Scattering and Sparse Partitions, and their Applications

Arnold Filtser

Columbia University

ICALP 2020 online presentation

Universal Steiner tree

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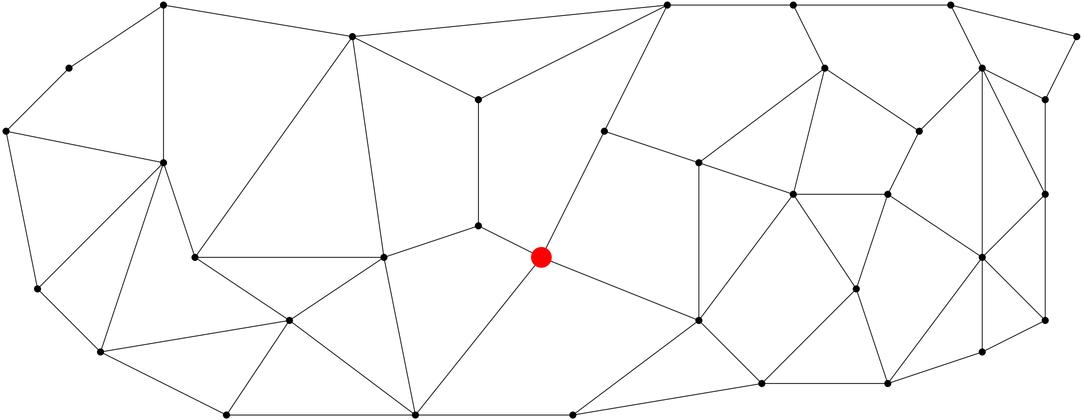

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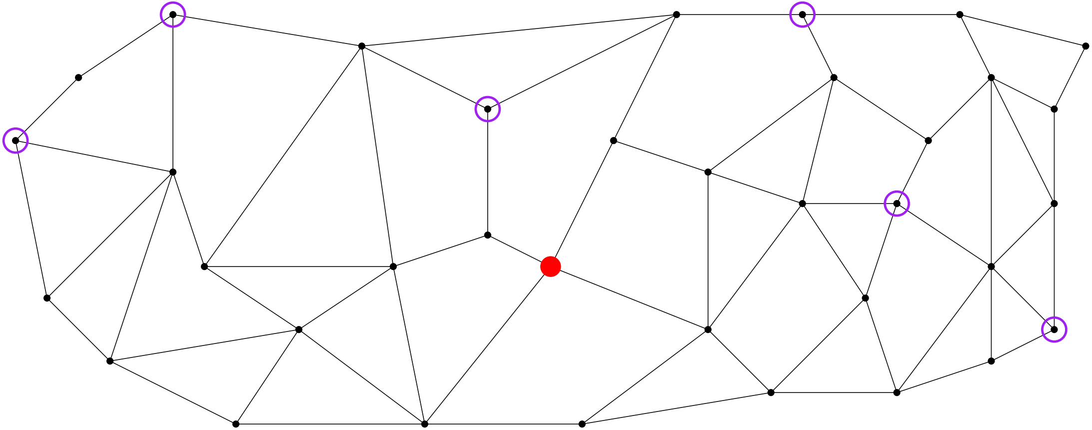

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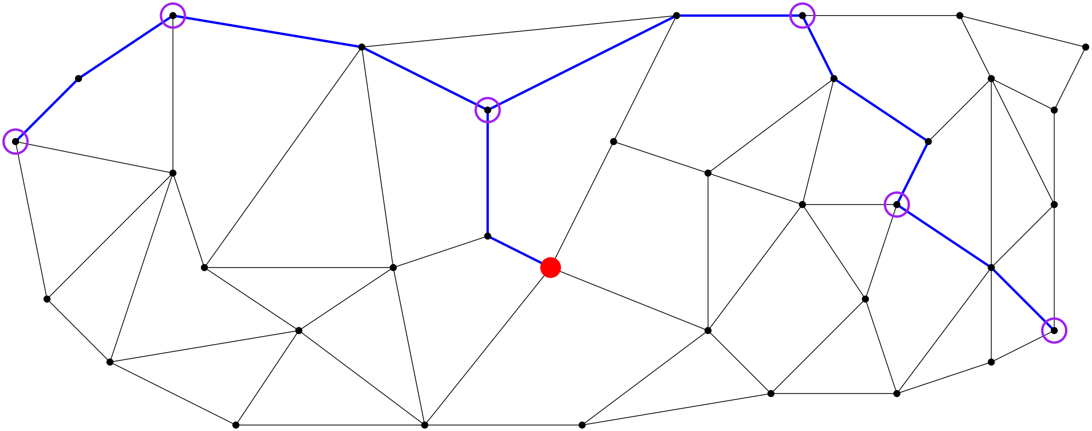

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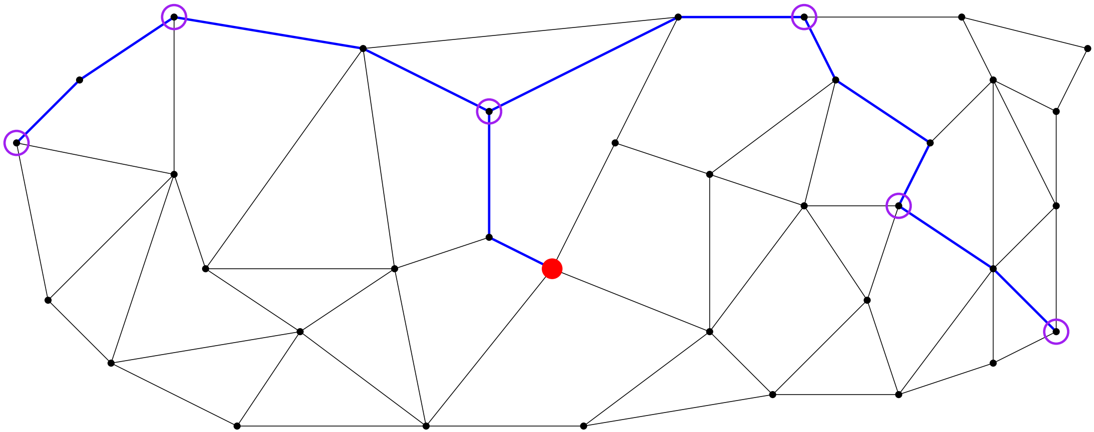

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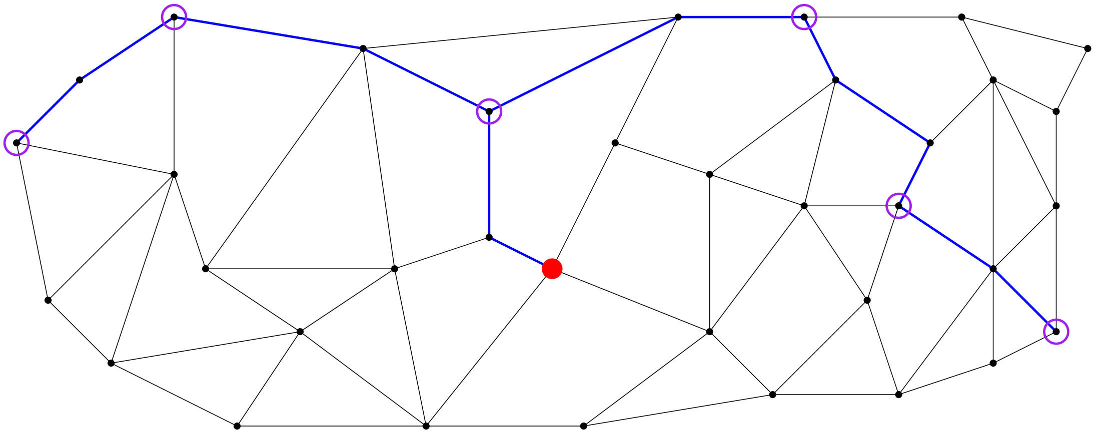

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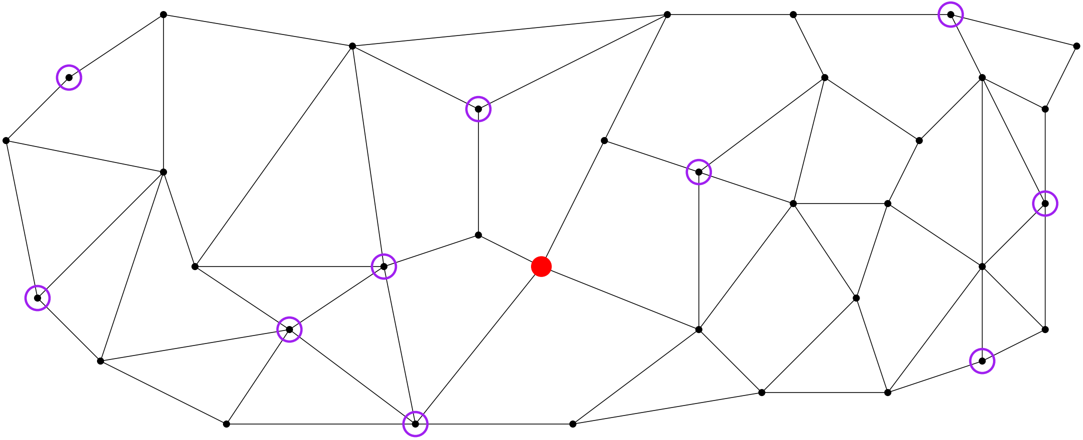

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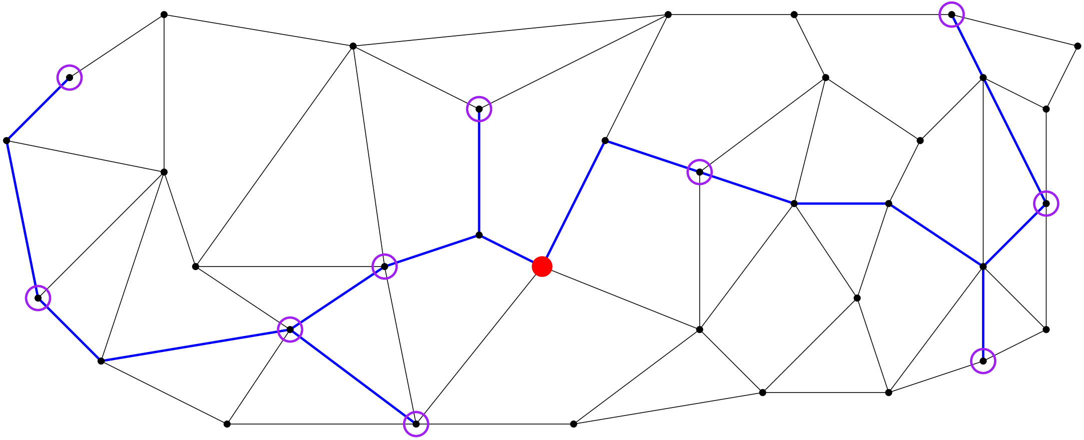

