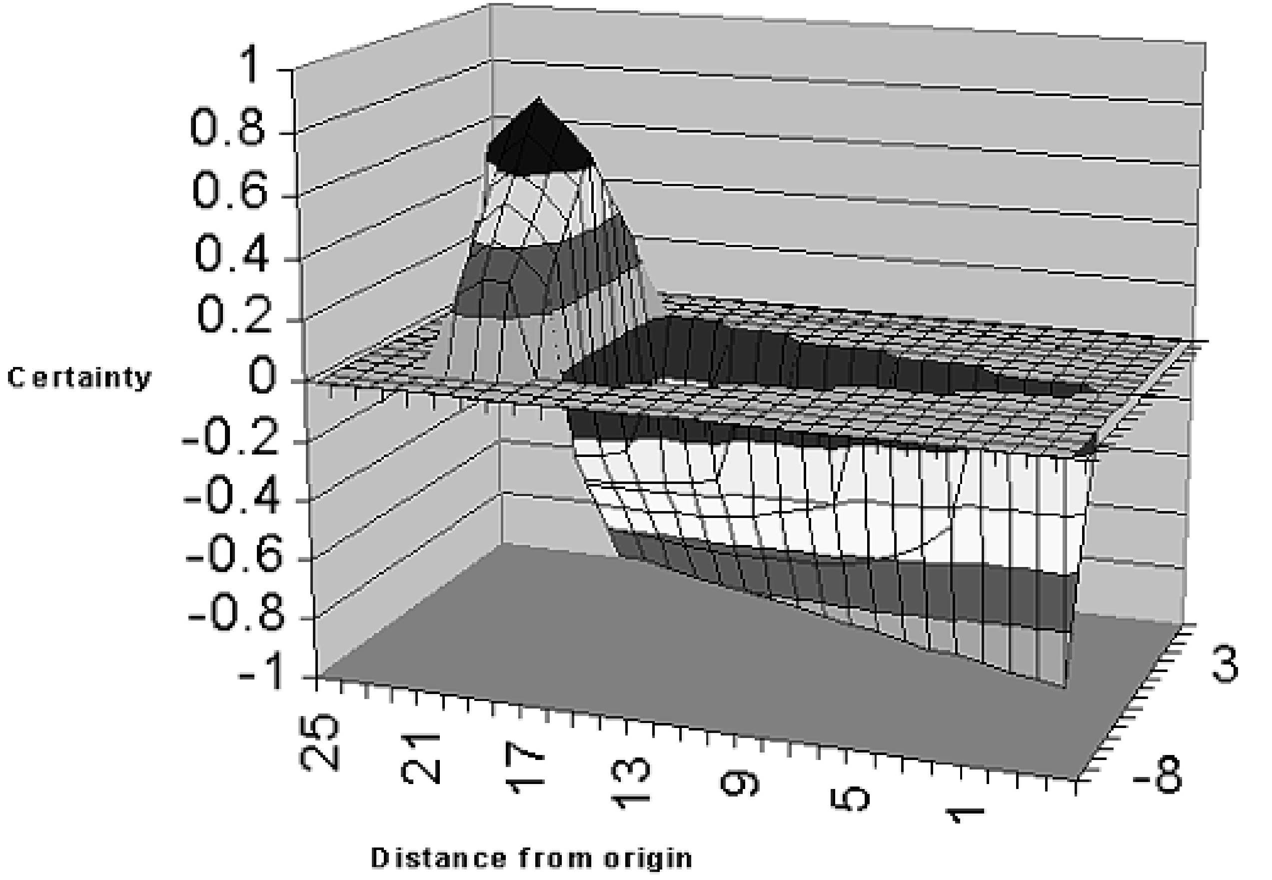
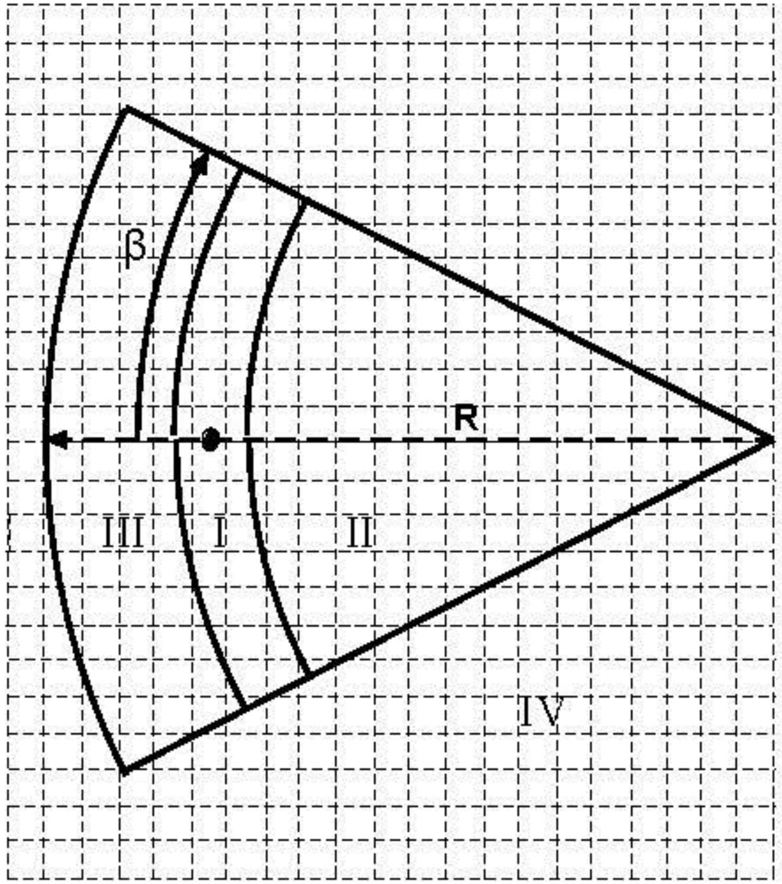
Updating Occupancy Grids Using Bayesian Estimation

