1 CS 4733 Notes: TRANSFORM GRAPHS/FRAME EQUATIONS

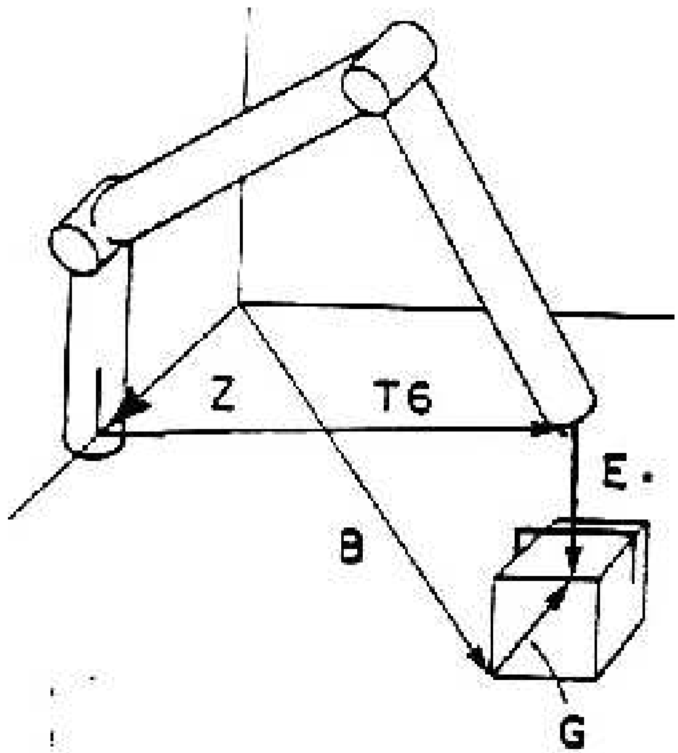

Figure 1: Transform graph for block assembly problem.

In the figure above, a robot manipulator is trying to pick up a block B. G is the transform from the block origin to the actual graspng point on the block. Z is the transform from the world coordinate system to the base of the robot, $T6$ is the transform from robot base to end-effector (tool), E is the end-effector (tool) transform from wrist of robot to gripper.

We can write a frame equation:

$$Z \,:\, T6 \,:\, E \,=\, B \,:\, G \tag{1}$$

If the location of object B is unknown, we can solve for it:

$$Z \,:\, T6 \,:\, E \,:\, G^{-1} \,=\, B \tag{2}$$

If we know the location of robot world frame Z, block B, end effector transform E, and grasp point on object G, we can solve for the the manipulator transform T6:

$$T6 \,=\, Z^{-1} \,:\, B \,:\, G \,:\, E^{-1} \tag{3}$$

2 Example of Transform Equation

Example 1: Below is a simple assembly problem. We are trying to put the piece labeled *block* onto the other piece labeled *assembly*. Each piece is defined as having its own local, internal coordinate system. We need to find out what transform will represent the movement of the block so that it will be aligned with the assembly such that segment rs will be coincident with segment pq.

