1 Configuration Space Path Planning

- Reference: 1) A Simple Motion Planning Algorithm for General Purpose Manipulators by T. Lozano-Perez, 2) Siegwart, section 6.2.1
- Fast, simple to implement
- Can handle multiple DOF Robots, complicated non-convex obstacles
- Can plan motion in cluttered environments
- A key component of task level programming "Plan Path (collision free) from start to goal"
- Use an approximate, discrete, quantized approach to reduce mathematical complexity. No attempt to model object surfaces
- Configuration: Set of parameters that completely specify the position of an object. We can use joint space for N-DOF Robot as its configuration. Cartesian not unique
- C-Space: Set of all possible configurations
- Key idea for Path Planning: Map obstacles into the robot's C-Space. Creates regions of C-Space that contain obstacles and free-space. Then plan path in C-Space.
- Problem: Joint Space can still be high dimensional.
- Solution: Use "slices" of C-Space for path planning. Projections of N-DOF space into set of N-1 Dimensional Projections.
- 2-D C-Space creation and Path Planning is simple. Represent 3-D as 2-D slices and plan in these spaces.
- Basic Idea: Determine range of legal joint values for a joint parameter given ranges of the previous joints.
- Advantages: Planning becomes simple search in 1 and 2-D space. Disadvantages: Loss of accuracy (granularity of projection), increased storage as DOF increases.
- Tradeoff: Simple vs. intersecting high DOF manifolds of objects and manipulators.

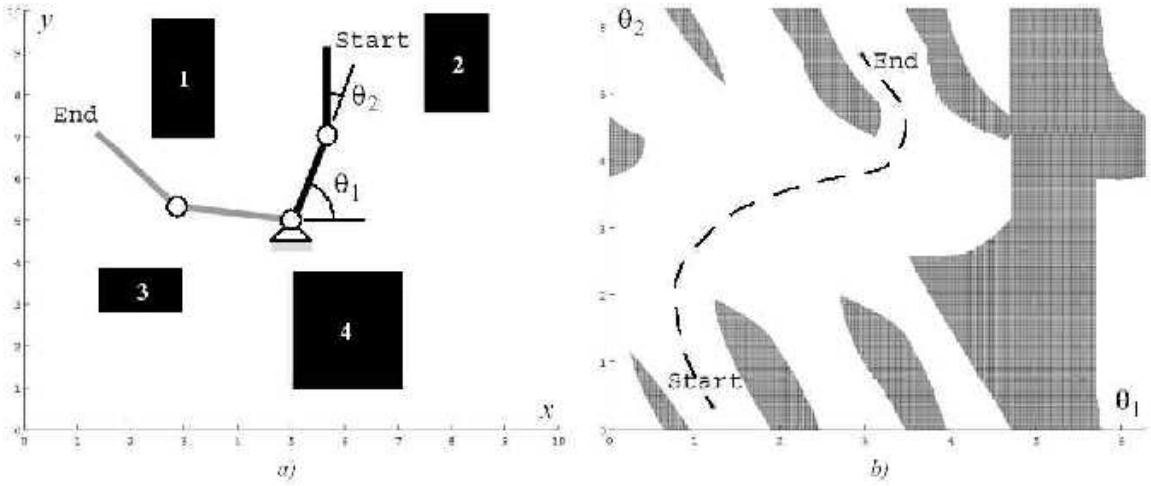
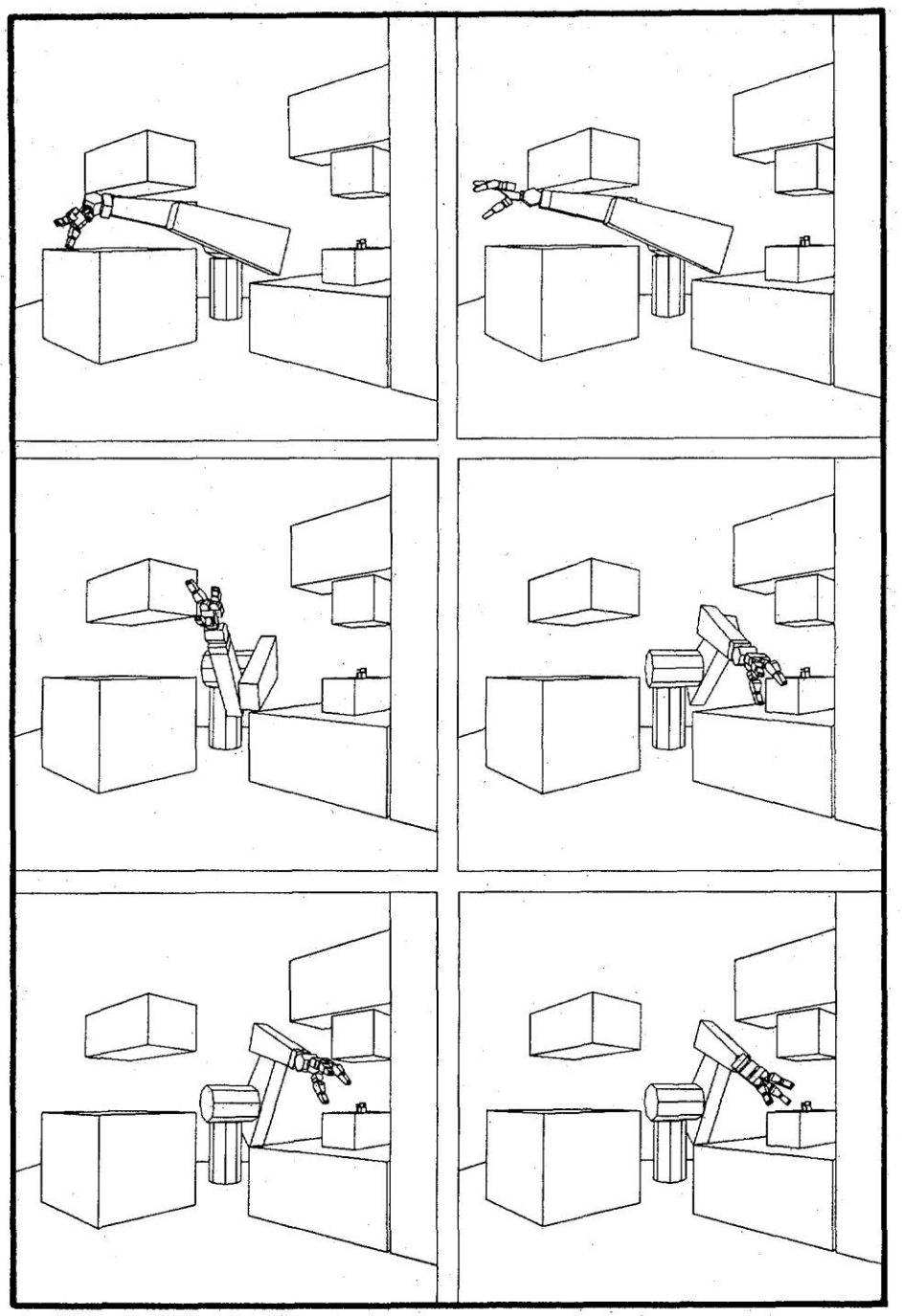