Reference:

An Introduction to AI Robotics by R. Murphy, MIT Press, chapter 11

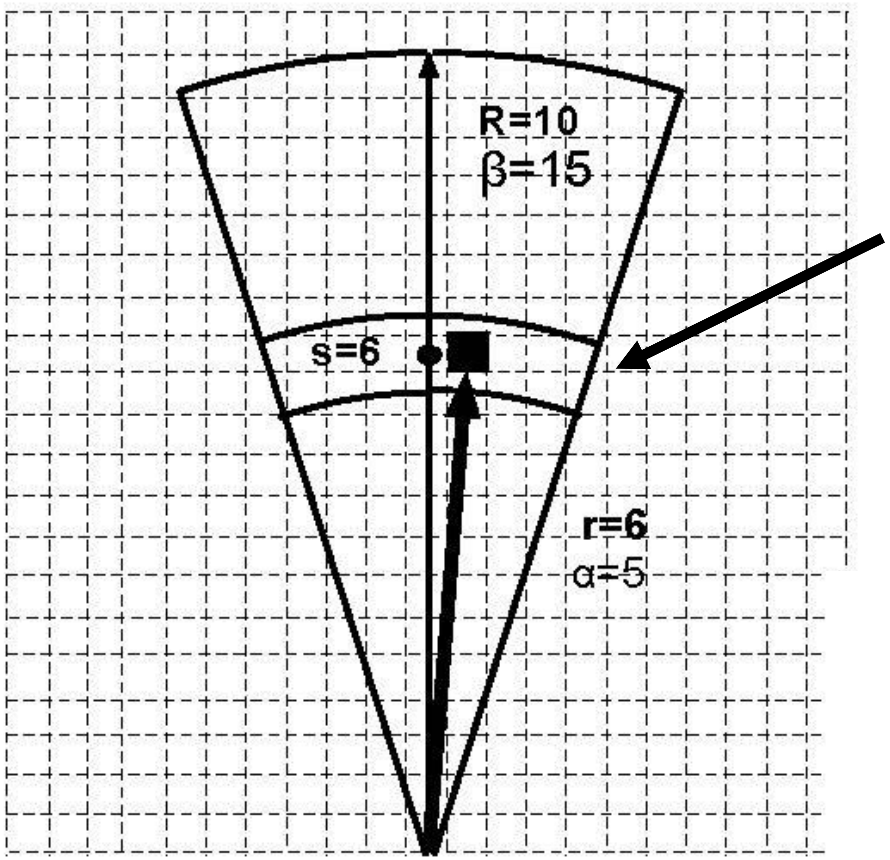
Sonar Sensor Model





Region I: Probable obstacle Region II: Probable free-space Region III: unknown

Mapping Sonar values to Occ grid values: Region I



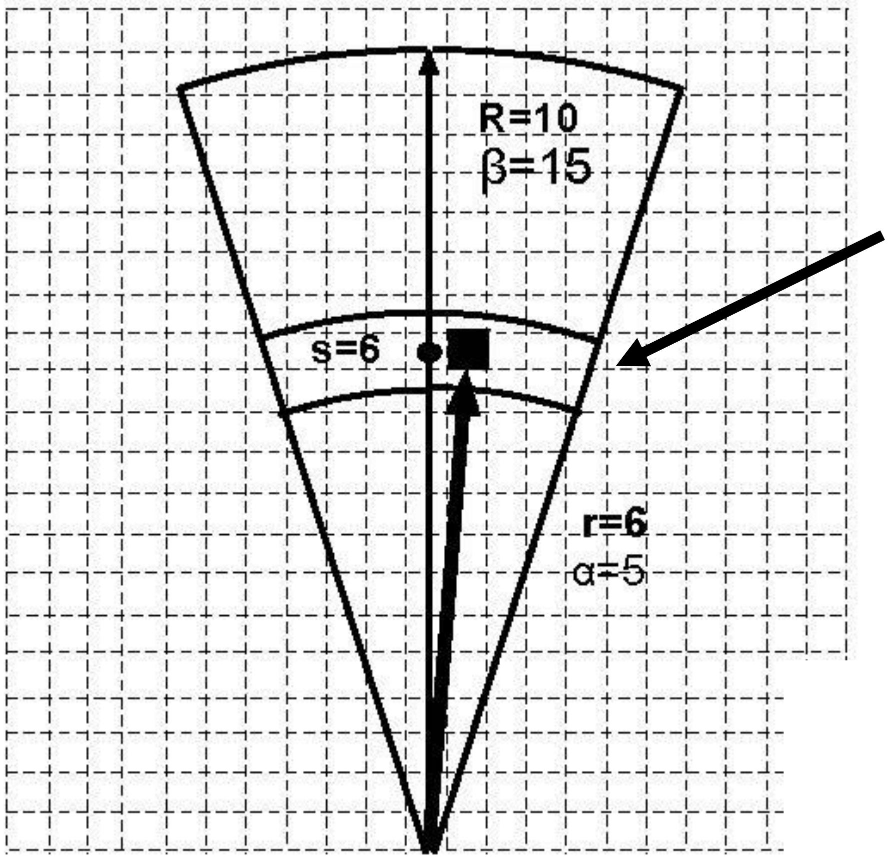
Region I: probable obstacle
R = max sensor range
β = Beam width (half-angle)
(r,α) = polar coordinates of grid point measured from sonar
s = sensor distance reading
ε = tolerance band for distance reading
Max_occupied = Maximum certainty of obstacle (0.0 – 1.0)

If Grid[i][j] iff: (r,a) within cone of uncertainty and

$$s - \varepsilon < r < s + \varepsilon$$

$$P(s|Occupied) = \frac{\frac{(R-r)}{R} + (\frac{\beta-\alpha}{\beta})}{2} \times Max_{occupied}$$

Mapping Sonar values to Occ. grid values: Region II



Region II: probable free-space

R = max sensor range

β = Beam width (half-angle)

