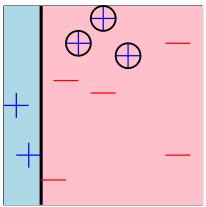
Execution of AdaBoost



Execution of AdaBoost

D_1

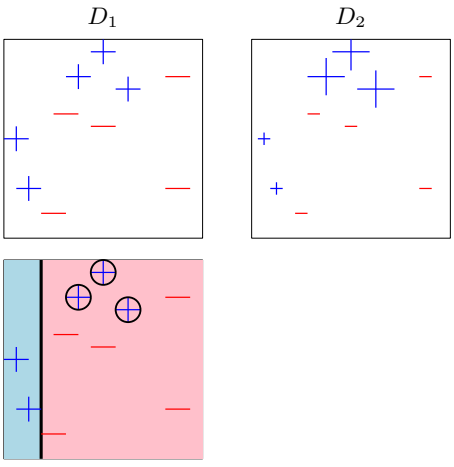




h_1

$\epsilon_1 = 0.30,\ \alpha_1 = 0.42$

Execution of AdaBoost



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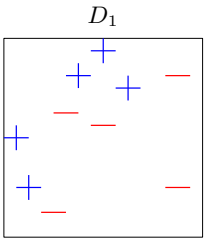

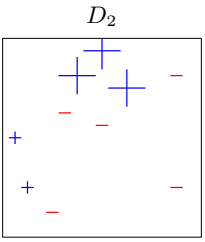

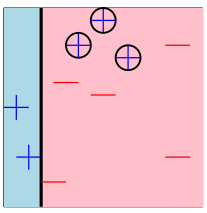

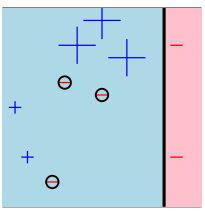

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h_2

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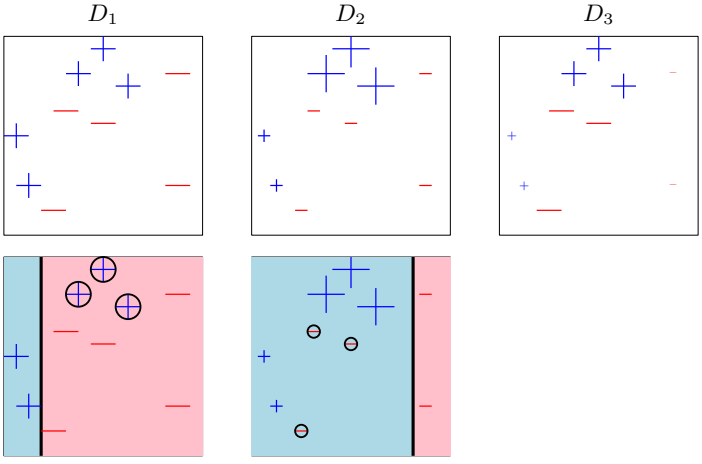

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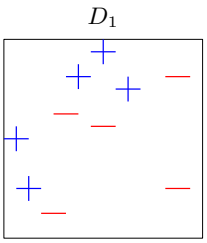

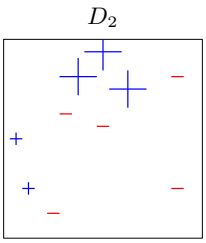

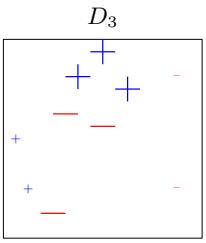

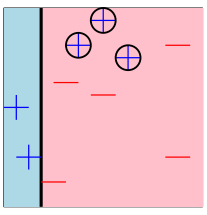

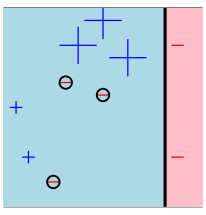

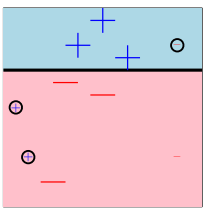

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h_3
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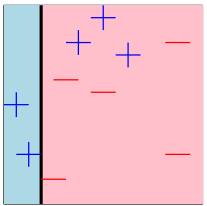
Final classifier from AdaBoost



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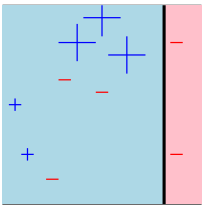

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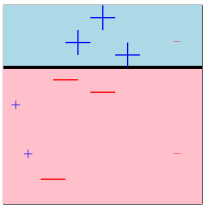