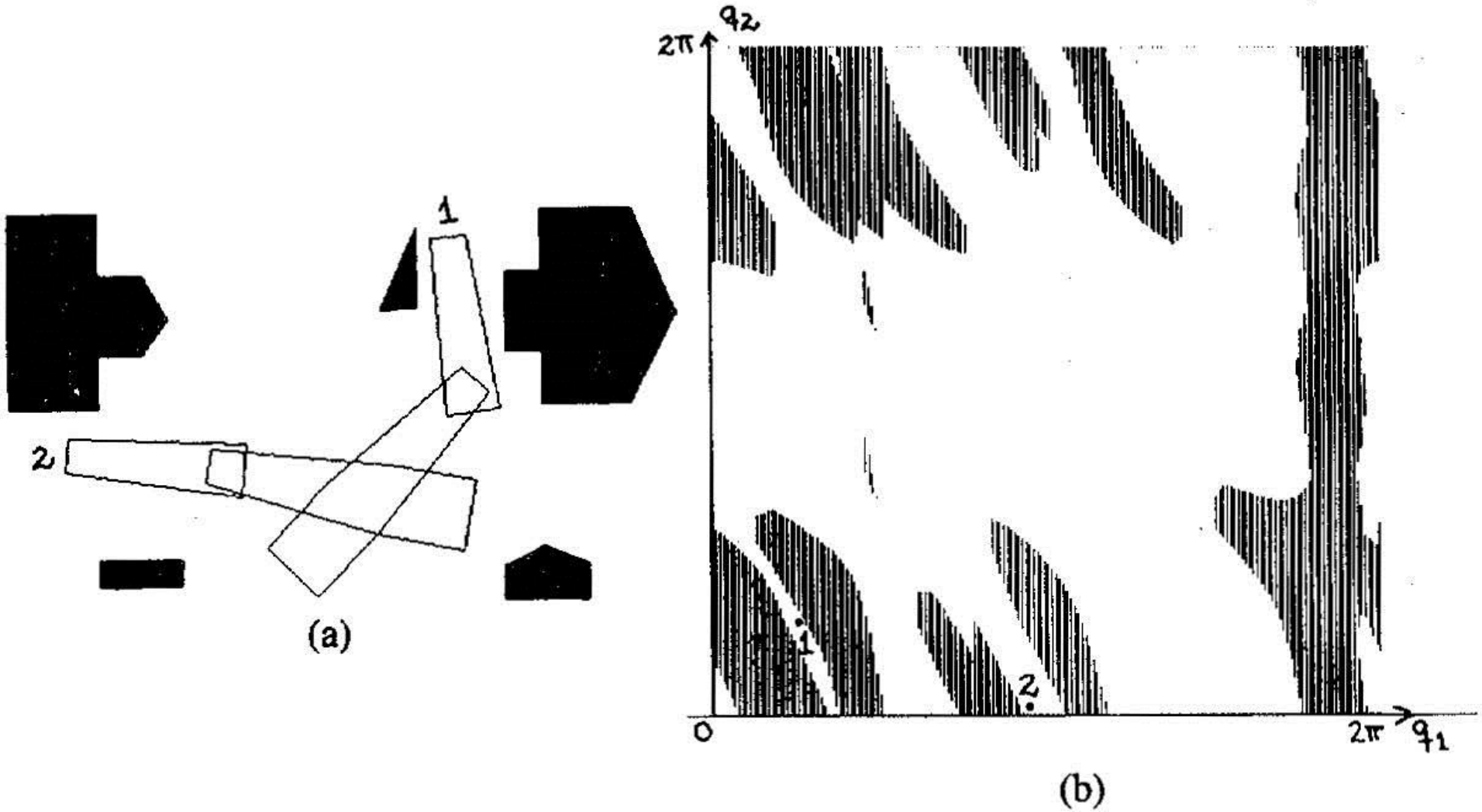
Figure 1: Configuration Space for Two Link Manipulator with obstacles

Manipulator Path Planning in 6-DOF with Clutter

- Exact solutions are computationally intractable
- Rather than plan in Cartesian Space, use the configuration space of the robot
- Set of 6 joint angles defines the robot's configuration
- Set of 3 joint angles (shoulder, elbow, wrist) defines MOST of the workspace asuuming a small gripper
- Can use simple bounding box to define the gripper

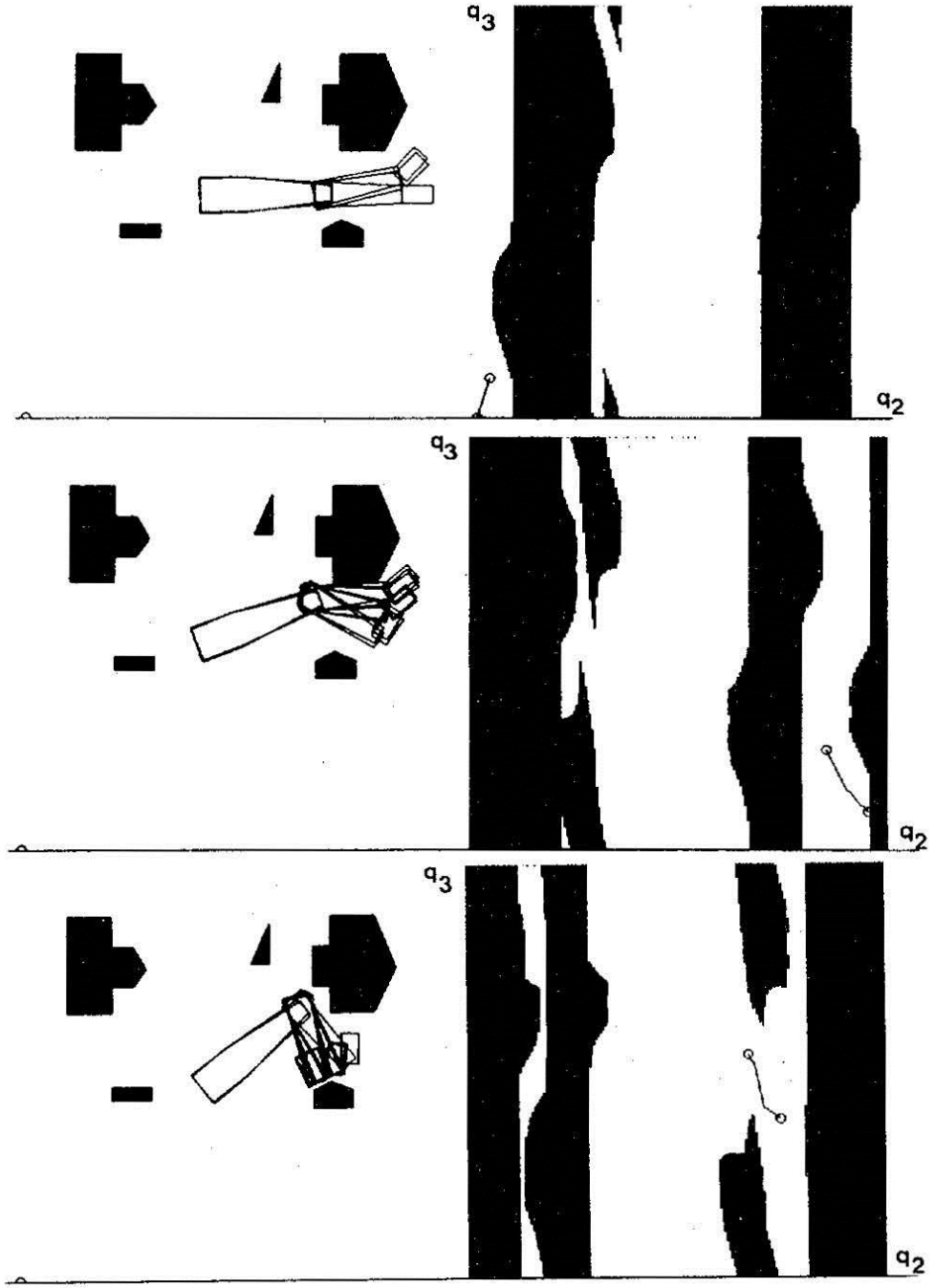


Obstacles mapped into the robot's C-Space



Using C-Space slices

- Each row shows Cartesian and C-Space constraints for a given value of joint 1 (q_1) – a Slice Projection
- Dotted path shows the Cartesian movement mapped to C-Space
- By creating multiple C-Space slices for a range of q_1, we can plan a collision free path in C-Space