(r,α) = polar coordinates of grid point measured from sonar

s = sensor distance reading

ε = tolerance band for distance reading

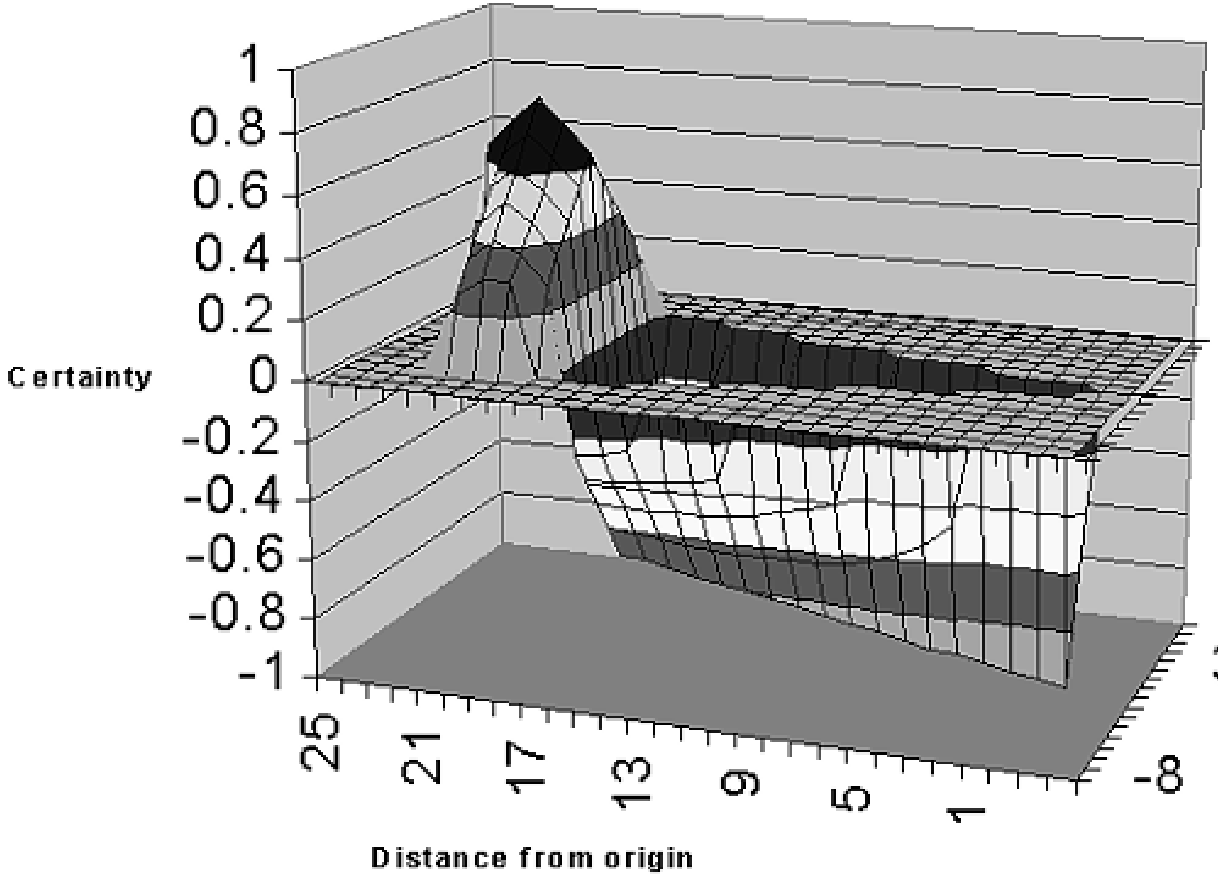
Max_occupied = Maximum certainty of obstacle (0.0 – 1.0)