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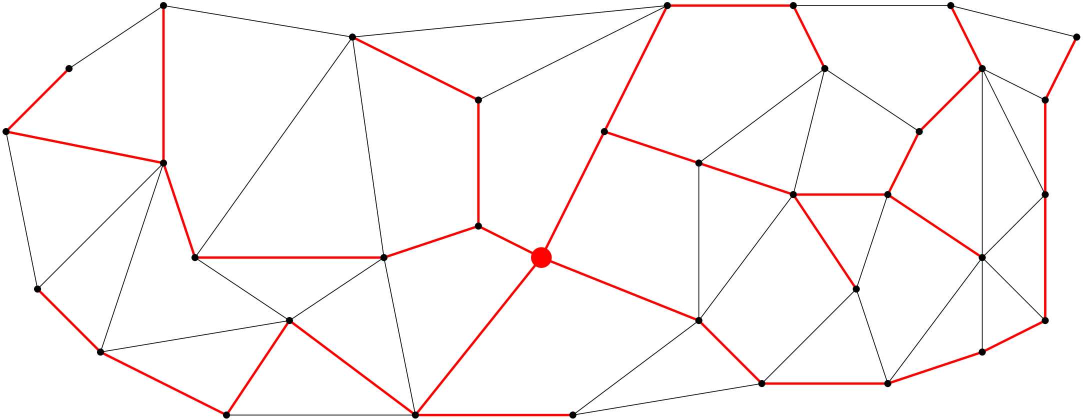

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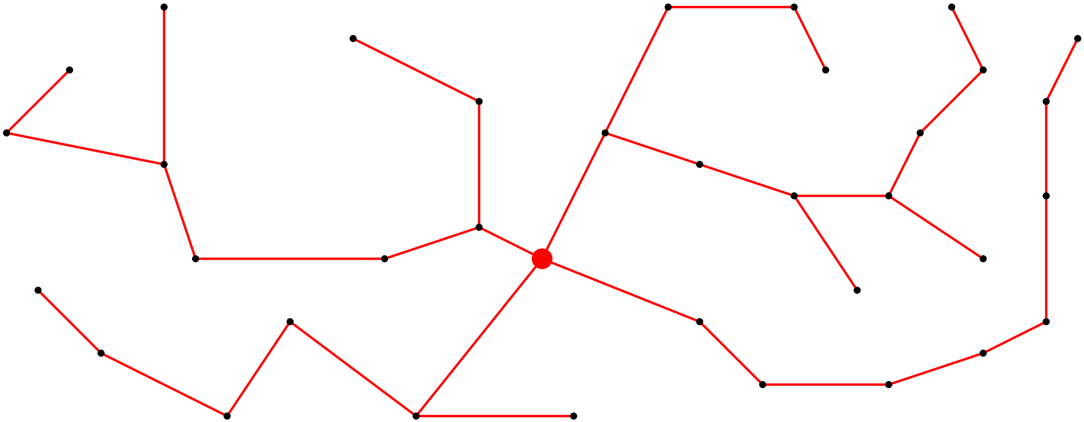

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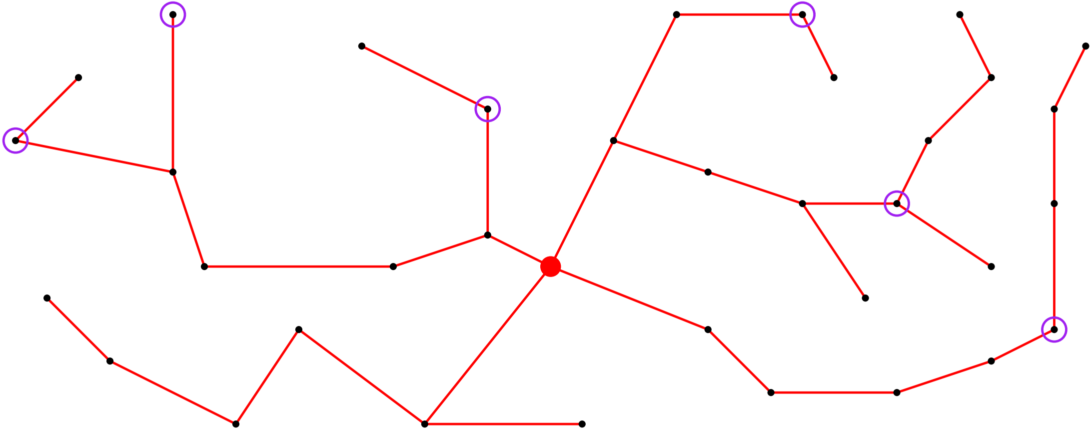

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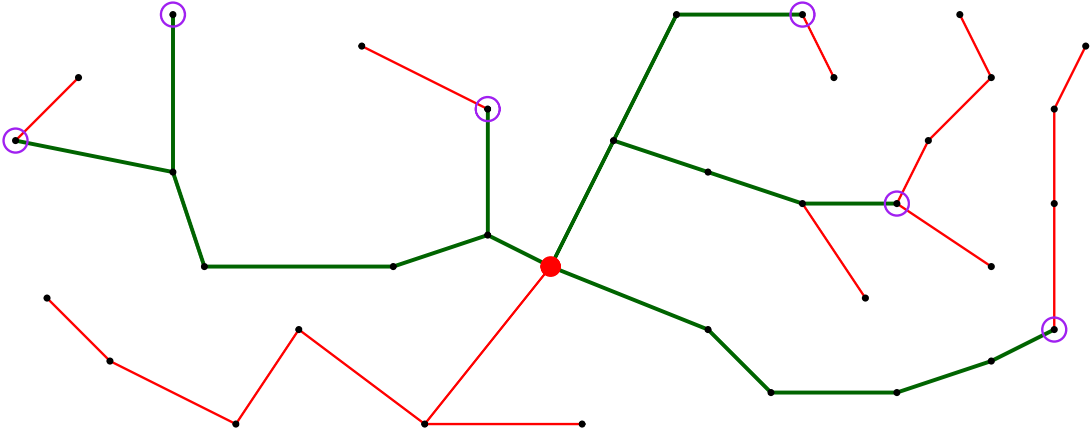

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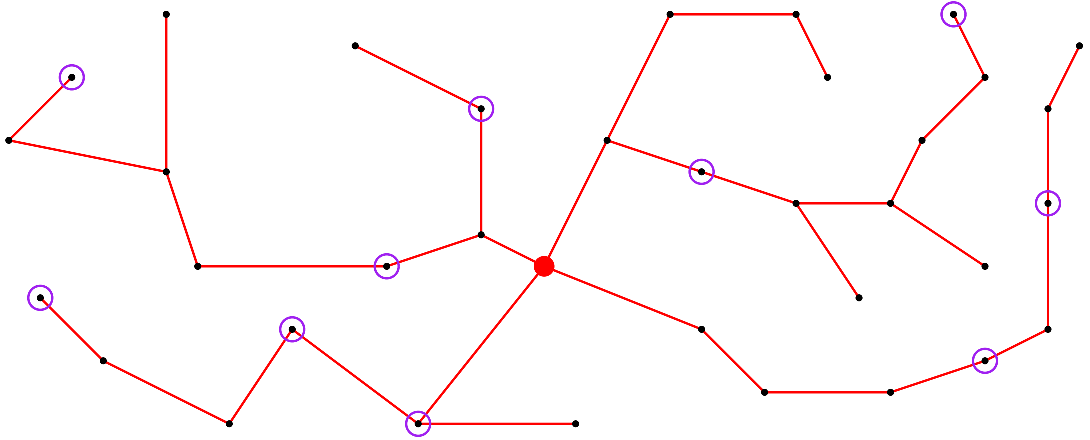

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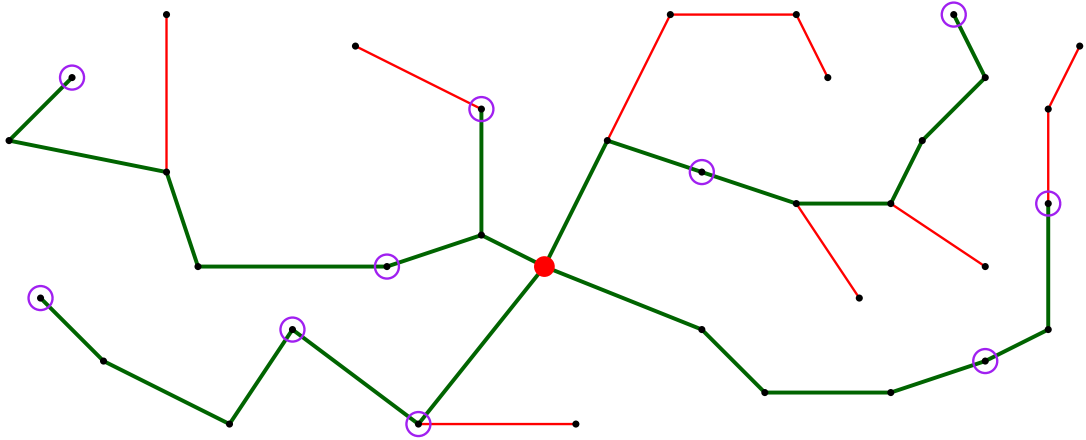

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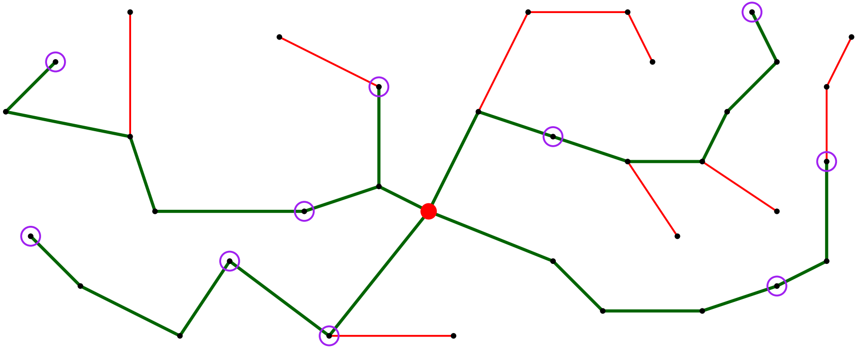

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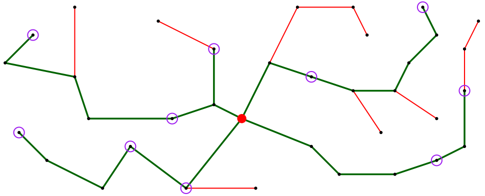

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Theorem ([Jia, Lin, Noubir, Rajaraman, Sundaram 05])

*Suppose G admits (σ, τ)-**sparse** partition scheme,*

$\Rightarrow$ *solution to the **UST** problem with stretch $O(\tau\sigma^2 \log_\tau n)$.*

Steiner Point removal problem

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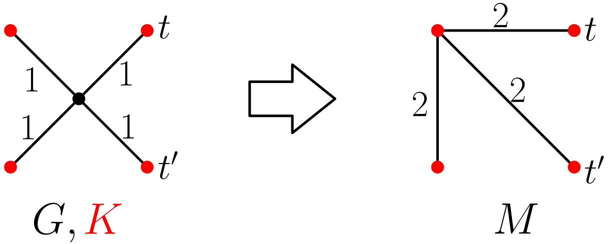

The distortion is: $\frac{d_M(t,t')}{d_G(t,t')} = \frac{4}{2} = 2$

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Theorem ([Fil 19] (improving [Kamma, Krauthgamer, Nguyen 15], [Cheung 18]))

Given G with $\boldsymbol{k}$ ***terminals****, there is a solution to the* **SPR** *problem with* **distortion** $O(\log k)$.