Slice Projections

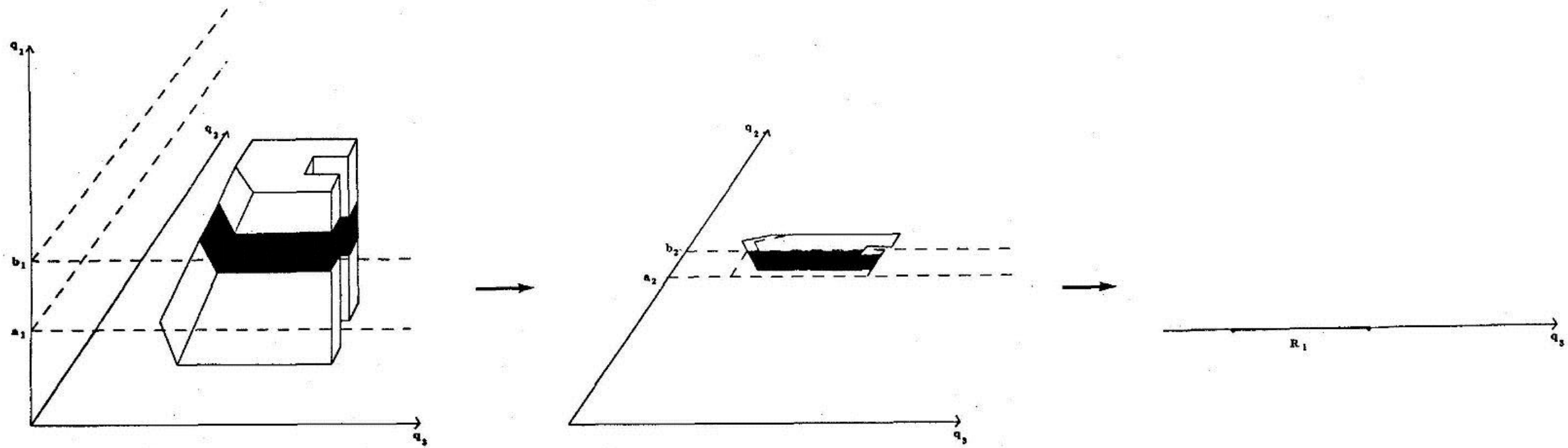


Fig. 3. Slice projection of three-dimensional obstacle into list of two-dimensional slices that are in turn represented by one-dimensional slices.

Representing C-Space: Multi-level Tree

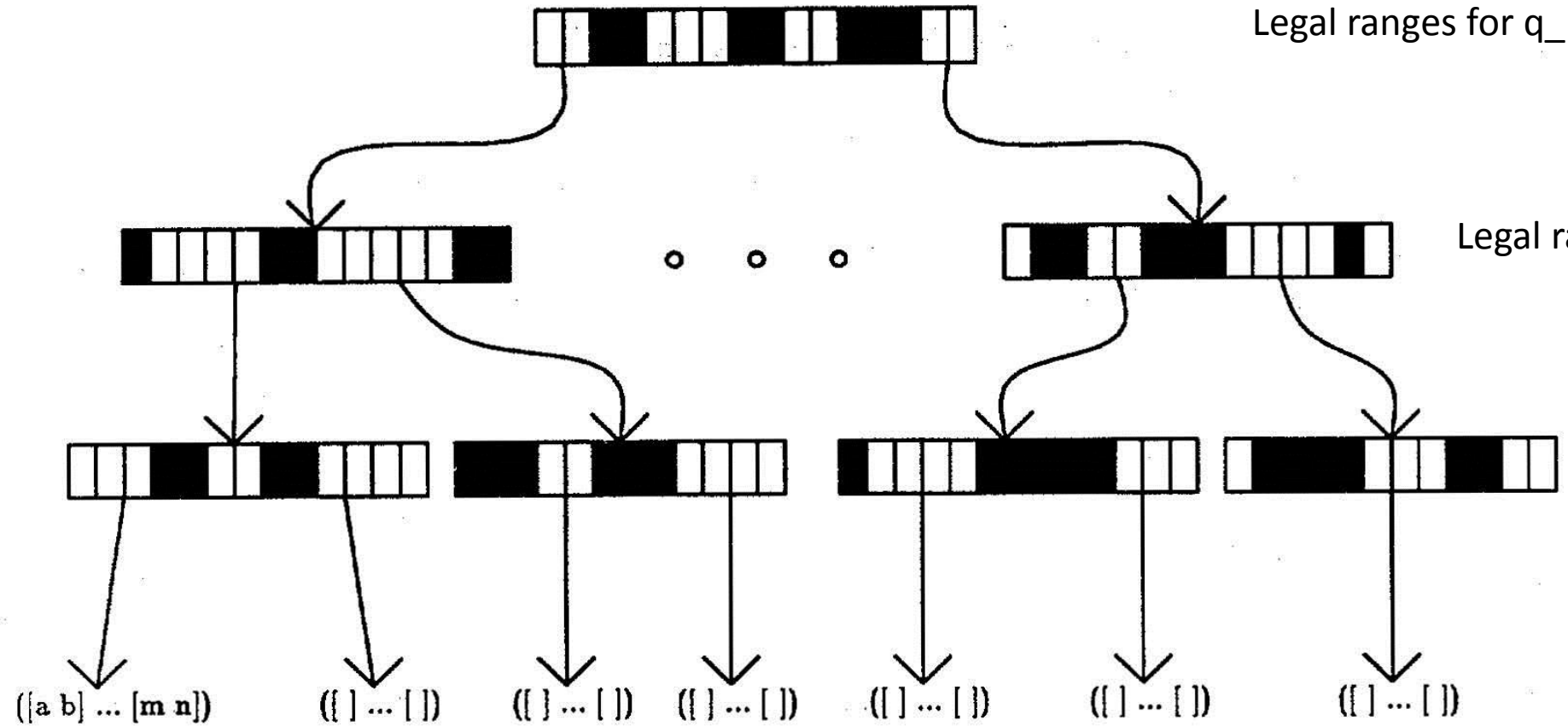


Fig. 6. Recursive nature of C space leads to recursive data structure: an n-level tree whose leaves represent legal ranges of configurations for robot manipulator.

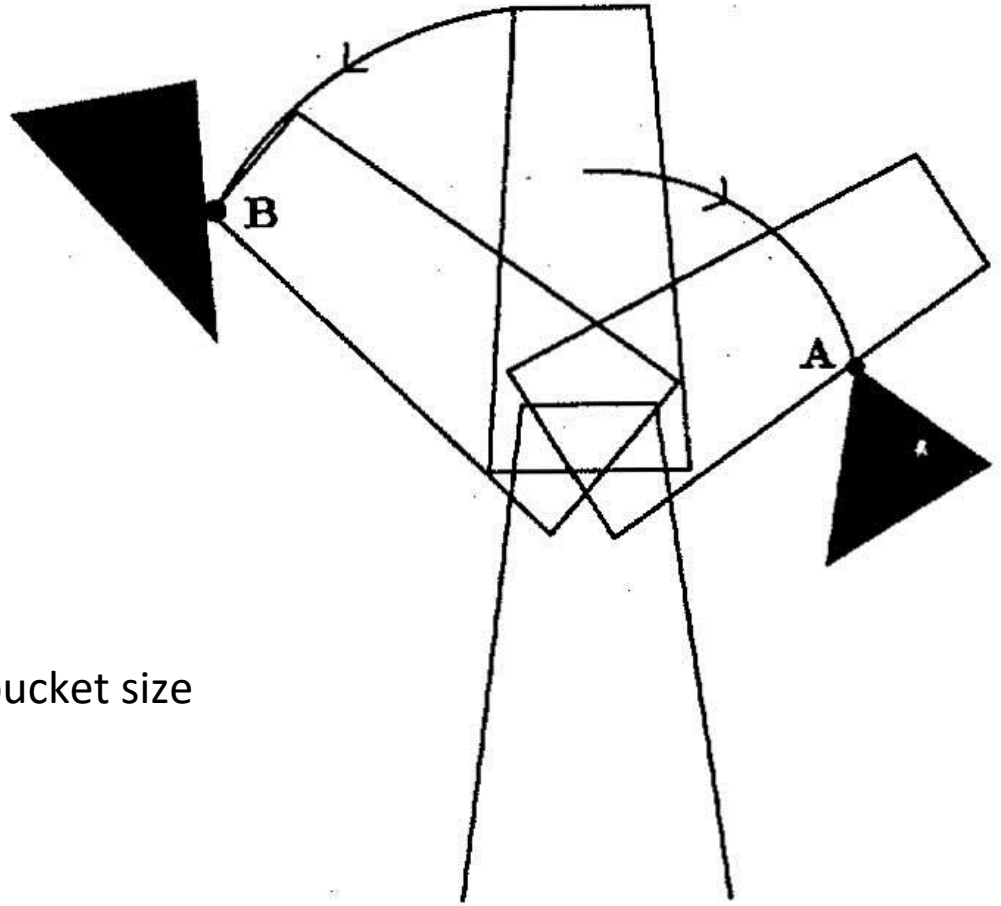
Computing legal ranges of joint angles

- Collisions are either:
  - Link vertex hitting obstacle edge – Contact B
  - Link edge hitting obstacle vertex – Contact A