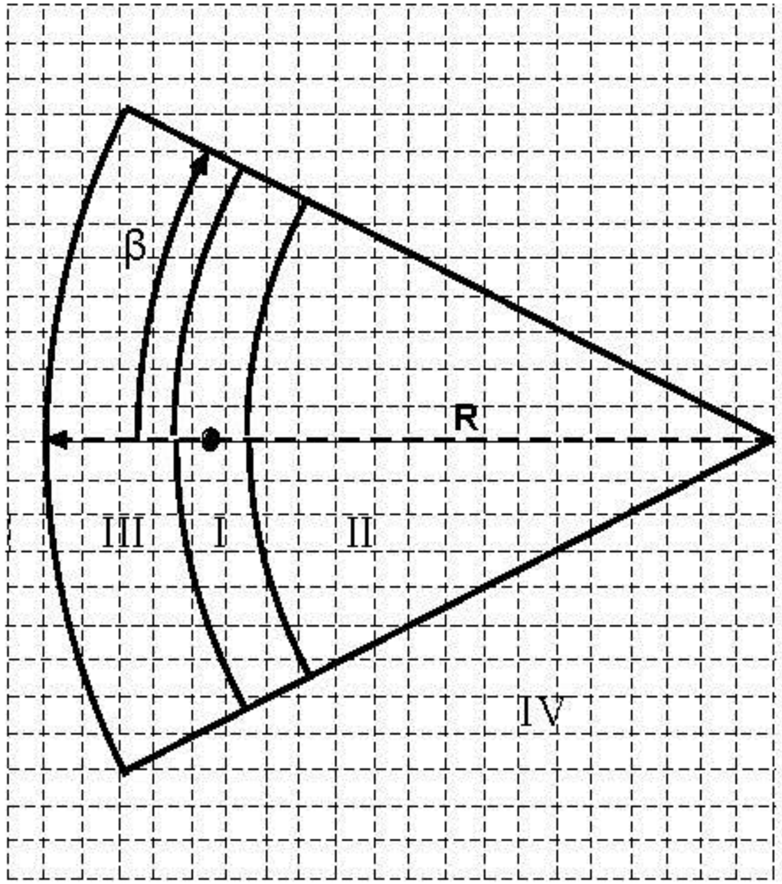
Update Grid[i][j] iff: (r,a) within cone of uncertainty and

$r < s - \varepsilon$

$$P(s|Empty) = \frac{\frac{(R-r)}{R} + (\frac{\beta-\alpha}{\beta})}{2}$$

Region III: Unknown, don't update these cells!