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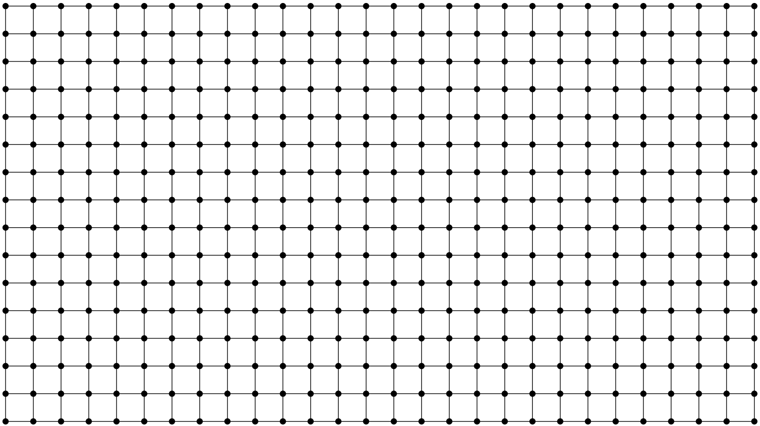
What about special graph families?

Theorem ([Fil 20])

*Suppose that every **induced subgraph** $G[A]$ of G admits (σ,τ)-scattering partition scheme, $\Rightarrow$ solution to the **SPR** problem with distortion $O(\tau^3\sigma^3)$.*

Sparse partitions

$\mathcal{P}$ is a (σ, τ, Δ)-**sparse partition** if:



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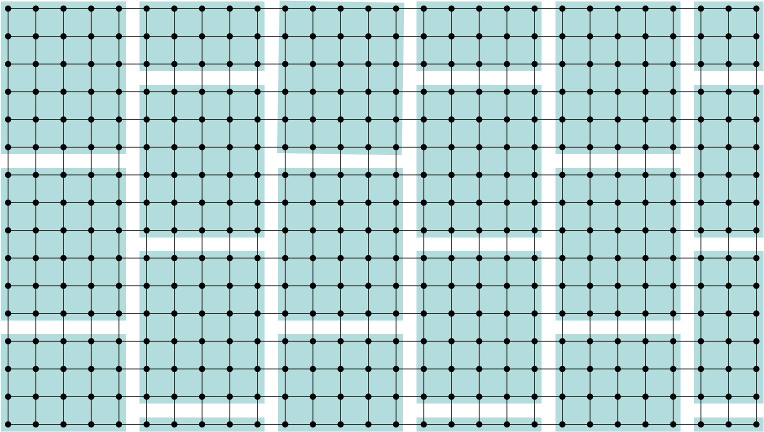

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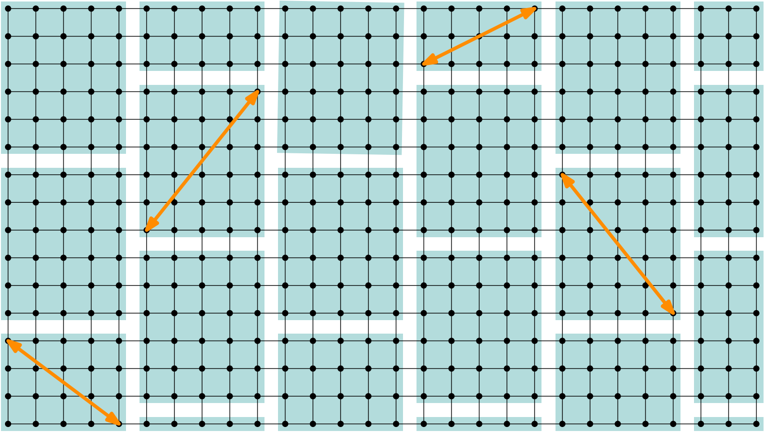

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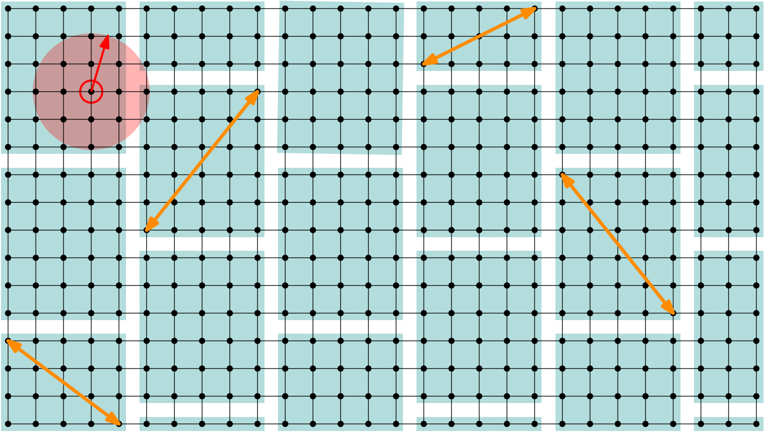

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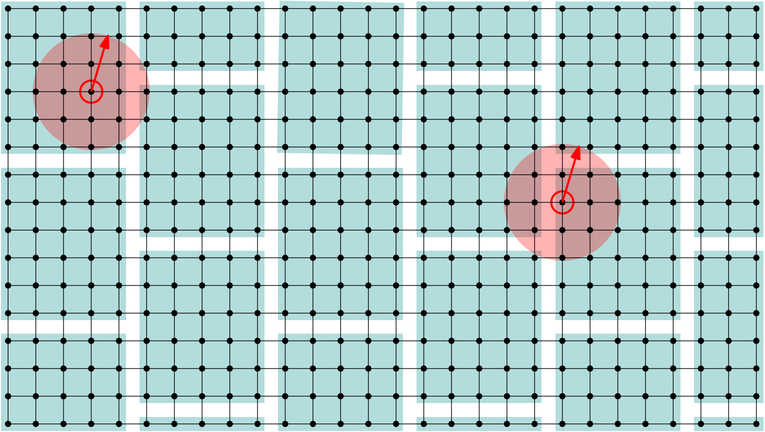

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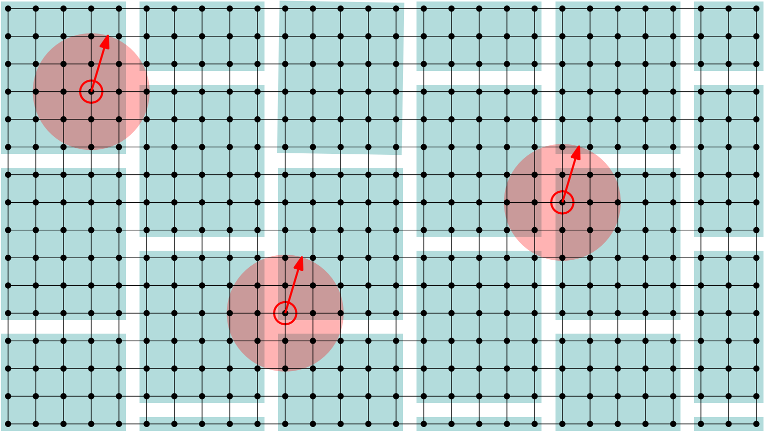

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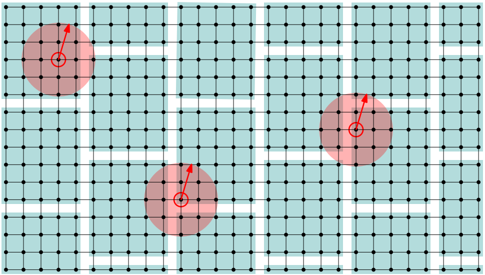

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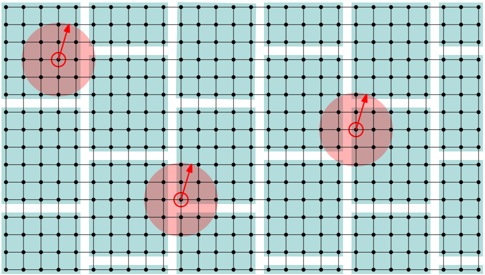

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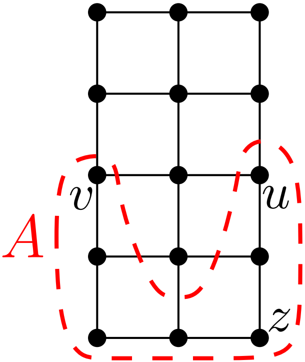

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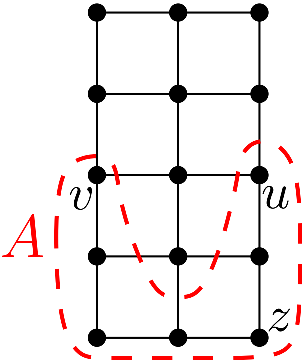

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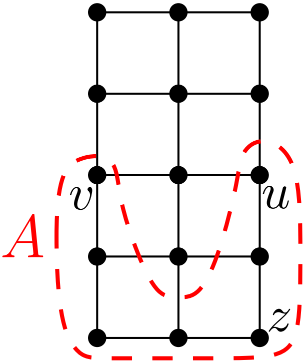

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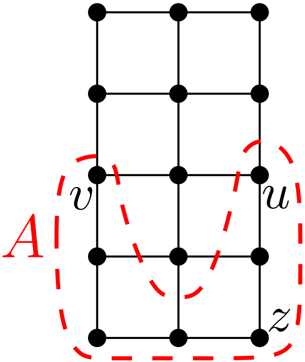

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Weak diameter of $A = 4$.
Strong diameter of $A = 6$.

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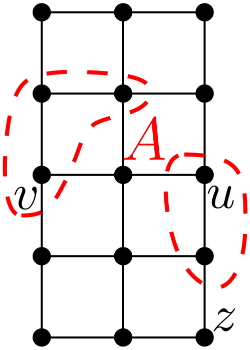

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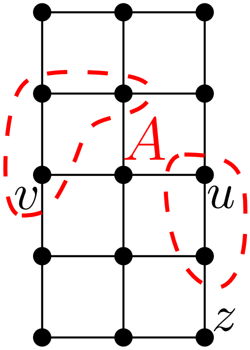

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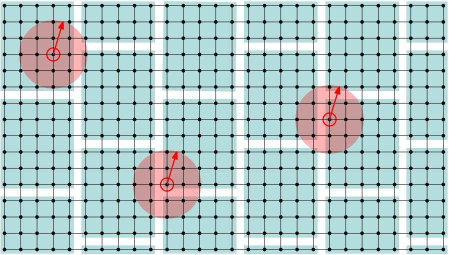

- The **strong/weak** diameter of each cluster $\leq \Delta$.
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(σ, τ)-**strong/weak** sparse partition scheme: $\exists$ (σ, τ, Δ)-**strong/weak** sparse partition for all $\Delta > 0$.