- Every vertex of a link follows a circular path
- Every obstacle vertex has circular path relative to the link
- Can easily compute these legal ranges, joint by joint

Problem: Discrete Bucket sizes - Link may be in contact

Solution: Grow links by small amount to cover range of discrete bucket size



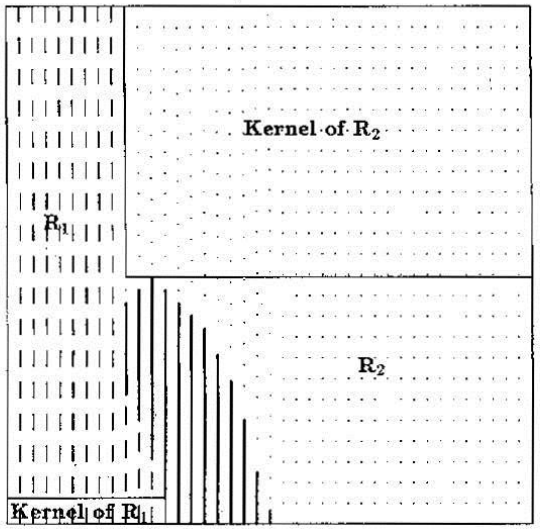


Fig. 13. Illustration of definition of free-space regions. Bold lines indicate configuration space obstacles. Two regions are indicated in dashed lines. Kernels are rectangular areas within regions corresponding to common intersection of all free ranges in region.

Region: Vertical adjacent slices with overlap

Kernel of a region: q_2 values common to all vertical slices of q_1 moving left to right

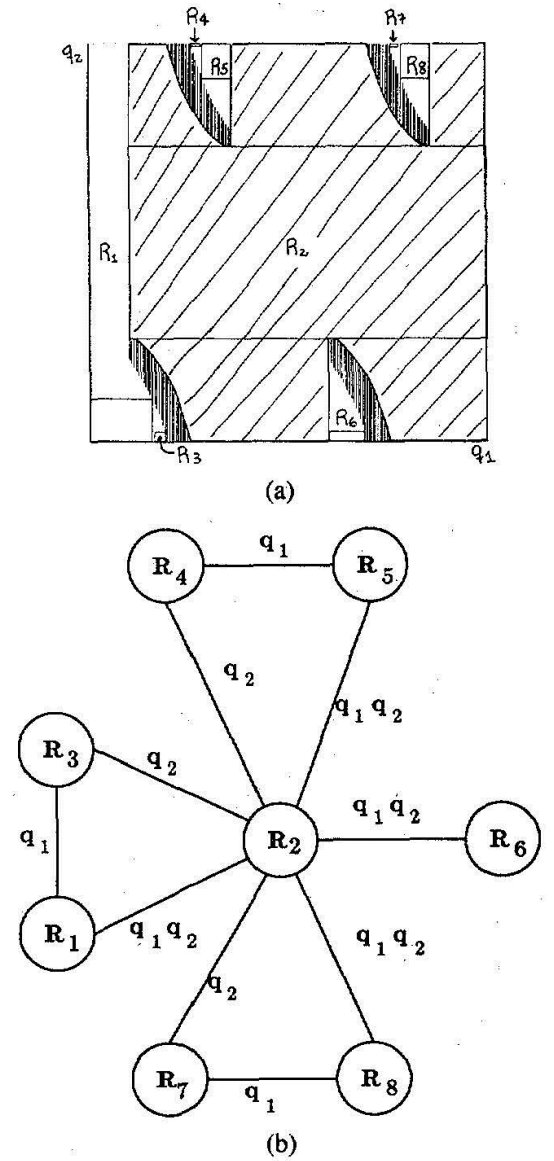


Fig. 14. (a) Regions for two-joint C space. Rectangles are region *kernels*. Hashed area shows region R_2. (b) Region graph corresponding to regions in part A. Link labels indicate existence of common boundary in q_1 and/or q_2 directions.

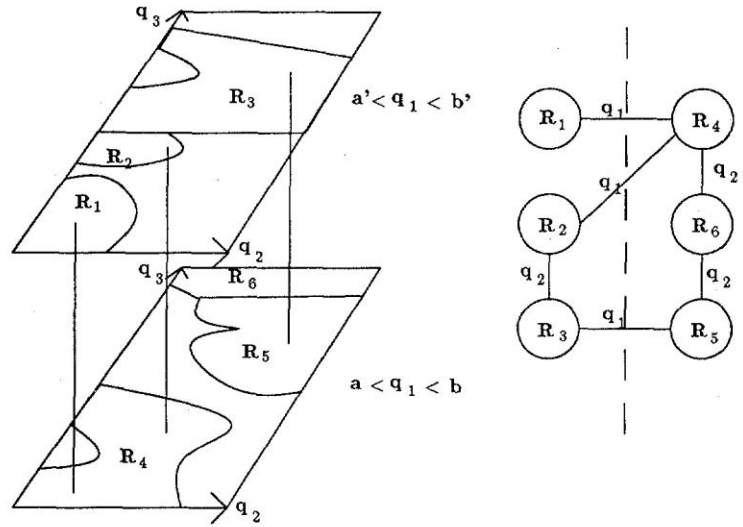


Fig. 15. Region connectivity for three dimensional slices; regions can have neighbors in q_1 direction.

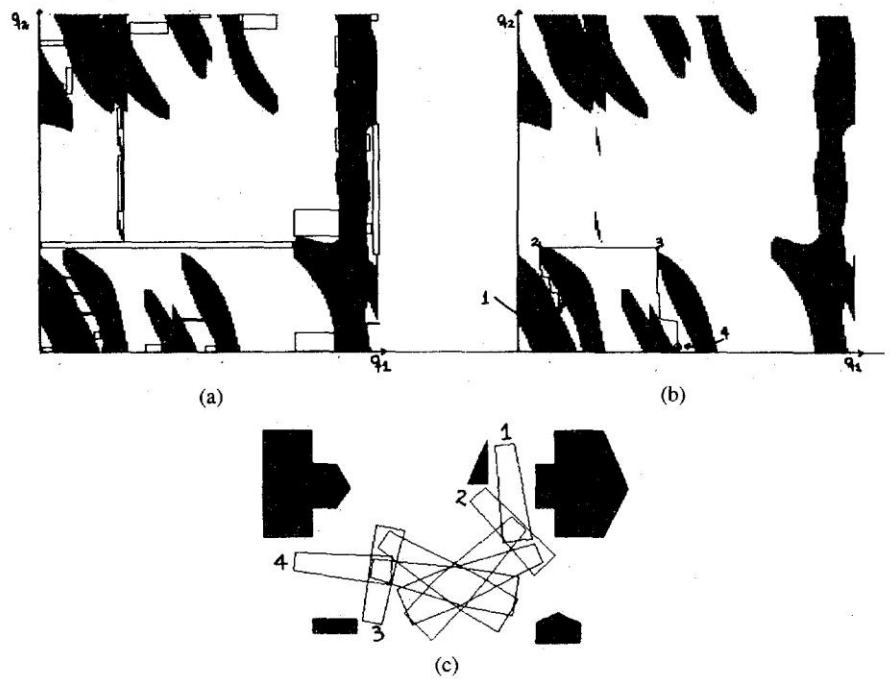


Fig. 16. (a) Regions kernels for example in Fig. 4. (b) Path found between start (1) and goal (4) configurations. (c) Some intermediate configurations.