Region I: Probable obstacle   Region II: Probable free-space   Region III: unknown

Bayes Rule for Sonar Updates

We want to find out the probability of the cell being occupied, given the new sensor reading s, and also knowing our **prior** probability of the cell being occupied:

Conditional probabilities for $P(H|s)$

The sensor model represents $P(s|H)$: the probability that the sensor would return the value being considered given it was really occupied. Unfortunately, the probability of interest is $P(H|s)$: the probability that the area at $grid[i][j]$ is really occupied given a particular sensor reading. The laws of probability don't permit us to use the two conditionals interchangeably. However, Bayes' rule does specify the relationship between them:

$$P(H|s) = \frac{P(s|H)P(H)}{P(s|H)P(H) + P(s|\neg H)P(\neg H)}$$

Substituting in *Occupied* for H, Eqn. 11.3 becomes:

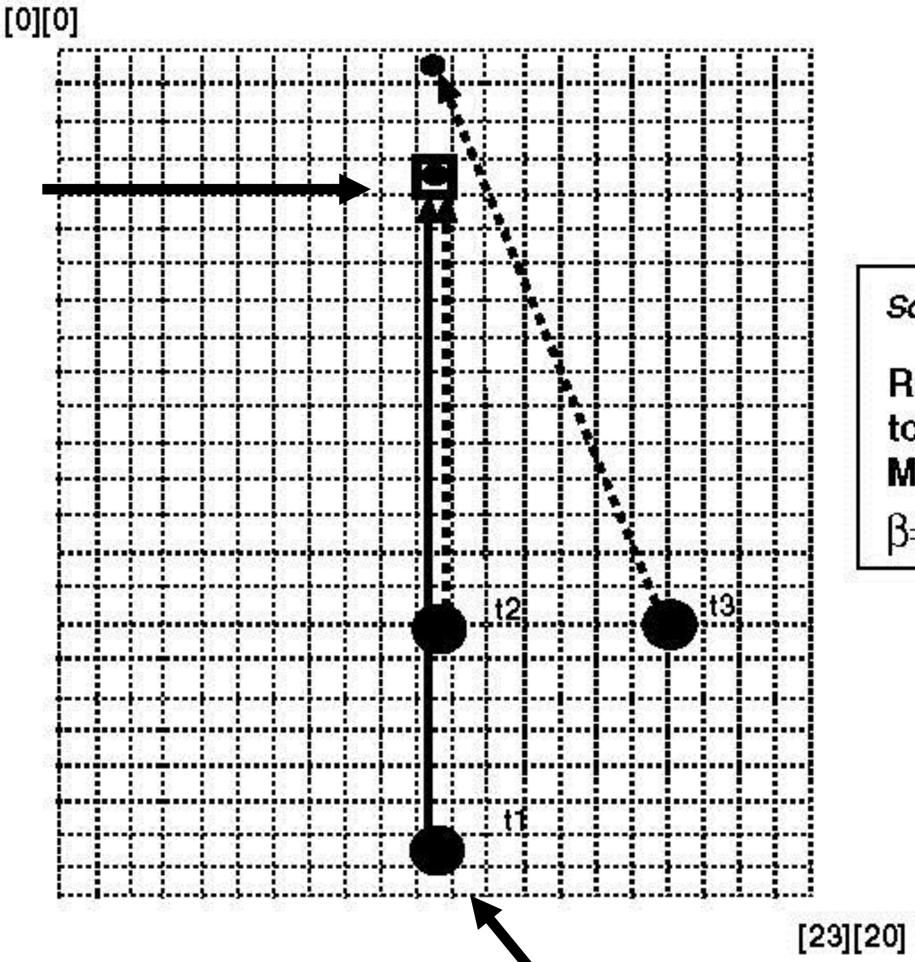
$$P(Occupied|s) = \frac{P(s|Occupied)\,\boxed{\text{P(Occupied)}}}{P(s|Occupied)\,\boxed{\text{P(Occupied)}} + P(s|Empty)\,\boxed{\text{P(Empty)}}}$$

$P(s|Occupied)$ and $P(s|Empty)$ are known from the sensor model. The other terms, $P(Occupied)$ and $P(Empty)$, are the unconditional probabilities, or *prior probabilities* sometimes called *priors*. The priors are shown in the boxes

Example: Initial Grid at t_0

- Occ. Grid of 24 x 21
- Each cell is 0.5 units square
- Robot at [21][10] at time t_1
- ε = 0.5 = tolerance
- Max_occupied = 0.98
- R =10 units = max sonar range

Obstacle
at [3][10]



Robot at [21][10]

Sonar model parameters:

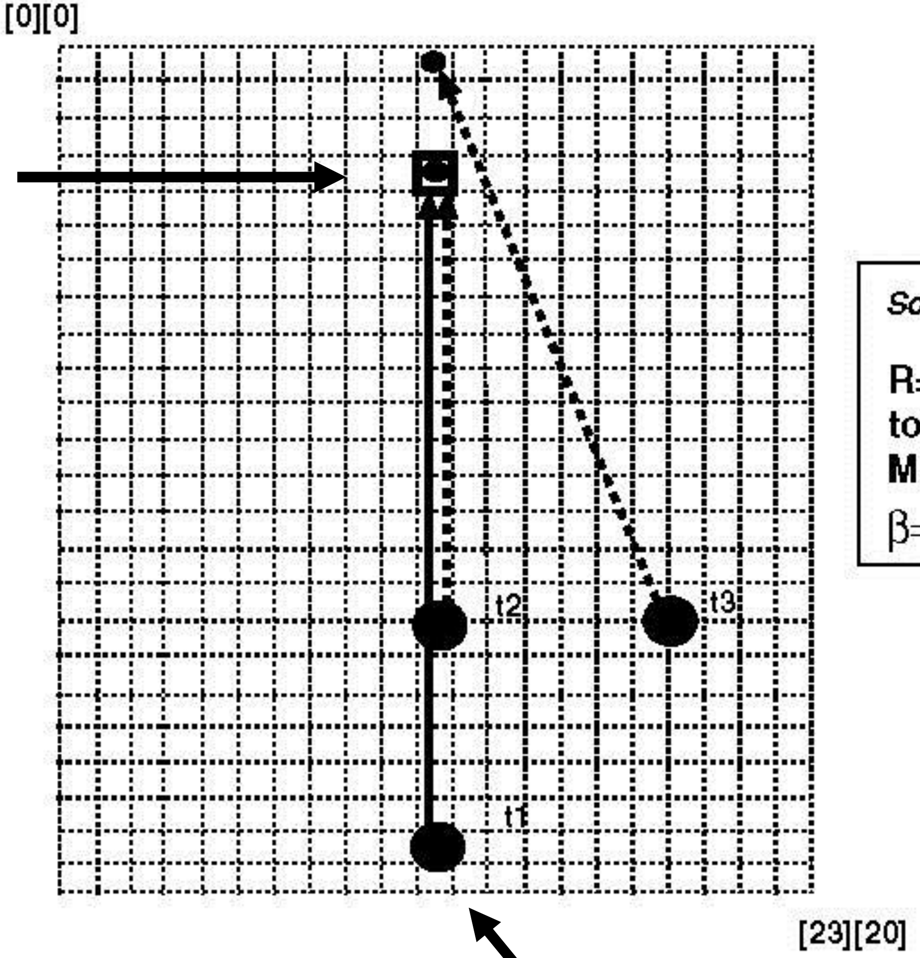
R=10
tolerance =+/- 0.5
Max_occupied = 0.98
β=15

At t_0, EVERY CELL IS INITIALIZED with P_occ = 0.5

Before we start sensing, every cell is equally
Likely to be empty or contain an obstacle

Example: Sensor Reading at t_1

Obstacle at [3][10]



Robot at [21][10]

Sonar model parameters:

R=10
tolerance =+/- 0.5
Max_occupied = 0.98
β=15

- Occ. Grid of 24 x 21
- Each cell is 0.5 units square
- Robot at [21][10] at time t_1
- s = 9 units = sonar reading
- ε = 0.5 = tolerance
- Max_occupied = 0.98
- R =10 units = max sonar range
- Cell [3][10] is in region I (obstacle)
- Cell [3][10] is r=9 units from robot at Angle α = 0 – in cone of uncertainty:
- r is in range: $s - \varepsilon < r < s + \varepsilon$

$$P(s|Occupied) = \frac{\left(\frac{R-r}{R}\right) + \left(\frac{\beta-\alpha}{\beta}\right)}{2} \times Max_{occupied}$$