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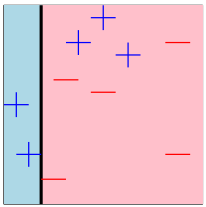

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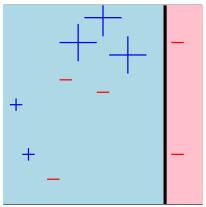

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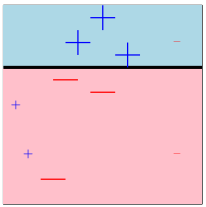

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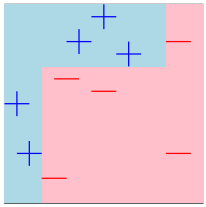
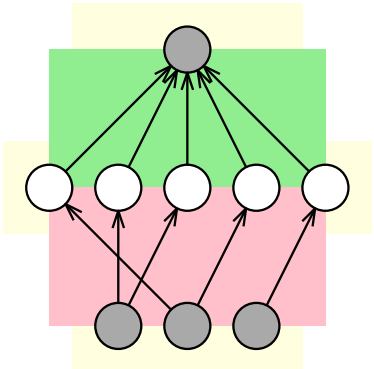
Final classifier

$f(x) = \text{sign}(0.42\, h_1(x) + 0.65\, h_2(x) + 0.92\, h_3(x))$

(Zero training error rate!)



Structure of final classifier



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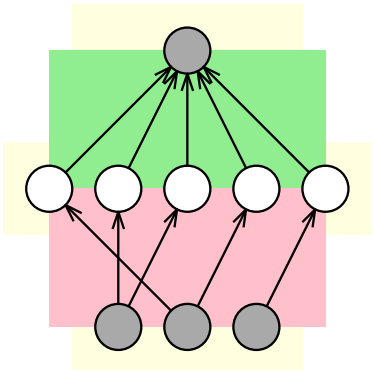

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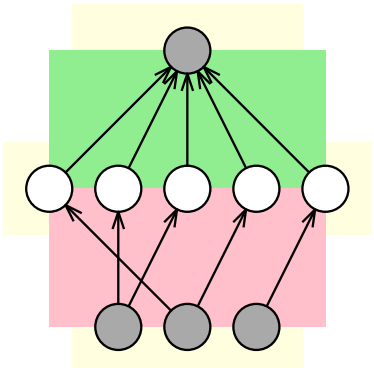

- **Middle layer:** predictions of "weak classifiers" h_t's

  e.g., $h_t(\vec{x}) = \text{sign}(\vec{x} \cdot \vec{w}_t - \theta_t)$

  (For decision stumps, $\vec{w}_t$ has exactly one non-zero entry)

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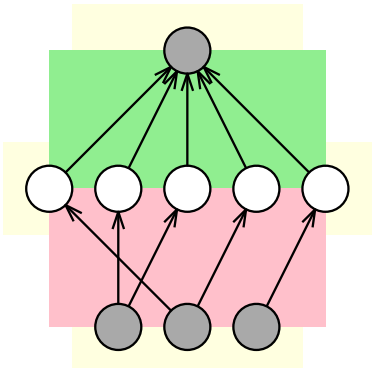

- **Top layer:** output of classifier
  Output is weighted majority vote of "weak classifiers"

$$f(\vec{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t \, h_t(\vec{x})\right)$$

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Training error rate of final classifier

Write D_t-weighted training error rate as $\epsilon_t = \frac{1}{2} - \gamma_t$

Theorem (Freund & Schapire, 1995). Training error rate AdaBoost's final classifier is

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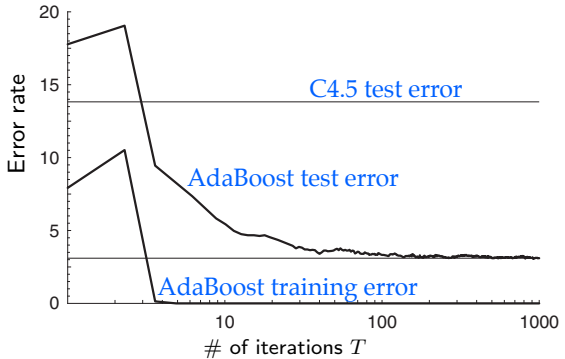
- Adaptivity: Don't care about how large any individual γ_t is; only care about aggregate $\sum_t \gamma_t^2$
- # iterations T = # constituent classifiers in final (ensemble) classifier
  (More iterations $\longrightarrow$ more complex classifier?)