4-DOF Manipulator Path Plan

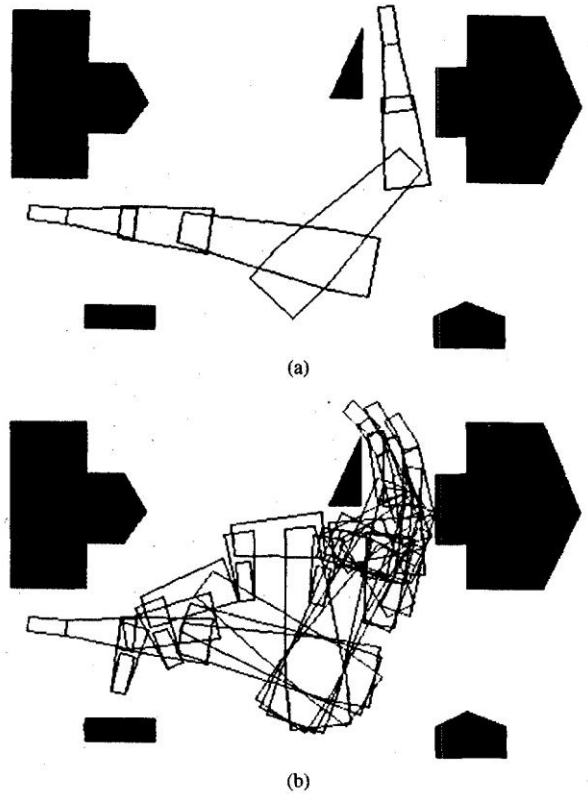


Fig. 18. (a) Initial and goal configurations for two-dimensional manipulator with four degrees of freedom. (b) Path found by algorithm in Section VII-B.

2 Configuration Space Path Planning Examples

Below are some simple examples of how Configuration Space (CSPACE) path planning works. We will be using a two-link, planar, pick-and-place manipulator as in figure 1. This design allows us to avoid computing arm collisions with the obstacles - we just have to worry about the gripper (assumed to be a point) moving into an obstacle.

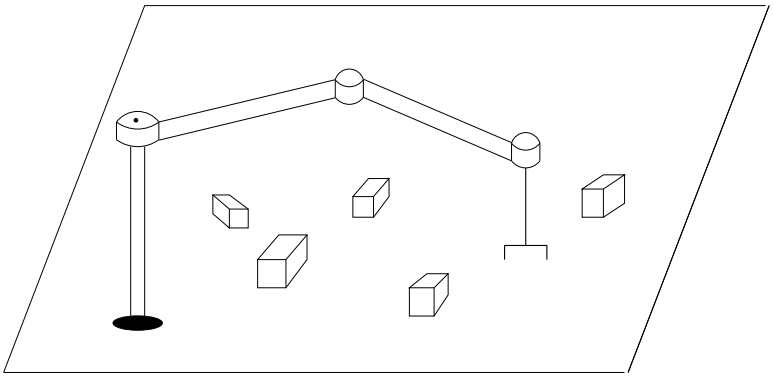


Figure 2: Two Link Manipulator used in examples

We give the system Cartesian start and goal positions. Each Cartesian position has 2 inverse kinematic solutions, so the path planner can plan 4 distinct paths from each distinct start and goal position. A set of known obstacles is given as part of the environment. Here is the algorithm:

1. Partition each of the robot's joints into discrete regions, say every 5 degrees. This creates a $72x72$ discrete angular grid.
2. Iterate over all combinations of joint 1 and joint 2 (we use the center of each 5 degree range as the test point). If the forward kinematics for these joint angles lies inside an obstacle, we classify this part of the $72x72$ grid as $forbidden$, otherwise it is a legal configuration.
3. Given a Cartesian start and end position, do the following:
   (a) Using Inverse Kinematics, find the solutions to $(\theta 1\ ,\ \theta 2)$ for the start and goal position. There will be 2 solutions for each position.
   (b) Choosing 1 of the 4 possible solutions, perform a breadth-first search in the joint-space grid from $(\theta 1_{start}, \theta 2_{start})$ to $(\theta 1_{goal}\ ,\ \theta 2_{goal})$. Mark the path.

We will use a simple Breadth First Search in Configuration Space of the manipulator to find a path in free space between start and goal positions represented as joint configurations. Pseudo-code below.

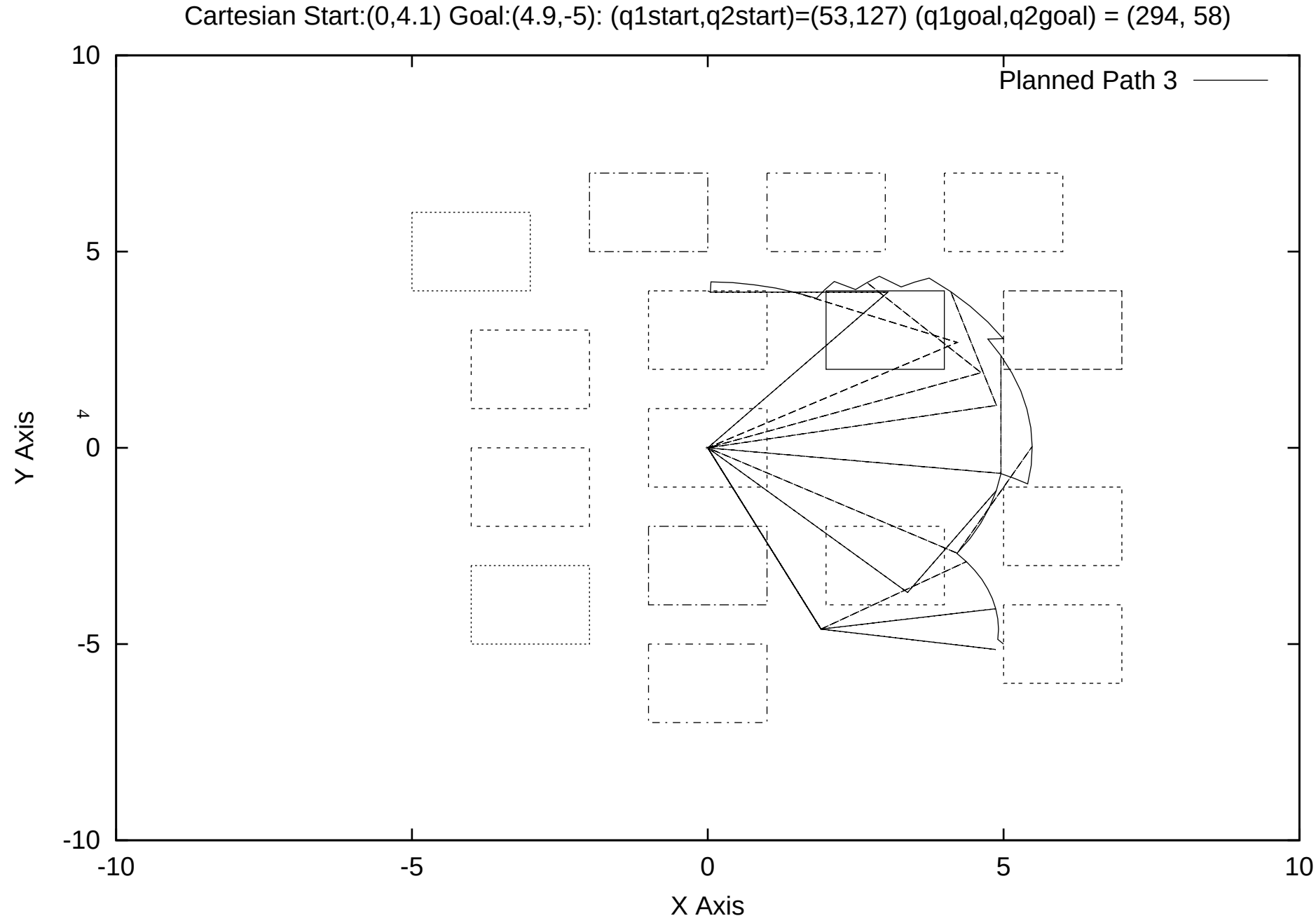
FINDPATH($\theta_{1start}\ ,\ \theta_{2start}\ ,\ \theta_{1goal}\ ,\ \theta_{2goal}$**)**
if($(\theta_{1start}\ ,\ \theta_{2start})\ \equiv\ (\theta_{1goal}\ ,\ \theta_{2goal})$**) then stop-GOAL=FOUND**
ADD($(\theta_{1start}\ ,\ \theta_{2start})$**) to QUEUE and mark as VISITED**
While QUEUE NOT EMPTY and GOAL NOT FOUND do
  POP top item in QUEUE and assign to ($\theta_1\ ,\ \theta_2$**)**
  if($(\theta_1\ ,\ \theta_2)\ \equiv\ (\theta_{1goal}\ ,\ \theta_{2goal})$**) then GOAL=FOUND**
    else ADD to QUEUE each Free Space UNVISITED 4-neighbor of ($\theta_1\ ,\ \theta_2$**)**
      Mark each neighbor added to QUEUE as VISITED
      and Remember which node Opened this node (its predecessor, ($\theta_1\ ,\ \theta_2$**)**
If GOAL==FOUND
Recreate path by following predecessor chain from ($\theta_{1start}\ ,\ \theta_{2start}$**) to (**$\theta_{1goal}\ ,\ \theta_{2goal}$**)**