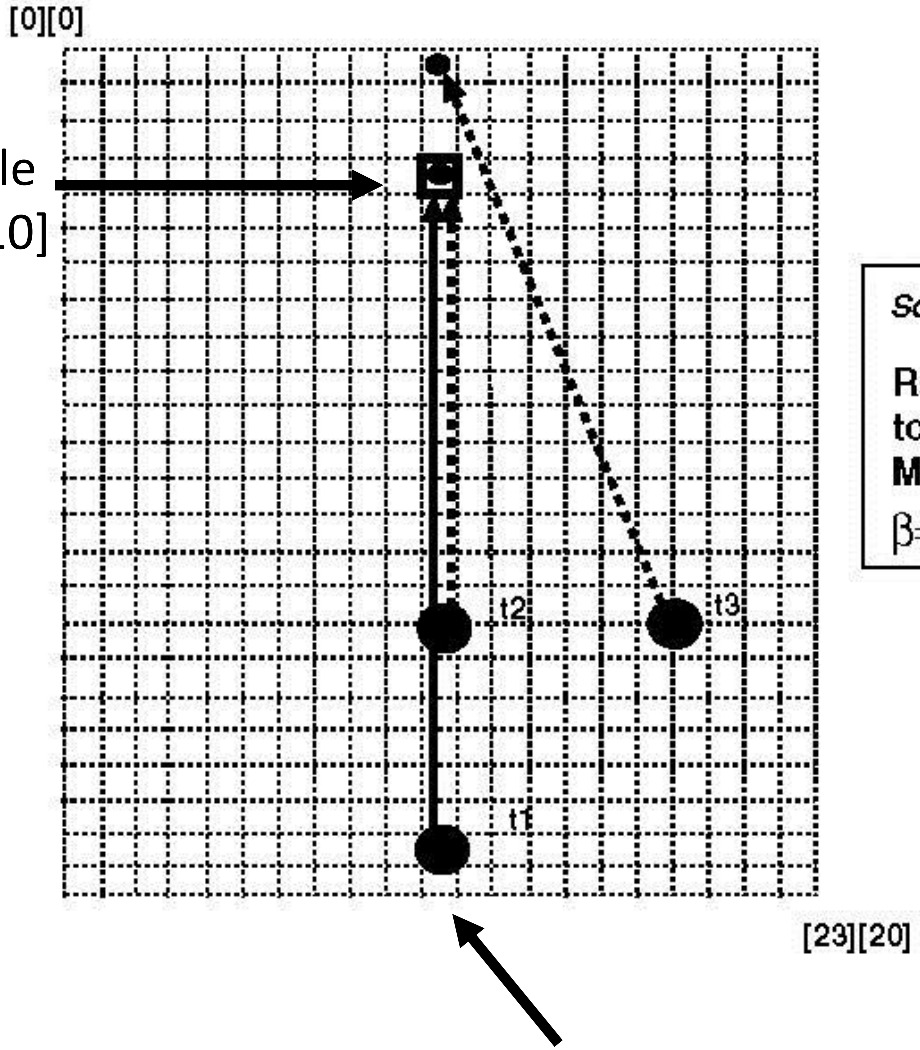
$$= \frac{\left(\frac{10-9}{10}\right) + \left(\frac{15-0}{15}\right)}{2} \times 0.98 = 0.54$$

$$P(s|Empty) = 1.0 - P(s|Occupied)$$

$$= 1.0 - 0.54 = 0.46$$

Example: Bayes Rule Update at t_1 after sensor read

Obstacle at [3][10]



Robot at [21][10]

Sonar model parameters:

R=10
tolerance =+/- 0.5
Max_occupied = 0.98
β=15

- Occ. Grid of 24 x 21
- Each cell is 0.5 units square
- Robot at [21][10] at time t_1
- s = 9 units = sonar reading
- ε = 0.5 = tolerance
- Max_occupied = 0.98
- R =10 units = max sonar range

Cell [3][10] is in region I (obstacle)