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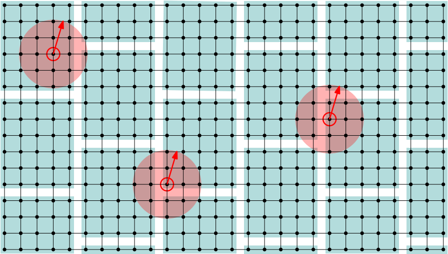

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[BDRRS 12]: subgraph solution using **hierarchy** of strong sparse partitions.

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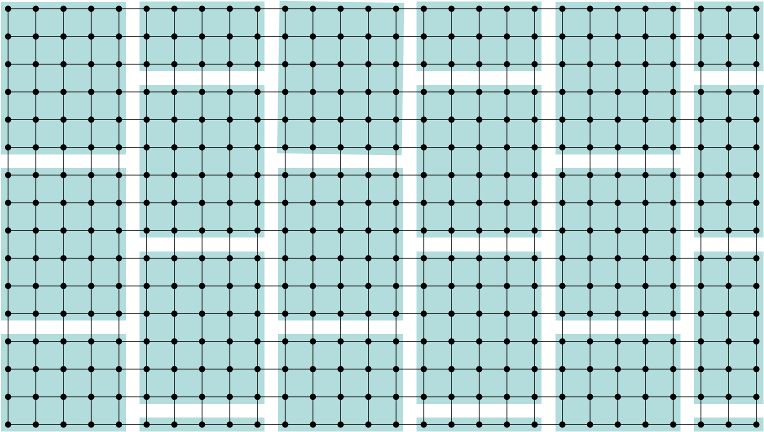

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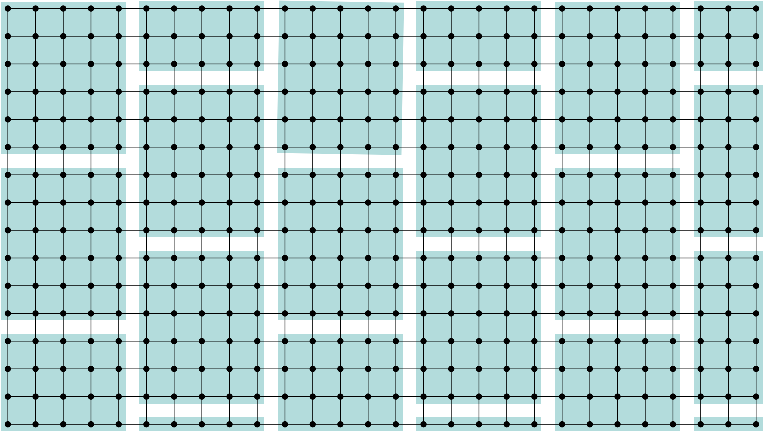

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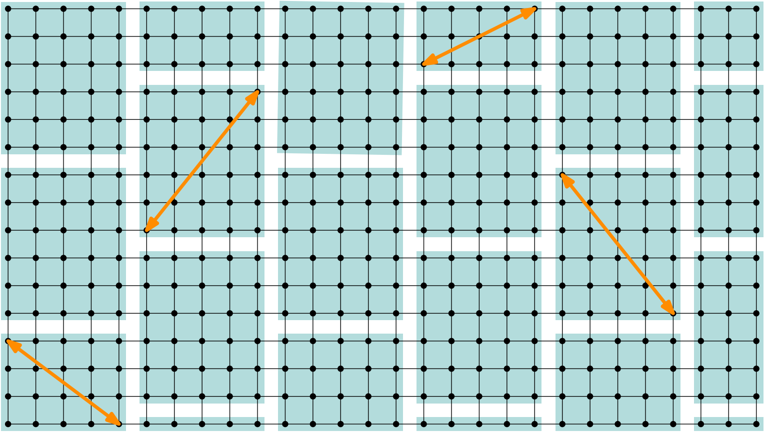

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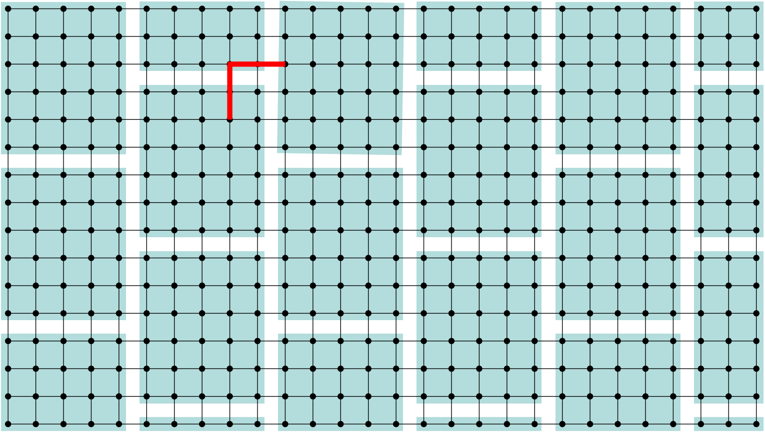

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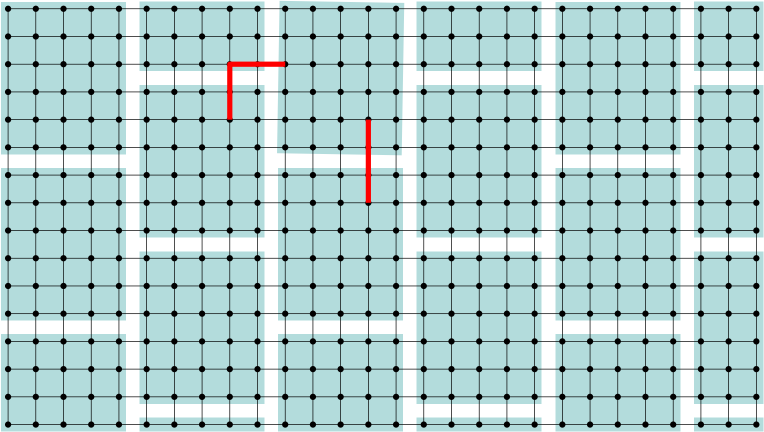

- Each cluster is connected.
- The weak-diameter of each cluster $\leq \Delta$.
- Every shortest path of length $\leq \frac{\Delta}{\sigma}$ intersects at most τ clusters.

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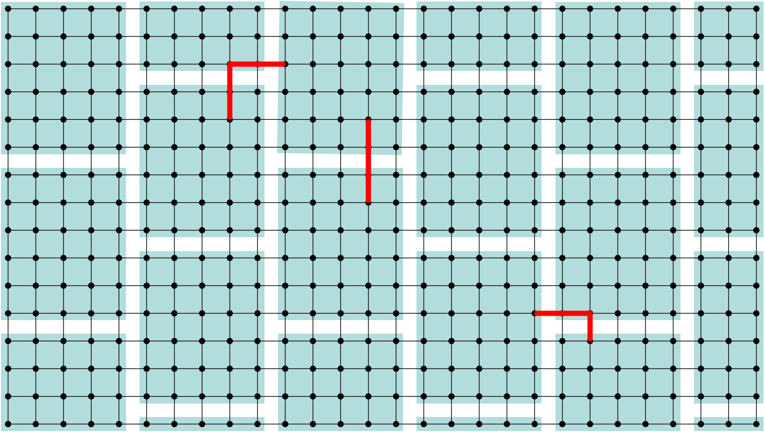

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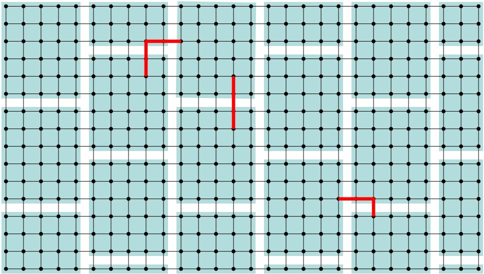

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(σ, τ)-**scattering partition scheme**: $\forall \Delta > 0 \quad \exists\, (\sigma, \tau, \Delta)$-scattering partition.

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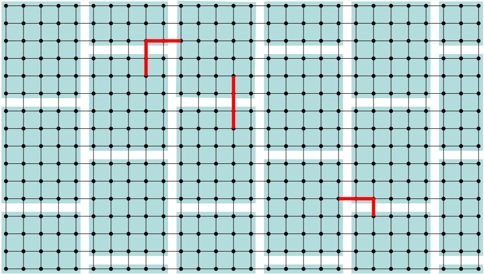

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(σ, τ)-**scattering partition scheme**: $\forall \Delta > 0 \quad \exists$ (σ, τ, Δ)-scattering partition.

Theorem ([Fil 20])

Suppose that every **induced subgraph** *$G[A]$ of G admits (σ, τ)-scattering partition scheme, $\Rightarrow$ solution to the* **SPR** *problem with distortion $O(\tau^3\sigma^3)$.*

Observations

$$(\sigma, \tau, \Delta)\text{-strong sparse} \quad \Rightarrow \quad (\sigma, \tau, \Delta)\text{-weak sparse}\ .$$

- Each cluster **strong** diameter $\leq \Delta$.
- Every ball of radius $\leq \frac{\Delta}{\sigma}$ intersects at most τ clusters.

- Each cluster **weak** diameter $\leq \Delta$.
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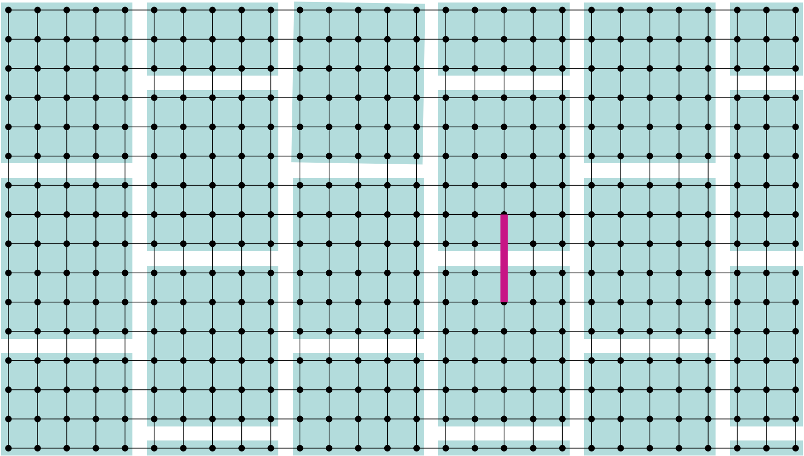
Observations

(σ, τ, Δ)-strong sparse $\Rightarrow$ (σ, τ, Δ)-scattering.