Typical run of AdaBoost

(Figure 1.7 from Schapire & Freund text)
AdaBoost+C4.5 on "letters" dataset: (C4.5 = greedy training heuristic for decision trees)



(# nodes across all decision trees in final f is $>2 \times 10^6$)

Training error rate is zero after just five rounds,
but **test error rate continues to decrease, even up to** 1000 **rounds**!

Boosting the margin

Final (ensemble) classifier from AdaBoost:

$$f(\vec{x}) \;=\; \operatorname{sign}\Bigg(\underbrace{\frac{\sum_{t=1}^{T} \alpha_t \, h_t(\vec{x})}{\sum_{t=1}^{T} |\alpha_t|}}_{g(\vec{x})} \Bigg)$$

Call $y \cdot g(\vec{x}) \in [-1, +1]$ the L_1-**margin** achieved on example $(\vec{x}, y) \in \mathbb{R}^d \times \{-1, 1\}$

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New theory [Schapire, Freund, Bartlett, and Lee, 1998]:

- **Larger L_1-margins $\Rightarrow$ "simpler" classifier vis-à-vis "Occam's razor", independent of T**
- AdaBoost tends to increase L_1-margins on training examples

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On "letters" dataset:

	$T = 5$	$T = 100$	$T = 1000$
training error rate	0.0%	0.0%	0.0%
test error rate	8.4%	3.3%	3.1%
% L_1-margins ≤ 0.5	7.7%	0.0%	0.0%
min. L_1-margin	0.14	0.52	0.55

Recap

- AdaBoost sequentially + adaptively constructs classifiers for an ensemble
- Subjectivity of Occam's razor:
  - Measure complexity by size of ensemble (akin to number of features)?
  - Or measure simplicity by size of L_1-margins?

Application: Face Detection

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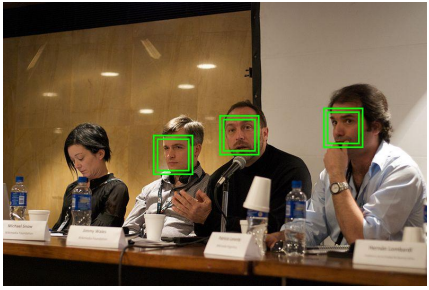
Problem: Given an image, locate all of the faces



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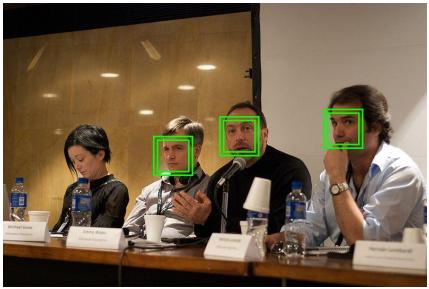

As a classification problem:

- Divide up images into patches (at varying scales, e.g., 24×24, 48×48)
- Make "face" vs "not face" prediction on each patch

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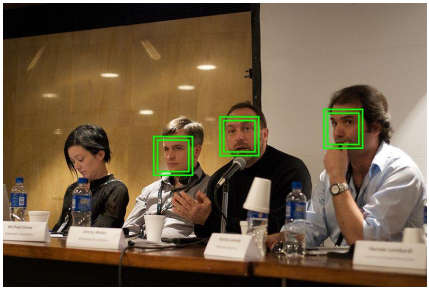

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Question: How to make something that works in real-time?

Face detectors via AdaBoost

Real-time face detector (Viola & Jones, 2001), major achievement in computer vision

Face detectors via AdaBoost

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- Think of each image patch ($d \times d$-pixel gray-scale) as a vector $\vec{x} \in [0,1]^{d^2}$.

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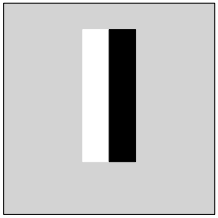

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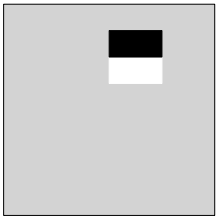

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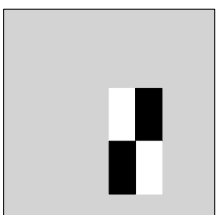

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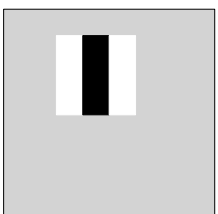

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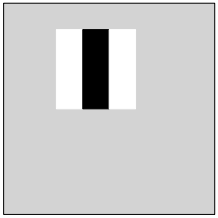