There are 4 examples that follow showing the joint-space and Cartesian-space paths for the same Cartesian start and goal positions using each of the 4 possible inverse kinematic solutions. In each example, the manipulator has link lengths $L_1 = 5$ and $L_2 = 3$.

In the joint-space diagrams, the path from start to goal is shown with asterisks. In the Cartesian-space plots, the path and obstacles are shown along with the manipulator links superimposed at every 5th point on the path.

```
       THETA1  THETA2
start  53      127
goal   294     58

Theta 1:                          Theta2
  0   5          11111111 *  1111111                 11111    11111111
  5  10         11111111  * 111111111                  11    11111111
 10  15        111111     *11111111111                     11111111
 15  20      111          ***1111111                     111111
 20  25    1                ***111                       1
 25  30        1111111        ***
 30  35     11111111    111     *                                      1
 35  40   11111111    111111    *  111                            11111111
 40  45 11111111    111111111   *11111111                       11111111     1
 45  50 111111    111111111     *11111111                     11111111    1111
 50  55 1111    11111111        *411111111                  111111111    111111
 55  60 1          11      11    111111111                   1111111    11111111
 60  65                11111    111111111                    1111    111111111
 65  70             111111111    1111111111          111           11111111
 70  75             111111111       1111111         1111111       111111
 75  80             11111111          11111       11111111111    11
 80  85            111                 11111       111111111          1
 85  90                                 1111 111    1111111        1111
 90  95               1111111     11    111 1111     11111    11111111
 95 100             11111111     1111       11111      11    11111111
100 105            11111111    1111111       11111          11111111
105 110           1111111     111111111      1111111       111111
110 115         11111111       11111111      1111111111    1
115 120       1                 11111111      111111111         1111
120 125                         11111111      111111111      11111111
125 130                          11   111     111111116    111111111
130 135                                       11111111    1111111
135 140                                       11111111    11111
140 145                                          111       1     11   111
145 150                          11111                          11111111
150 155                          111111                        11111111
155 160                          11111111                     11111111
160 165                         111111111           1      1111111
165 170                         1111111111        1111        111
170 175                          1111111111    111111111
175 180                            111111111    1111111111
180 185                     1      1111111     1111111111
185 190                    1111     1111   1    11111111
190 195                   1111111           1    111111
195 200                 1111111111          11     1111
200 205                1111111111           11
205 210                  11111                111
210 215                   11                  1111
215 220                         111          111111111
220 225                       11111111        11111111
225 230                   11    11111111      11111111
230 235                 11111    111111111     11111111
235 240              11111111    111111111      11111111
240 245             111111111    111111111      11111111
245 250             11111111     1111111111      11111
250 255             111             1111111      111      1
255 260                            11111              11111
260 265                       11      11111           111111111
265 270                      11111     1111 111      111111111
270 275                     1111111    111 1111     111111111
275 280                    111111111       11111      111111
280 285                   11111111111       11111
285 290                     1111111         1111111             111
290 295            3********** 111           1111111111     11111111
295 300         1111111       *               111111111    111111111
300 305     11111111    111  *                111111111    11111111
305 310   11111111    111111 *                 111111111    11111
310 315 11111111    111111111*                  11111111    11                  1
315 320 111111    111111111***                  11111111                     1111
320 325 1111    11111111   *                     111                       111111
325 330 1          11      *                                             11111111
330 335                    *                                           111111111
335 340                    *                          111           51111111
340 345                    *                        1111111       111111
345 350                    *                       11111111111    11
350 355              11111*   11                   111111111           1
355 360           11111111**  11111                 1111111        1111