Observations

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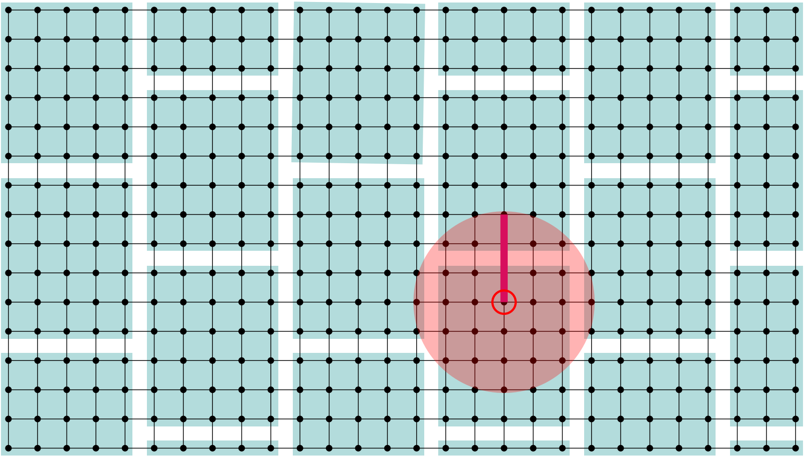
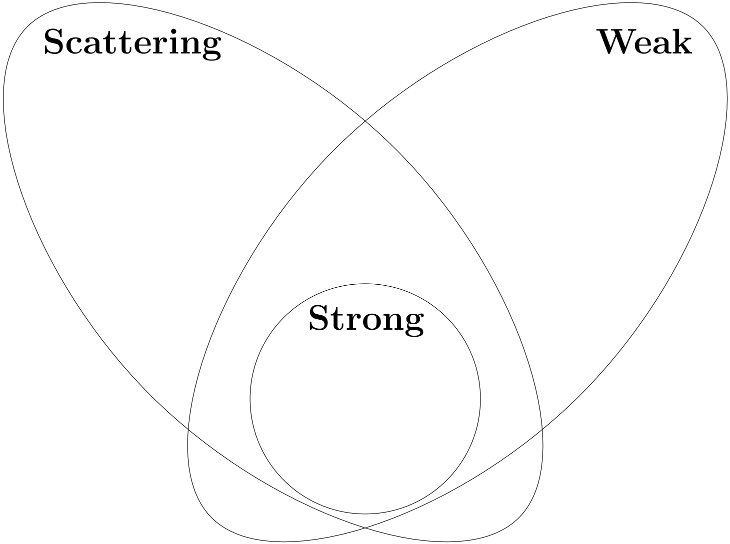
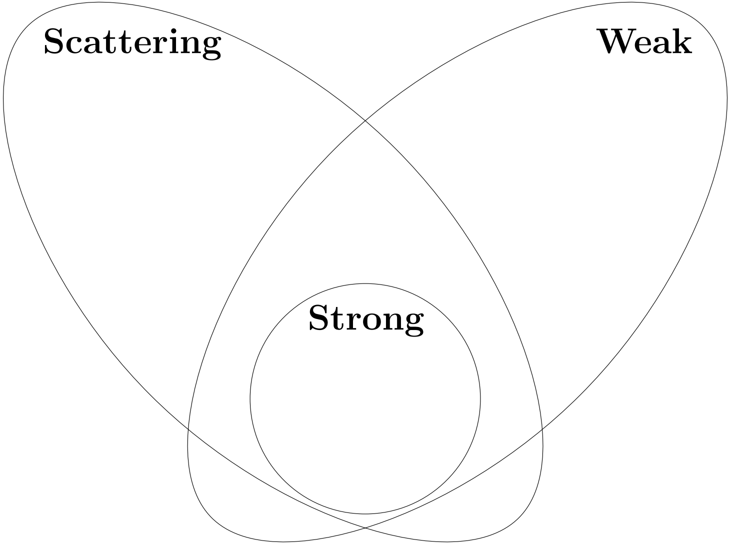
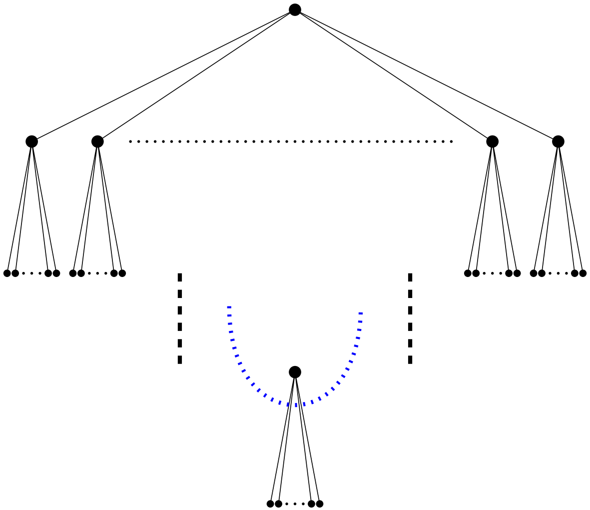



Scattering
Weak
Strong



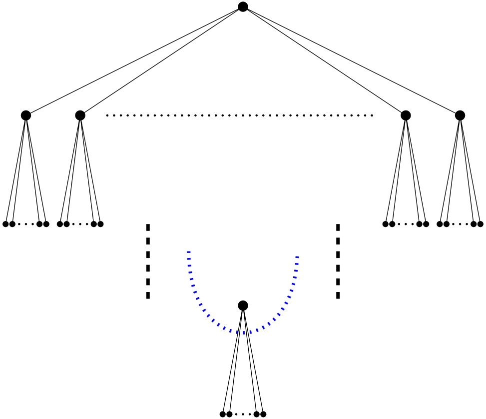
Trees?



Theorem ([**Fil** 20])

Suppose all n-vertex **trees** *admit a* (σ, τ)-**strong** *sparse partition scheme.*

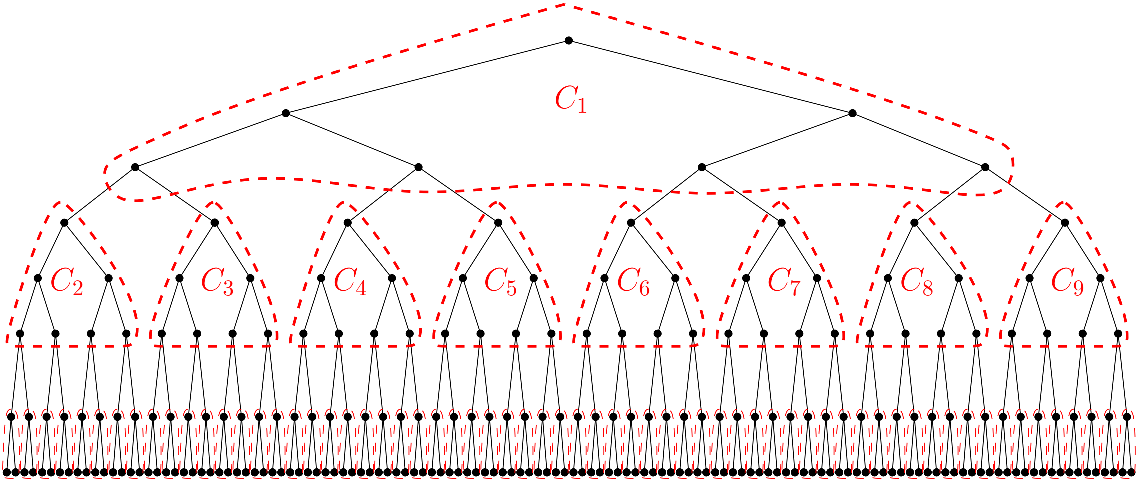
Then $\boldsymbol{\tau} \geq \frac{1}{3} \cdot n^{\frac{2}{\sigma+1}}$.



Corollary

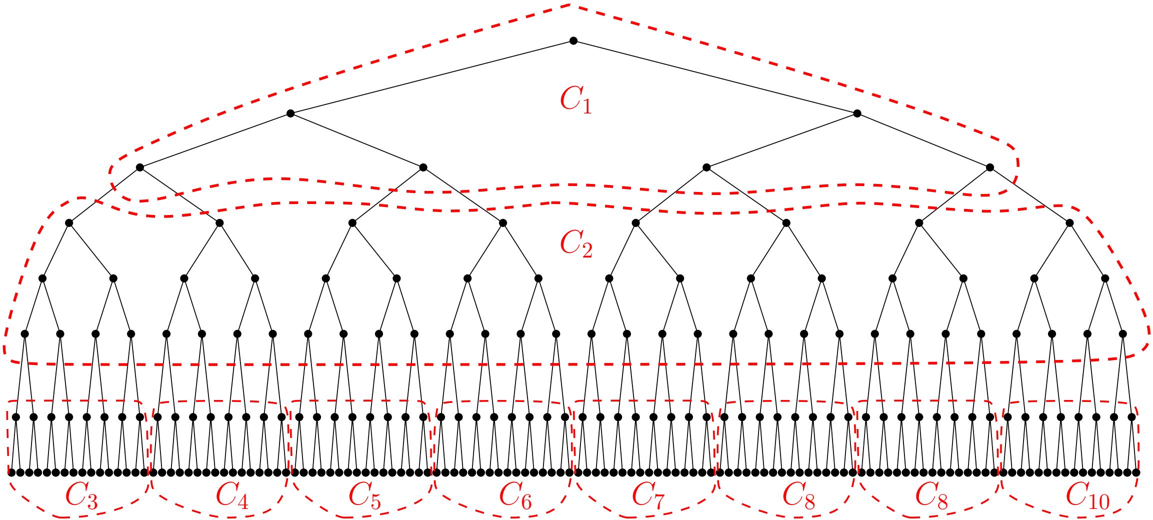
$\forall n > 1$, there are trees T_1, T_2 such that,

- *T_1 do not admit $\left(\frac{\log n}{\log\log n}, \log n\right)$-strong sparse partition scheme.*
- *T_2 do not admit $\left(\sqrt{\log n}, 2^{\sqrt{\log n}}\right)$-strong sparse partition scheme.*



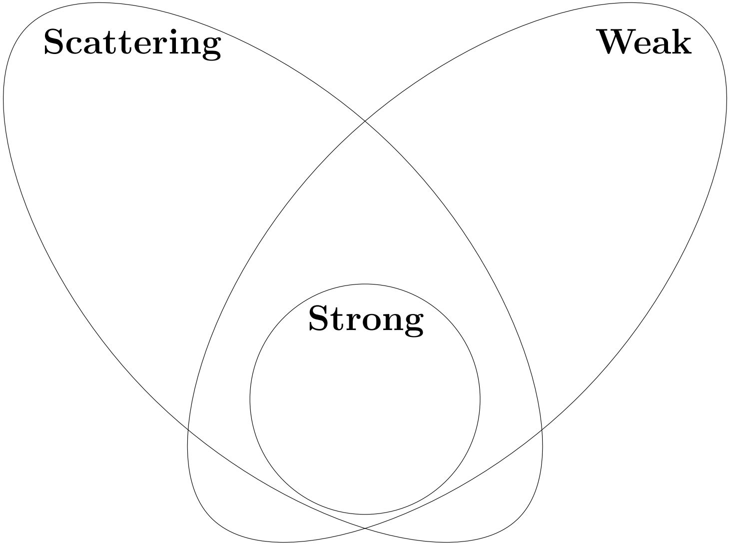
Theorem ([Fil 20])

Every **tree** *is* $(2,3)$-**scatterable**.