Cell is 9 units from robot at

Angle α = 0

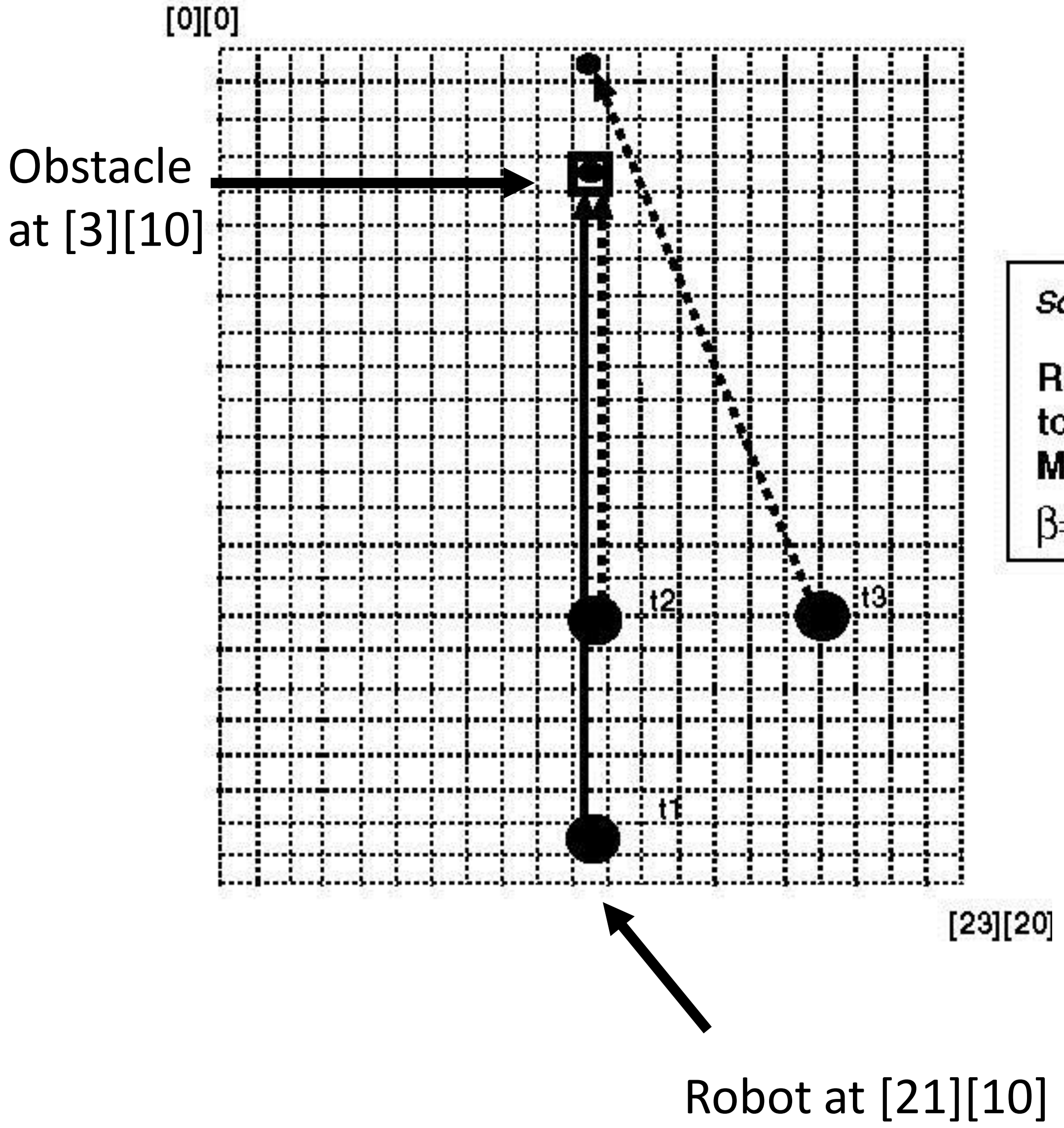
$$P(s_{t_1}|O) = 0.54$$
$$P(s_{t_1}|E) = 0.46$$
$$P(s_{t_0}|O) = 0.50$$
$$P(s_{t_0}|E) = 0.50$$

This yields:

$$P(O|s_{t_1}) = \frac{P(s_{t_1}|O)P(O|s_{t_0})}{P(s_{t_1}|O)P(O|s_{t_0}) + P(s_{t_1}|E)P(E|s_{t_0})} = \frac{(0.54)(0.50)}{(0.54)(0.50) + (0.46)(0.50)} = 0.54$$

$$P(E|s_{t_1}) = 1 - P(O|s_{t_1}) = 0.46$$

Example: Bayes Rule Update at t_2 after 2nd sensor read



Sonar model parameters:

R=10
tolerance =+/- 0.5
Max_occupied = 0.98
β=15

- Robot at [15][10] at time t_2
- s = 6 units = sonar reading
- ε = 0.5 = tolerance
- Max_occupied = 0.98
- R =10 units = max sonar range

Cell [3][10] is in region I (obstacle)

Cell is 6 units from robot at

Angle α = 0

$$P(s_{t2}|Occupied) = 0.69$$
$$P(s_{t2}|Empty) = 0.31$$

$$P(O|s_{t_2}) = \frac{P(s_{t_2}|O)P(O|s_{t_1})}{P(s_{t_2}|O)P(O|s_{t_1}) + P(s_{t_2}|E)P(E|s_{t_1})}$$
$$= \frac{(0.69)(0.54)}{(0.69)(0.54) + (0.31)(0.46)}$$
$$= 0.72$$
$$P(E|s_{t_2}) = 1 - P(O|s_{t_2}) = 0.28$$

Bayes Updating

- Cell [3][10] had P(occ) = 0.5 at t_0
- Cell [3][10] had P(occ) = 0.54 at t_1 after first sensor read
- Cell [3][10] had P(occ) = 0.72 at t_2 after second sensor read
- Successive sensor readings provide confirmation of obstacle
- Note: can use other sensors to update the grid (e.g stereo vision)
- Note: need to update cells in Region II (freespace) as well!