Theta 1 0  20  40  60  80 100 120 140 160 180 200 220 240 260 280 300 320 340
                             Theta 2
```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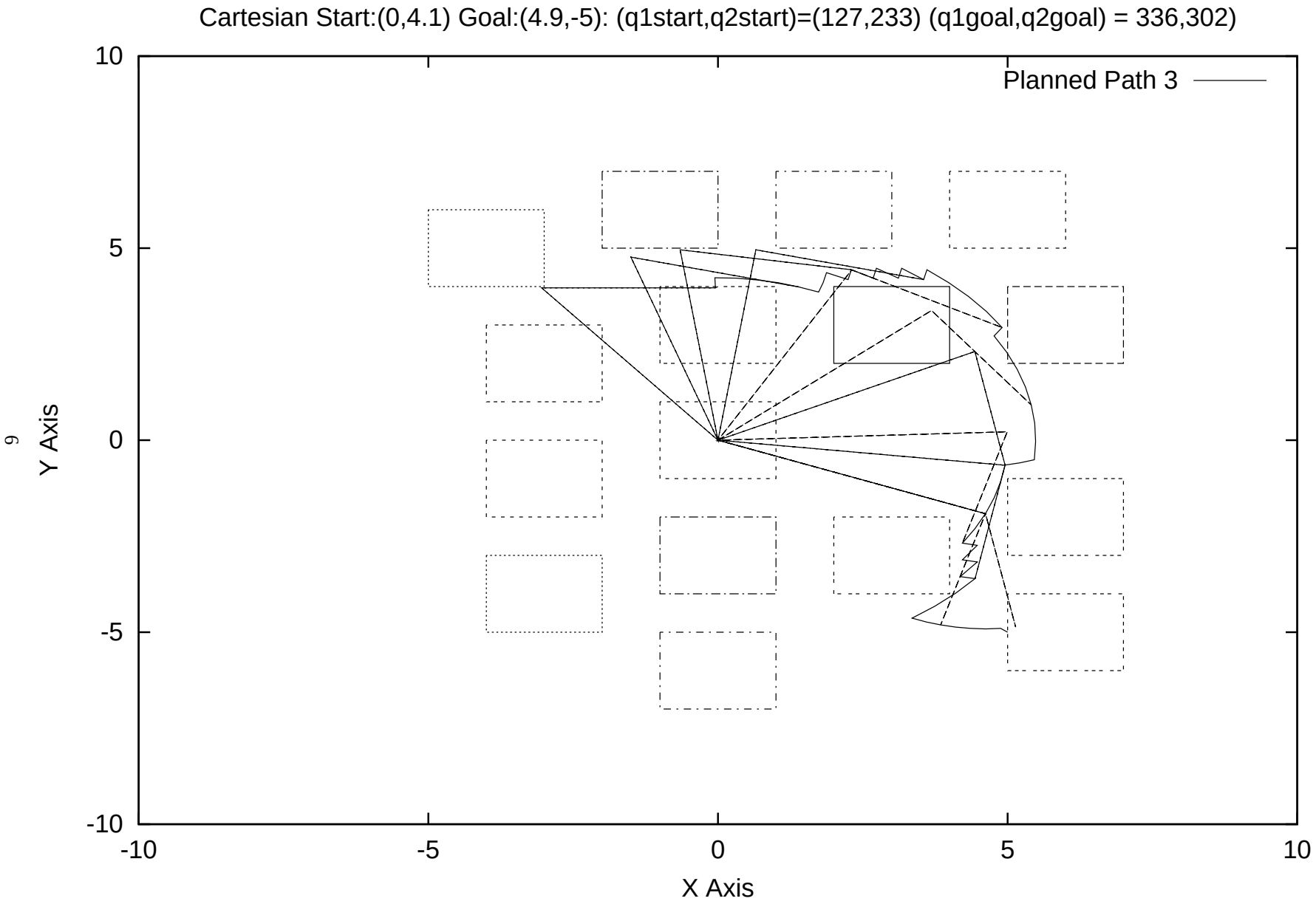
Planned Path 3
10
5
4
0
-5
-10
-10
-5
0
5
10
Y Axis
X Axis

```
      THETA1  THETA2
start  127  233
goal   336  302
Theta 1:                        Theta 2
  0   5          11111111    1111111                 11111**  11111111
  5  10         11111111    111111111                  11 *  11111111
 10  15        111111      11111111111                    * 11111111
 15  20      111             1111111                     *111111
 20  25    1                   111                       *1
 25  30        1111111                                   ***
 30  35     11111111    111                                *                1
 35  40   11111111    111111       111                     *       11111111
 40  45 11111111    111111111    11111111                  *     11111111     1
 45  50 111111    111111111      11111111                  *   11111111    1111
 50  55 1111    11111111         411111111                 *111111111    111111
 55  60 1          11      11    111111111                 *1111111    11111111
 60  65               11111    111111111                  **1111    111111111
 65  70             111111111    1111111111           111    *      11111111
 70  75             111111111       1111111         1111111  *    111111
 75  80             11111111          11111       11111111111*   11
 80  85            111                 11111       111111111**         1
 85  90                                 1111 111    1111111**      1111
 90  95               1111111     11    111 1111     11111**  11111111
 95 100             11111111     1111       11111      11**  11111111
100 105            11111111    1111111       11111     ***  11111111
105 110           1111111     111111111      1111111   *   111111
110 115         11111111       11111111      1111111111*   1
115 120       1                 11111111      111111111*        1111
120 125                         11111111      111111111*     11111111
125 130                          11   111     111111116*   111111111
130 135                                       11111111    1111111
135 140                                       11111111    11111
140 145                                          111       1     11   111
145 150                           11111                        11111111
150 155                           111111                     11111111
155 160                           11111111                  11111111
160 165                          111111111           1      1111111
165 170                          1111111111        1111        111
170 175                           1111111111    111111111
175 180                             111111111    1111111111
180 185                     1      1111111     1111111111
185 190                    1111     1111   1    11111111
190 195                  1111111           1    111111
195 200                 1111111111          11     1111
200 205                1111111111           11
205 210                  11111               111
210 215                  11                  1111
215 220                         111         111111111
220 225                      11111111      11111111
225 230                   11    11111111     11111111
230 235                 11111    111111111     11111111
235 240              11111111    111111111      11111111
240 245             111111111    111111111      11111111
245 250             11111111     1111111111      11111
250 255             111             1111111      111      1
255 260                            11111              11111
260 265                       11      11111           111111111
265 270                     11111     1111 111      111111111
270 275                    1111111    111 1111     111111111
275 280                   111111111       11111      111111
280 285                  11111111111       11111
285 290                    1111111         1111111             111
290 295            3         111           1111111111     11111111
295 300         1111111                     111111111    111111111
300 305     11111111    111                  111111111    11111111
305 310   11111111    111111                 111111111    11111
310 315 11111111    111111111                 11111111    11                  1
315 320 111111    111111111                   11111111                    1111
320 325 1111    11111111                        111                     111111
325 330 1          11                                                11111111
330 335                                                             111111111
335 340                                            111    *******51111111
340 345                                           1111111  *    111111
345 350                                          11111111111*   11
350 355              11111    11                  111111111**         1
355 360           11111111    11111                1111111**      1111
        0  20  40  60  80 100 120 140 160 180 200 220 240 260 280 300 320 340
                                Theta 2
```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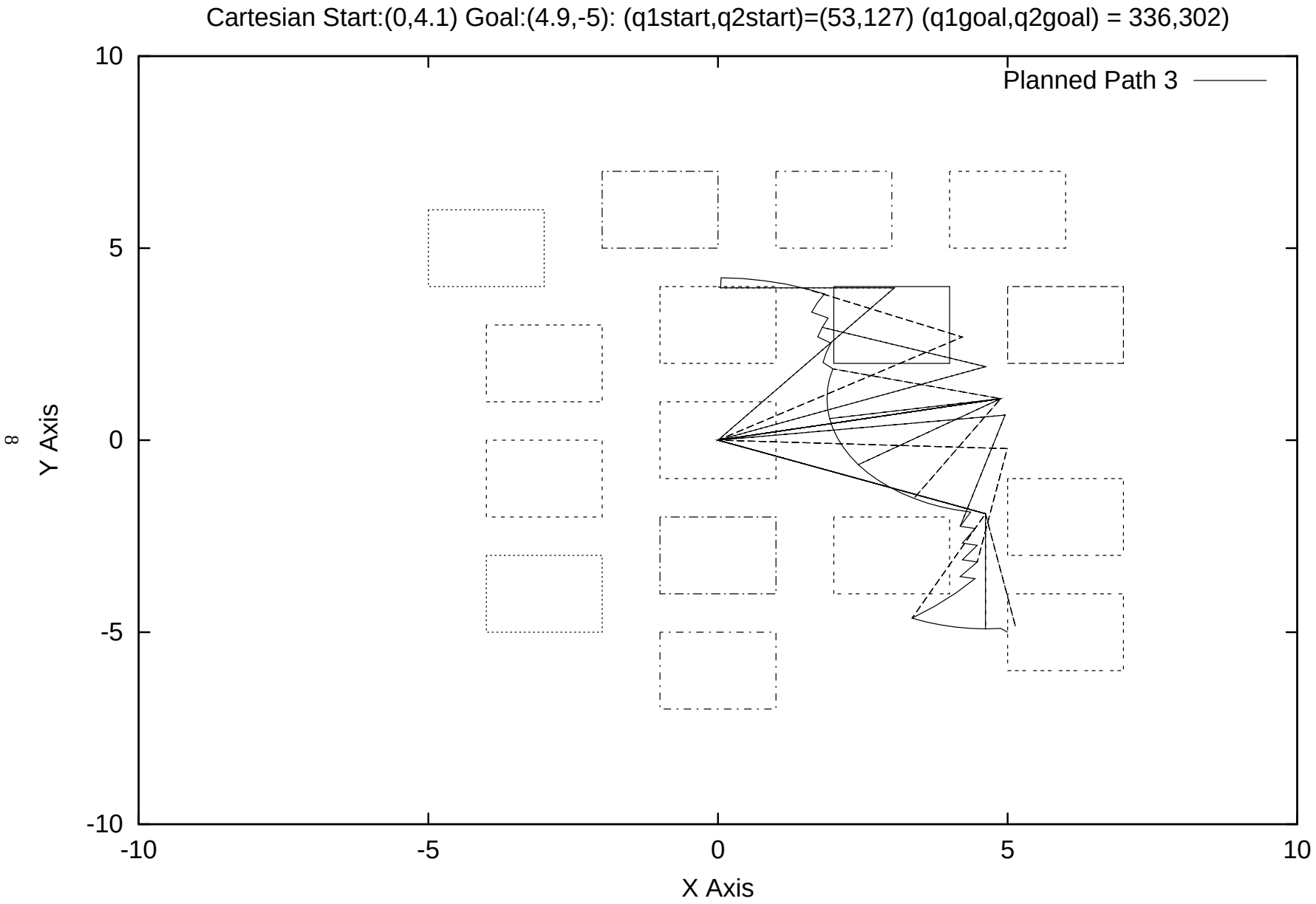
Planned Path 3
Y Axis
X Axis
10
5
0
-5
-10
-10
-5
0
5
10

```