Theorem ([Fil 20])

Every **tree** *admits a* $(4,3)$-**weak** *sparse partition scheme.*


Scattering
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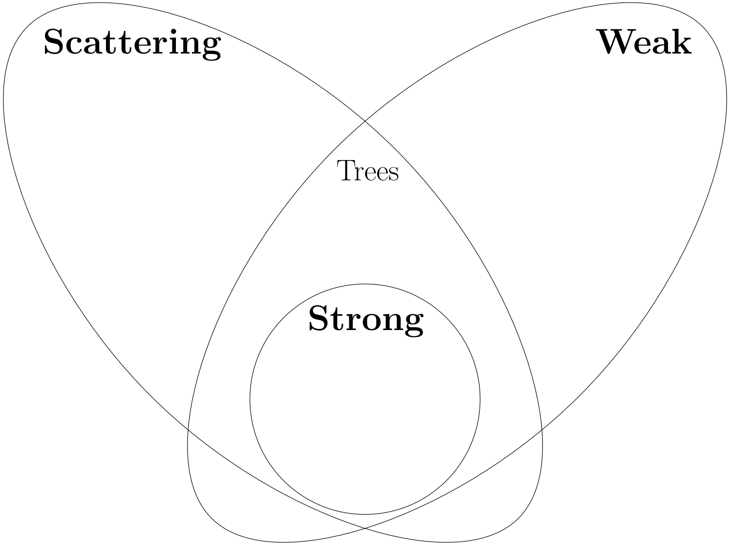
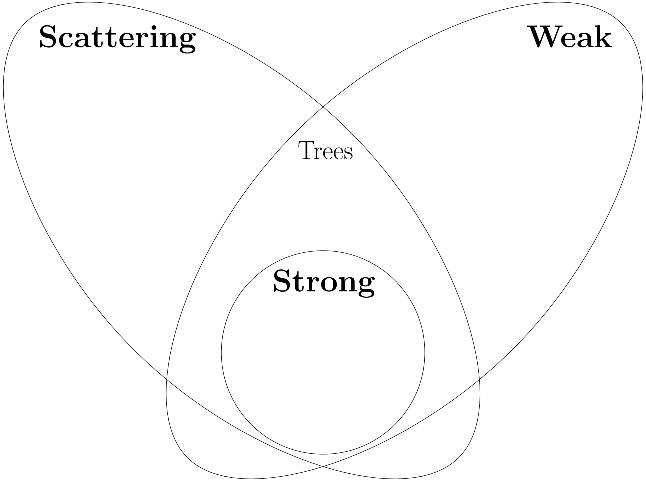
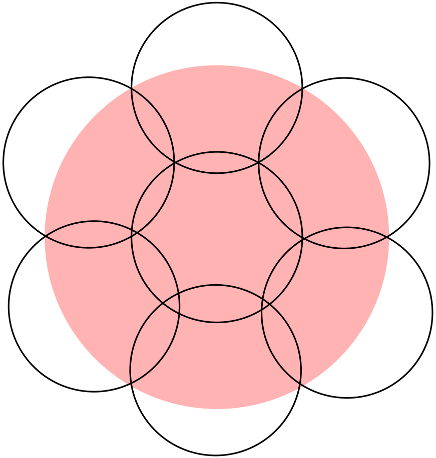
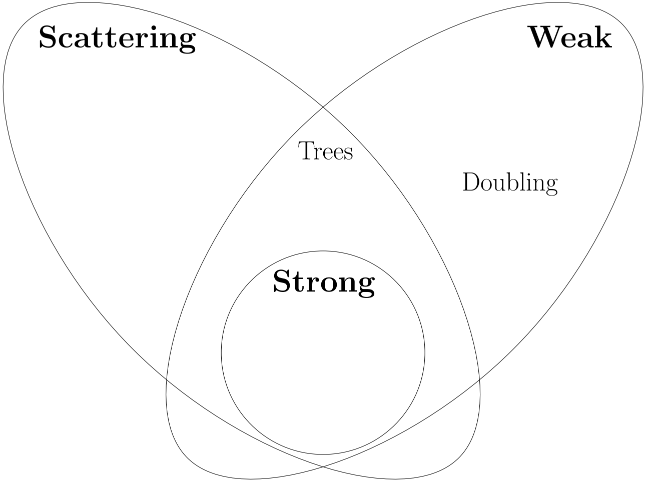

Scattering
Weak
Trees
Strong

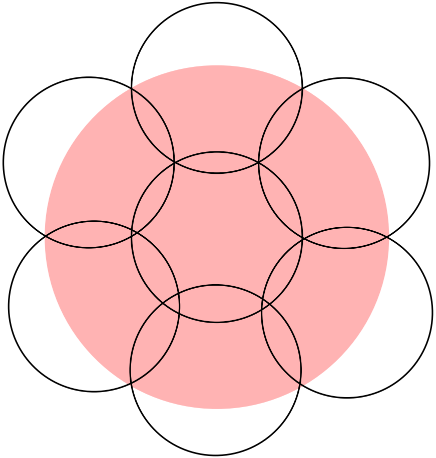




Theorem ([JLNRS 05])

Every graph with **doubling dimension** $\boldsymbol{d}$ *admits a* $(1, 2^{O(d)})$-**weak** *sparse partition scheme.*





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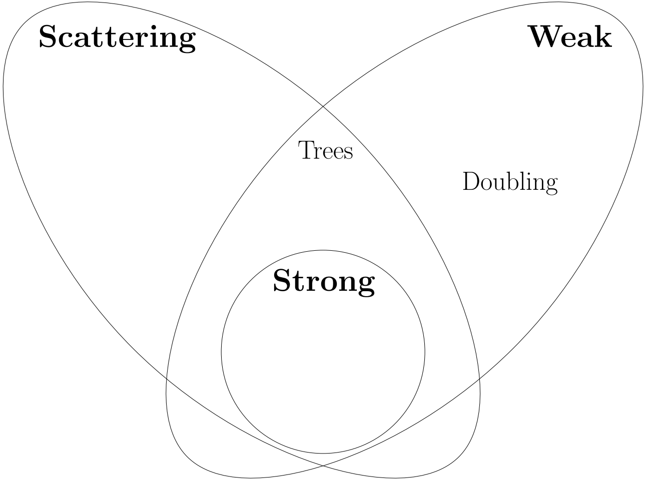
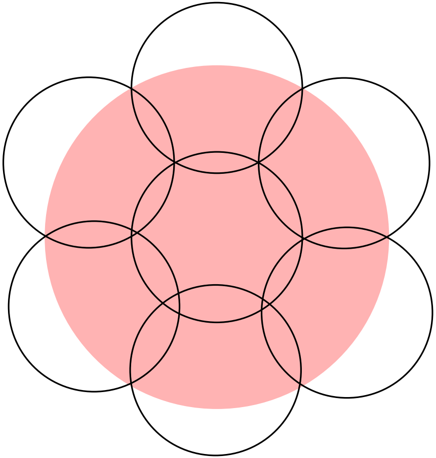
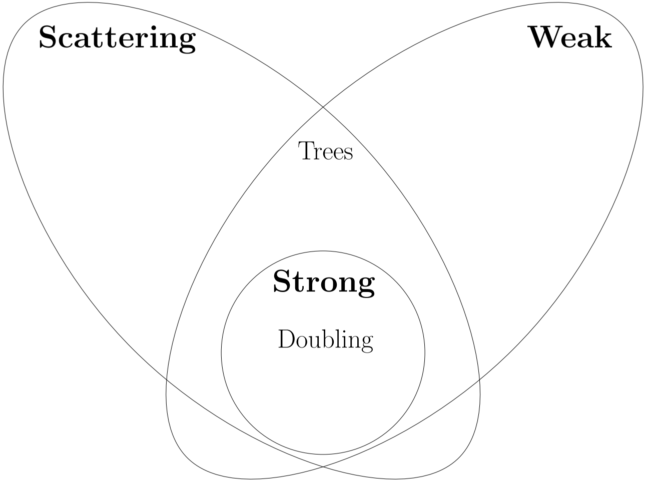
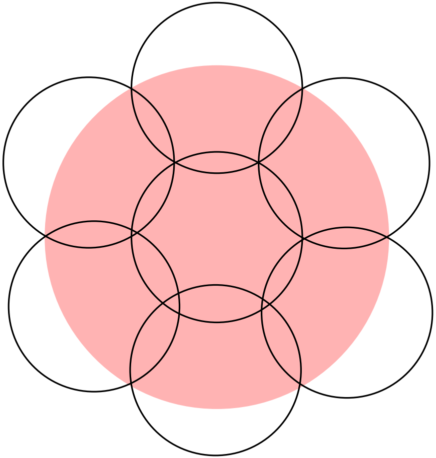




Theorem ([Fil 20])

Every graph with **doubling dimension d** *admits a $(O(d), \tilde{O}(d))$-***strong** *sparse partition scheme.*





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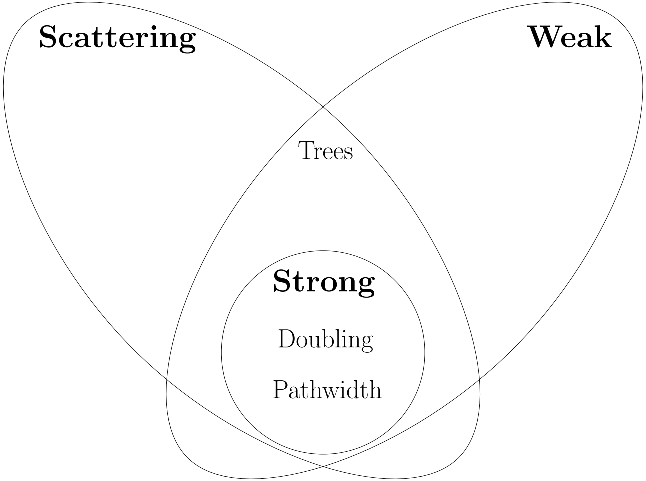
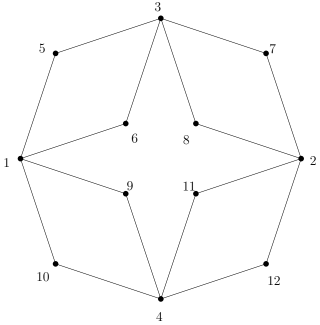
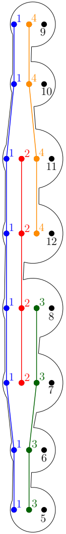
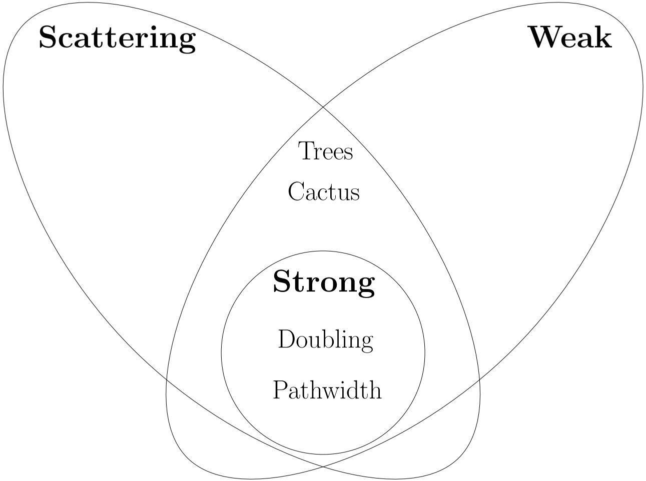