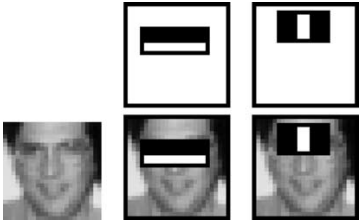

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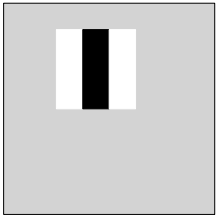
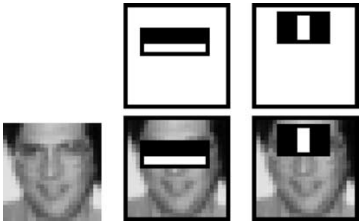
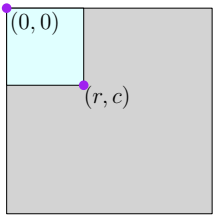




- AdaBoost combines many simple weak classifiers of this form
  - Extremely fast to evaluate via pre-computation ("integral image" trick)

Viola & Jones “integral image” trick

“Integral image” trick:



For given image, pre-compute

$$s(r, c) = \text{sum of pixel values in rectangle from } (0, 0) \text{ to } (r, c)$$

Can be done in a single pass through the image (i.e., linear time)

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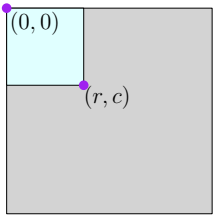

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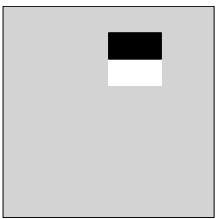

To compute inner product

$$\vec{w} \cdot \vec{x} = \text{average pixel value in black box} - \text{average pixel value in white box}$$

just need to add and subtract a few $s(r,c)$ values

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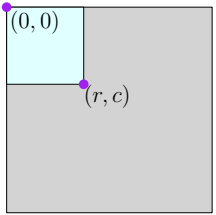

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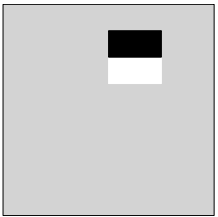

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$\Rightarrow$ Evaluating weak classifiers is extremely fast

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Observation: Most patches don't contain a face

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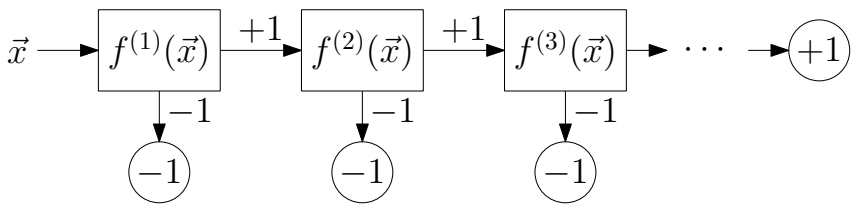
Idea: Train several classifiers (each using AdaBoost); arrange in a **decision list** (a.k.a. cascade)



- Each $f^{(\ell)}$ is trained using AdaBoost; but modify (e.g., by adjusting threshold before sign) to *minimize false negative rate*
- Can make $f^{(\ell)}$ in later stages more "complex" (i.e., larger T) than in earlier stages, since most examples don't make it to the end

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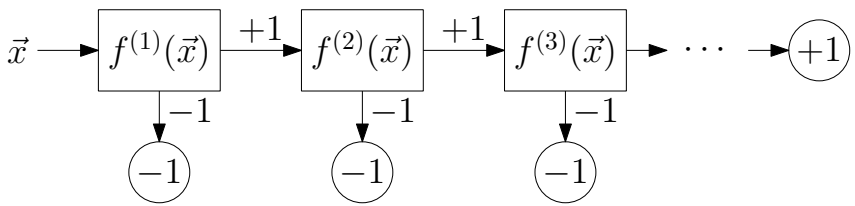

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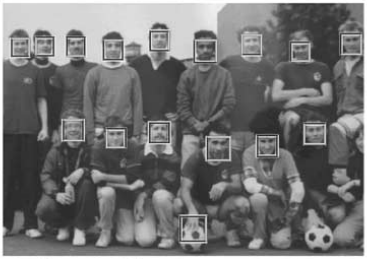
$\Rightarrow$ (Cascade) classifier evaluation is extremely fast

Viola & Jones detector: example results



Recap

- Viola-Jones face detector: Takes problem-specific requirements into account for
  - structuring the final classifier (cascade),
  - setting up the learning objective (focus on FNR),
  - using the learning algorithm (AdaBoost with particular weak learner),
  - etc.

  Really require domain-knowledge to make these choices!