       THETA1  THETA2
start   53      127
goal   336      302
                                Theta2
  0   5          11111111    1111111                 11111**  11111111
  5  10         11111111    111111111                  11**  11111111
 10  15        111111      1111111111*******************  11111111
 15  20      111             1111111***                  111111
 20  25    1                   111***                    1
 25  30         1111111          ***
 30  35     11111111    111     *                                        1
 35  40   11111111    111111    *  111                            11111111
 40  45 11111111    111111111   *11111111                       11111111     1
 45  50 111111    111111111     *11111111                     11111111    1111
 50  55 1111    11111111        *411111111                  111111111    111111
 55  60 1           11      11    111111111                  1111111    11111111
 60  65                11111    111111111                   1111    111111111
 65  70             111111111    1111111111           111           11111111
 70  75             111111111       1111111         1111111       111111
 75  80             11111111           11111       11111111111    11
 80  85            111                   11111       111111111           1
 85  90                                   1111 111    1111111        1111
 90  95               1111111     11    111 1111     11111    11111111
 95 100             11111111     1111       11111      11    11111111
100 105            11111111    1111111       11111          11111111
105 110           1111111     111111111      1111111       111111
110 115         11111111       11111111      1111111111    1
115 120       1                 11111111      111111111        1111
120 125                         11111111      111111111      11111111
125 130                          11   111     111111116    111111111
130 135                                       11111111    1111111
135 140                                       11111111    11111
140 145                                          111       1     11   111
145 150                           11111                       11111111
150 155                           111111                    11111111
155 160                           11111111                 11111111
160 165                          111111111           1      1111111
165 170                          1111111111        1111        111
170 175                           1111111111    111111111
175 180                             111111111    1111111111
180 185                     1      1111111     1111111111
185 190                    1111     1111   1    11111111
190 195                   1111111           1    111111
195 200                  1111111111          11     1111
200 205                 1111111111           11
205 210                   11111               111
210 215                   11                  1111
215 220                         111          111111111
220 225                      11111111         11111111
225 230                   11    11111111     11111111
230 235                  11111    111111111    11111111
235 240              11111111    111111111      11111111
240 245             111111111    111111111      11111111
245 250             11111111     1111111111      11111
250 255             111             1111111      111      1
255 260                            11111              11111
260 265                       11      11111           111111111
265 270                      11111     1111 111      111111111
270 275                     1111111    111 1111     111111111
275 280                    111111111       11111      111111
280 285                   11111111111       11111
285 290                     1111111         1111111             111
290 295            3          111           1111111111     11111111
295 300         1111111                      111111111    111111111
300 305     11111111    111                  111111111    11111111
305 310   11111111    111111                 111111111    11111
310 315 11111111    111111111                 11111111    11                  1
315 320 111111    111111111                   11111111                    1111
320 325 1111    11111111                        111                     111111
325 330 1          11                                                11111111
330 335                                                             111111111
335 340                                           111    *******51111111
340 345                                          1111111  *    111111
345 350                                         11111111111*   11
350 355              11111    11                 111111111**         1
355 360           11111111    11111               1111111**      1111
        0  20  40  60  80 100 120 140 160 180 200 220 240 260 280 300 320 340
                              Theta 2


       THETA1  THETA2
```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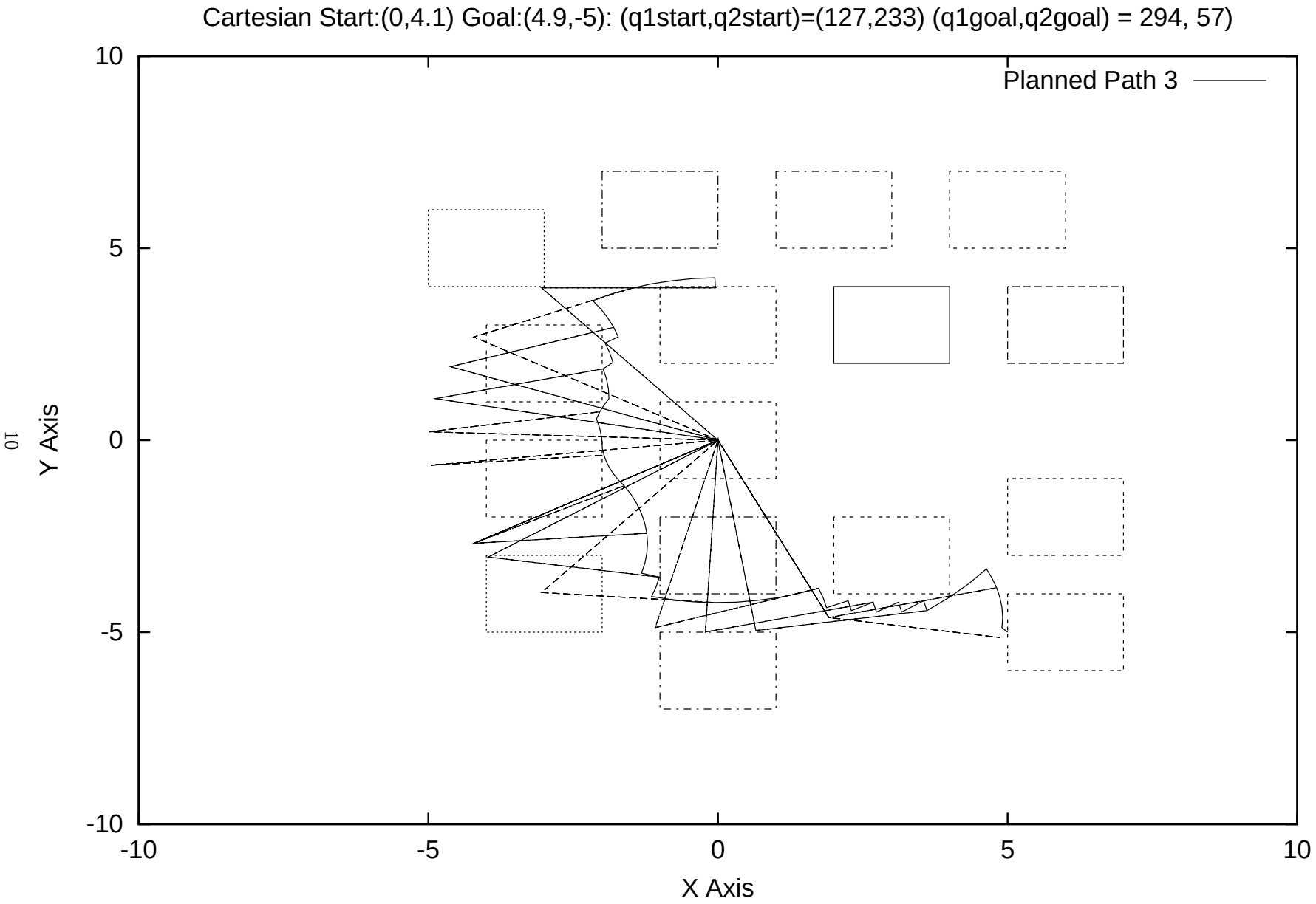
Planned Path 3
Y Axis
X Axis
10
5
0
-5
-10
-10
-5
0
5
10


start 127 233
goal 294 57
Theta1:
Theta 2
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340
Theta 2


Cartesian Start:(0,4.1) Goal:(4.9,-5): (q1start,q2start)=(127,233) (q1goal,q2goal) = 294, 57)
Planned Path 3
Y Axis
X Axis
10
5
0
-5
-10
-10
-5
0
5
10