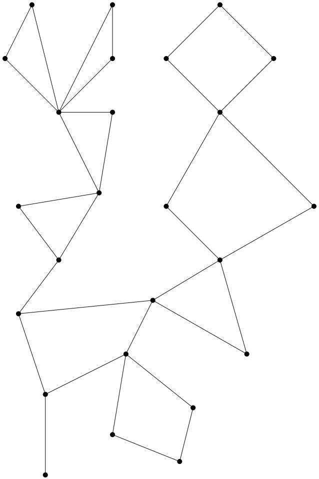




Theorem ([Fil 20])

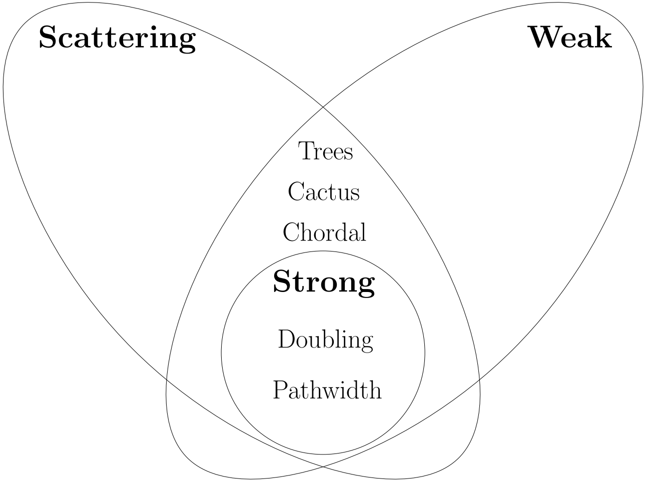
Every graph with **pathwidth ρ** *admits a* $(O(\rho), O(\rho^2))$-**strong** *sparse partition scheme, and a* $(8, 5\rho)$-**weak** *sparse partition scheme.*

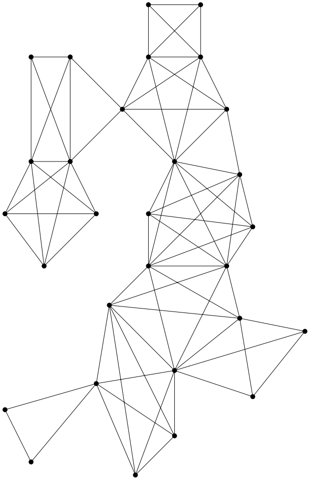




Theorem ([Fil 20])

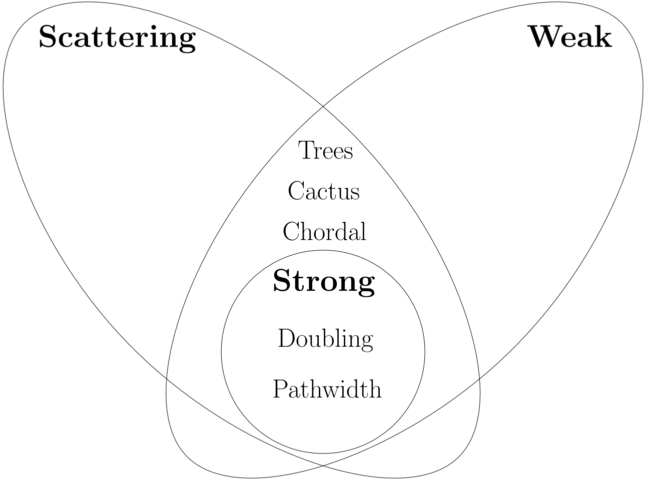
Every **cactus** *graph admits a* $(4, 5)$-**scattering** *partition scheme,*
and a $(O(1), O(1))$-**weak** *sparse partition scheme.*

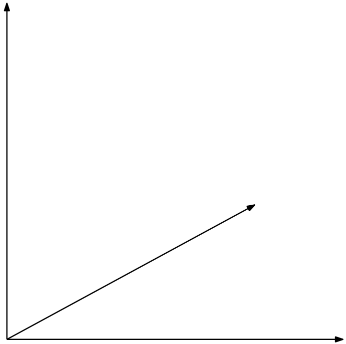




Theorem ([Fil 20])

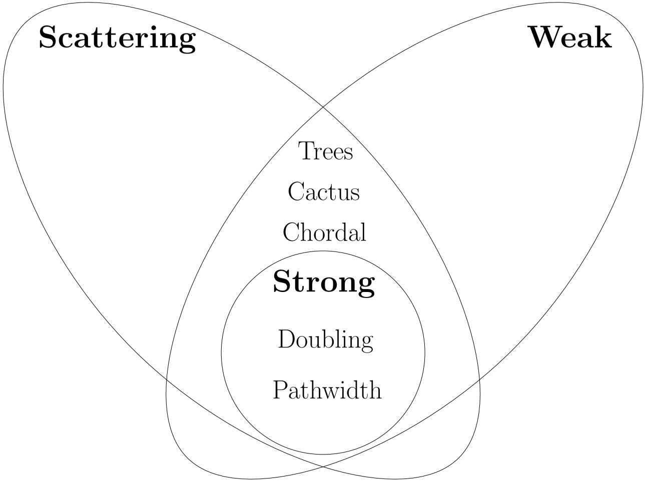
Every **chordal** *graph admits a* $(2,3)$-**scattering** *partition scheme, and a* $(24,3)$-**weak** *sparse partition scheme.*

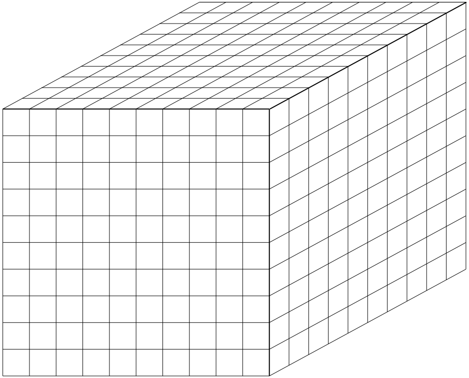




Theorem ([**Fil** 20])

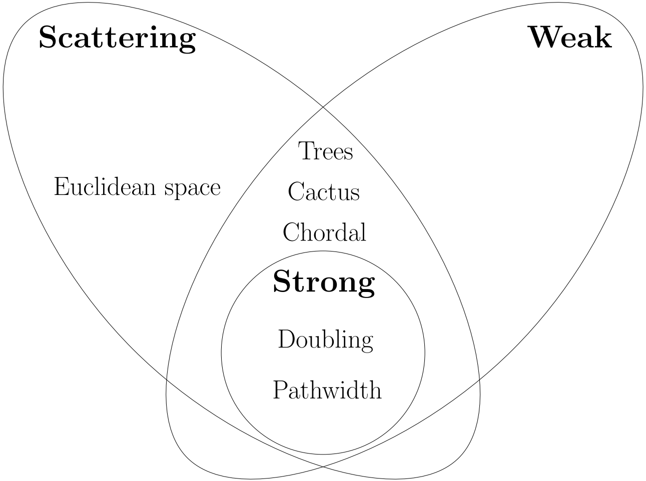
Suppose that $(\mathbb{R}^d, \|\cdot\|_2)$ *admits a* (σ, τ)-**weak** *sparse partition scheme.*
Then $\boldsymbol{\tau} \geq (1 + \frac{1}{2\sigma})^d$ *(alternatively* $\sigma > \frac{d}{4\ln\tau}$*).*

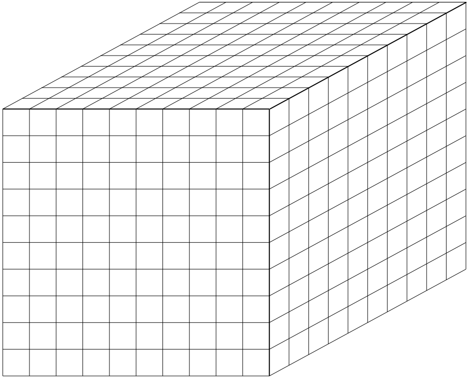




Theorem ([Fil 20])

The space $(\mathbb{R}^d, \|\cdot\|_2)$ *admits a* $(1, 2d)$-**scattering** *partition scheme.*

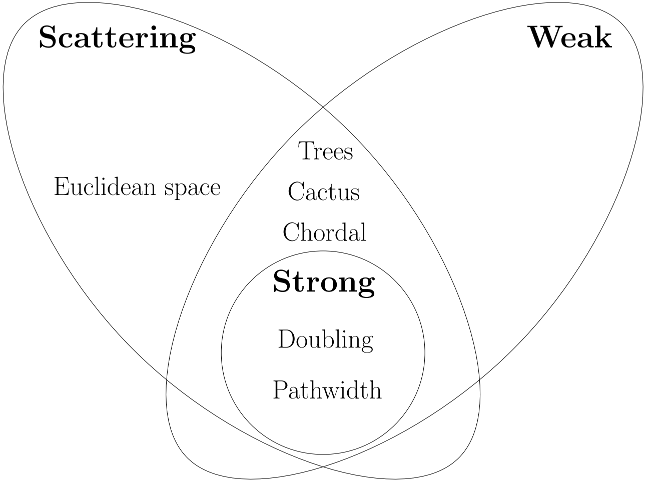


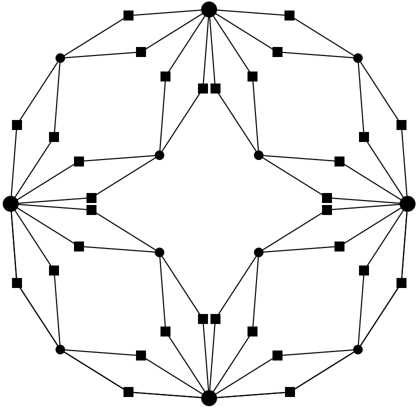


Theorem ([Fil 20])

The space $\left(\mathbb{R}^d, \|\cdot\|_2\right)$ *admits a* $(1, 2d)$-**scattering** *partition scheme.*

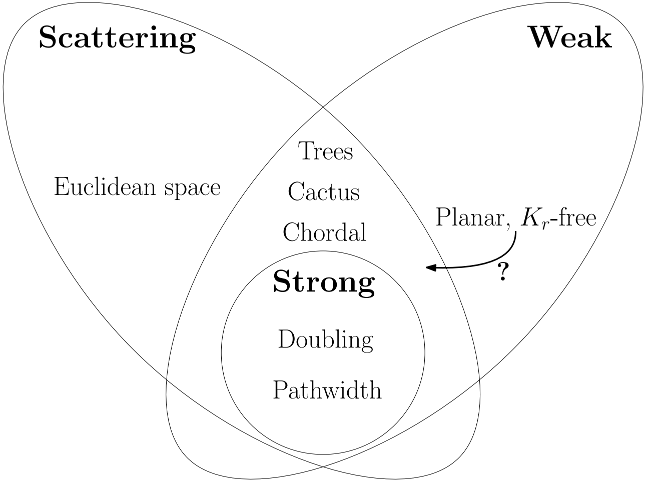
(For weak: $\boldsymbol{\tau} \geq (1 + \frac{1}{2\sigma})^d \Rightarrow$ *no* $(O(1), 2^{\Omega(d)})$*-weak partition scheme).*

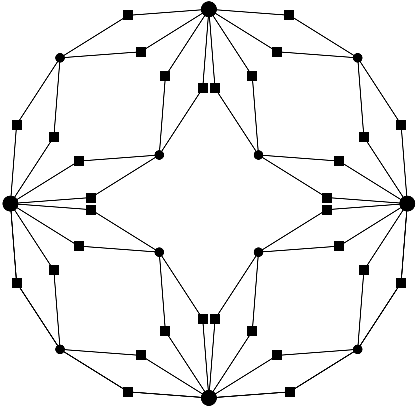




Theorem ([Fil 20])

*Every $K_{r,r}$-free graph admits an $(O(r^2), 2^r)$-**weak** sparse partition scheme.*





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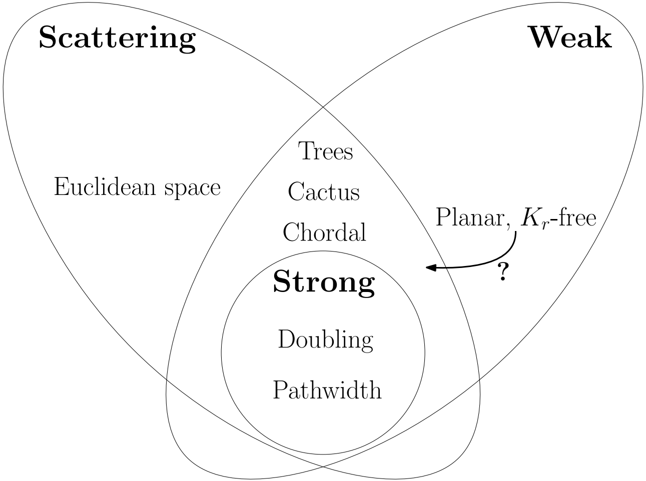
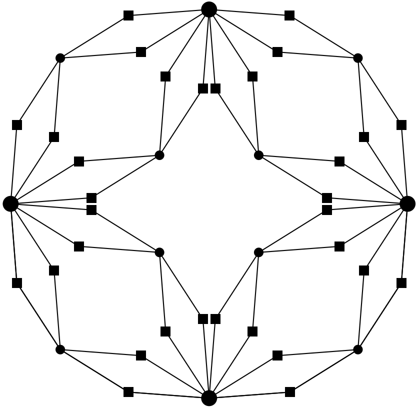
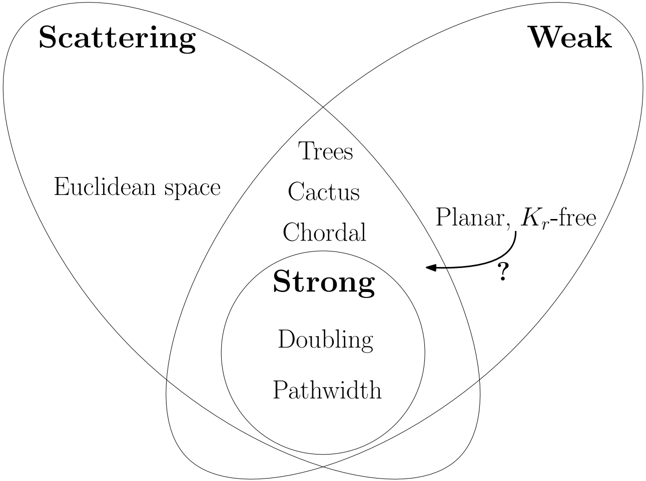
*What about **scattering**?*

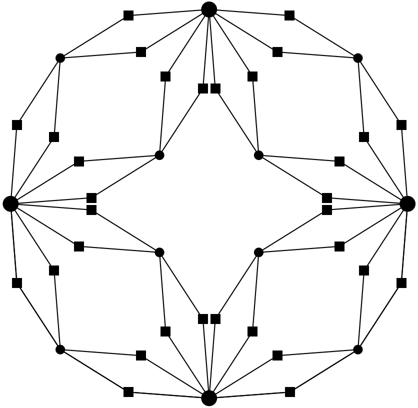




Conjecture

Planar *graphs are* $(O(1), O(1))$-**scattering**.





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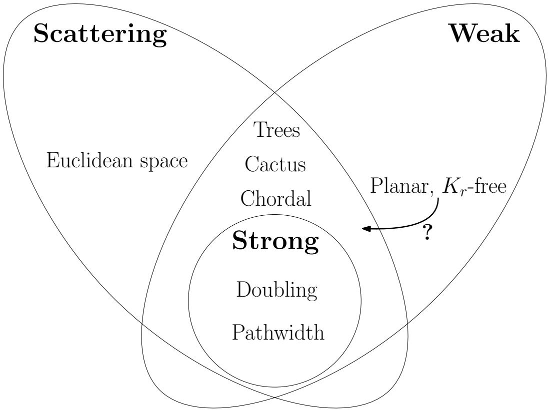
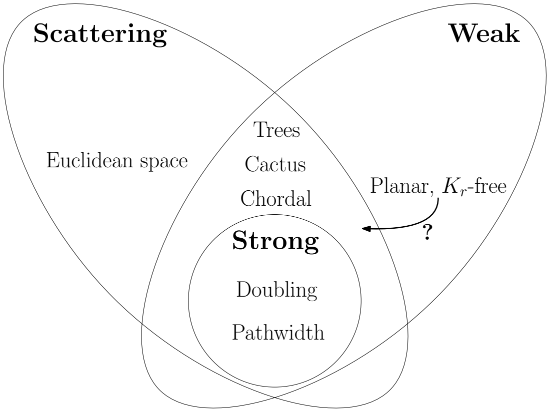
Will imply a solution for the **SPR** problem with **distortion** $\boldsymbol{O(1)}$ for **planar** graphs!



Consider a **general** weighted n vertex graph G:

- [JLNRS 05]: G admits $(O(\log n), O(\log n))$-**weak** sparse partition scheme.



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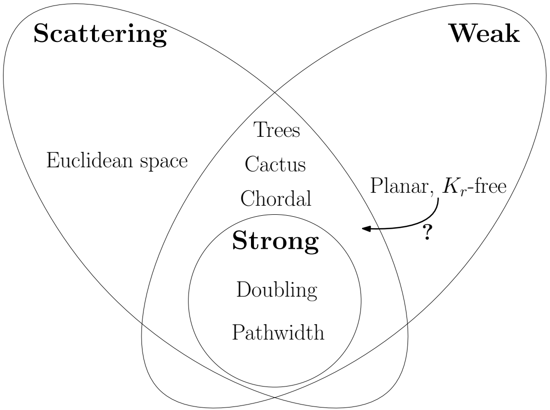

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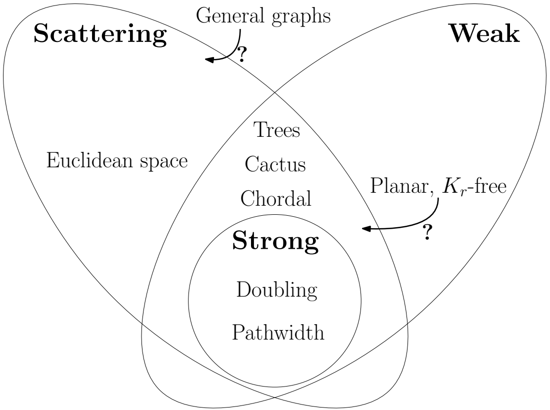

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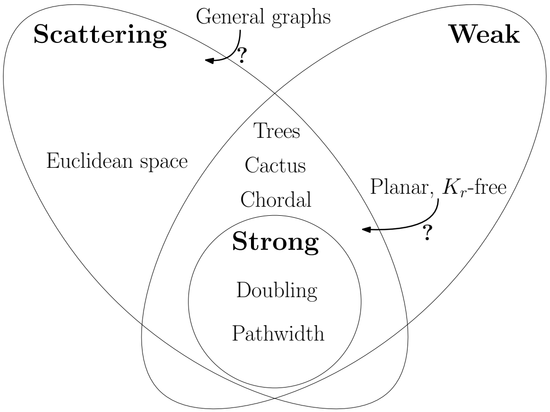
- [**Fil** 20]: G admits $(O(\log n), O(\log n))$-**strong** sparse partition scheme.
- [**Fil** 20]: $\exists G$ which **do not** admit $(O(\frac{\log n}{\log\log n}), O(\log n))$-**weak** sparse partition scheme.



Conjecture

*Every n vertex graph admits $(O(1), O(\log n))$-***scattering** *partition scheme. Furthermore, this is* **tight**.

Theorem ([JLNRS 05])

*Suppose G admits (σ, τ)–***weak sparse*** *partition scheme,*
$\Rightarrow$ *solution to the* **UST** *problem with stretch $O(\tau\sigma^2 \log_\tau n)$.*

Theorem ([Fil 20])

Suppose that every **induced subgraph** *$G[A]$ of G admits (σ, τ)-scattering partition scheme,* $\Rightarrow$ *solution to the* **SPR** *problem with distortion $O(\tau^3\sigma^3)$.*

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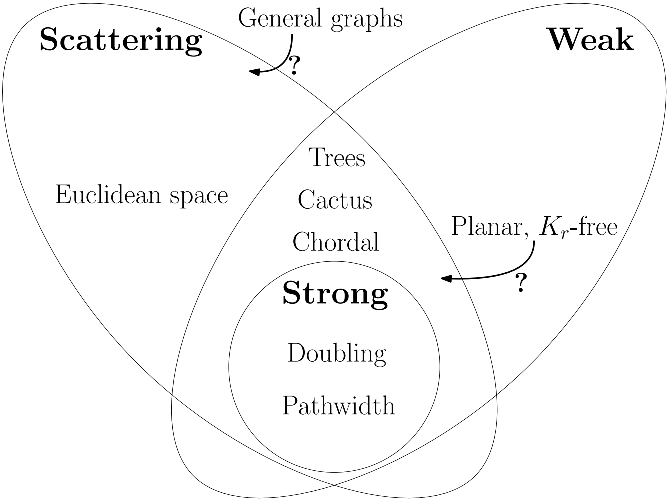
Conjecture

Treewidth k *graphs are* $(f(k), g(k))$-**scattering**.

Conjecture

General n *vertex graph are* $(O(1), O(\log n))$-**scattering**. *Furthermore, this is tight.*



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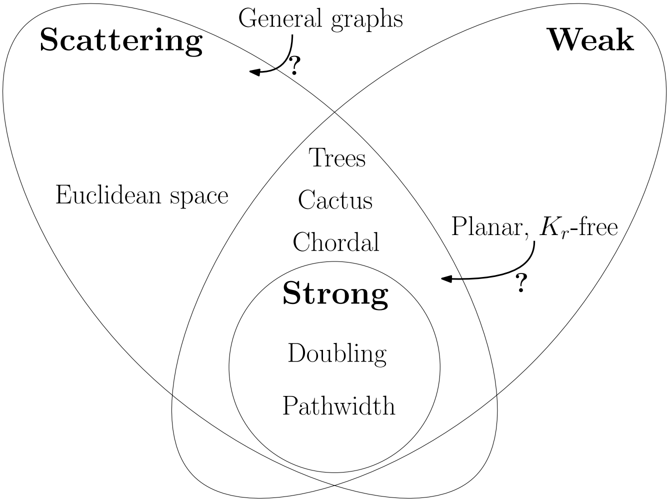

Thank you for listening!