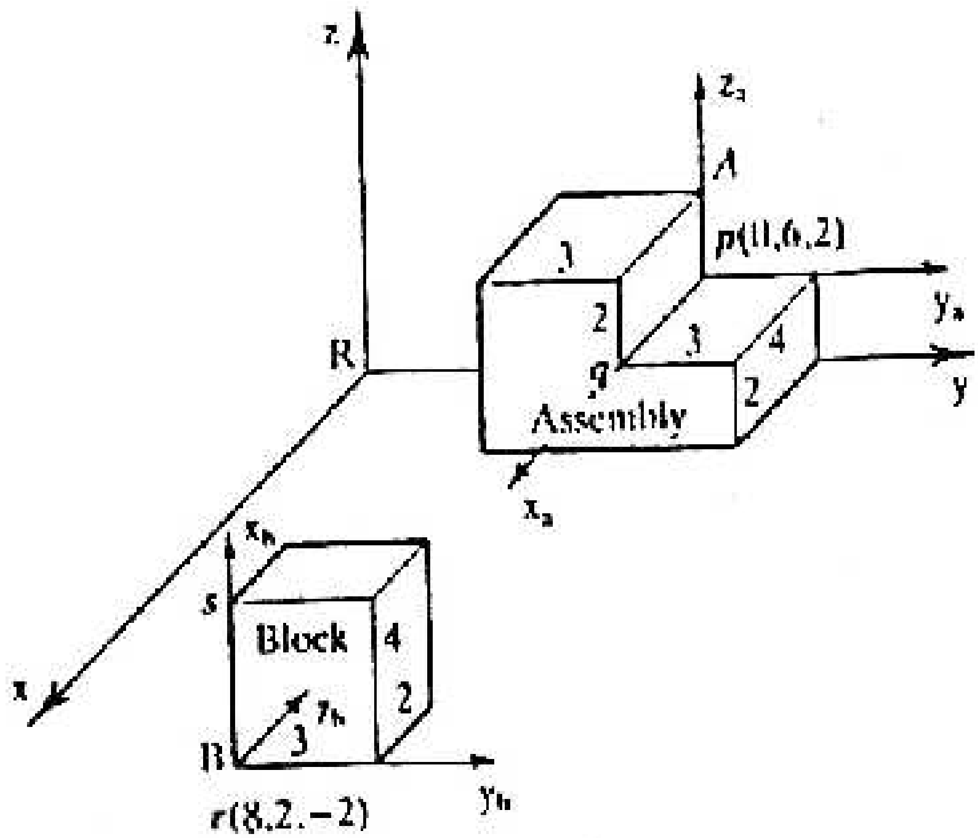

Figure 2: Block assembly problem.

Using the diagram, we can define these 4x4 transforms:

$$T^R_{block} = \begin{bmatrix} 0 & 0 & -1 & 8 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \; (T^R_{block})^{-1} = T^{block}_R = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \; T^R_{assembly} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

We can now write a frame equation that expresses our desire that the block frame is to be made coincident with the assembly frame. This means finding the transform $T^{block}_{assembly}$. The equation is:

$$T^R_{assembly} = T^R_{block}\, T^{block}_{assembly} \tag{5}$$

We can also draw this as a **transform graph** (fig.3):

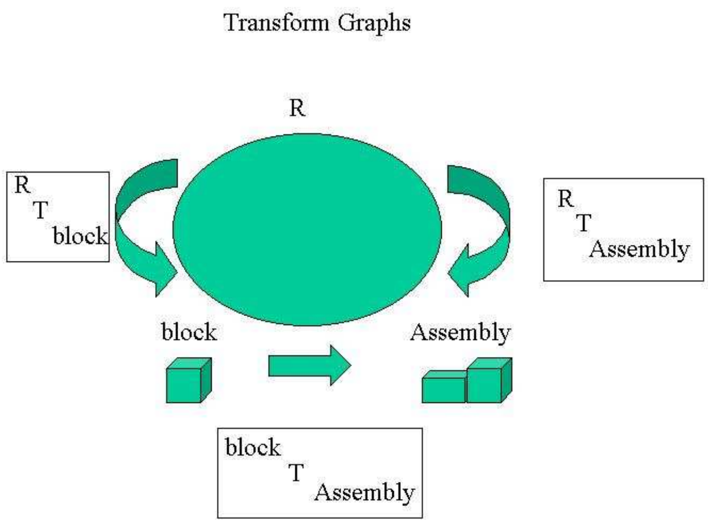

Figure 3: Transform graph for block assembly problem.

We can solve this equation for $T^{block}_{assembly}$ by multiplying both sides of the equation by $(T^R_{block})^{-1}$ on the left hand side (remember: order is important in multiplying matrices; they are not commutative). This yields:

$$(T^R_{block})^{-1}\; T^R_{assembly} = (T^R_{block})^{-1}\; T^R_{block} T^{block}_{assembly} = T^{block}_{assembly} \tag{6}$$

$$T^{block}_{assembly} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 4 \\ -1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

We can now find the new block position after we move it using our calculated transform:

$$T^R_{block-moved} = T^R_{block}\; T^{block}_{assembly} = \begin{bmatrix} 0 & 0 & -1 & 8 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 4 \\ -1 & 0 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T^R_{assembly} \tag{8}$$

Example 2: A robot work station has been set up with a TV camera (see figure 2. The camera can see the origin of the base coordinate frame where a 6-jointed robot is attached. It can also see the center of an object (assumed to be a cube)to be manipulated by the robot. If a local coordinate system has been established at the center of the cube, this object as seen by the camera can be represented by a homogenous transform matrix T_{obj}^{cam}. The origin of the base coordinate system as seen by the camera can also be expressed by a homogeneous transform matrix T_{base}^{cam}.

$$T_{obj}^{cam} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & -1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{base}^{cam} = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the transform T_{obj}^{base}, and the positon of the center of the cube with respect to the base coordinate frame.

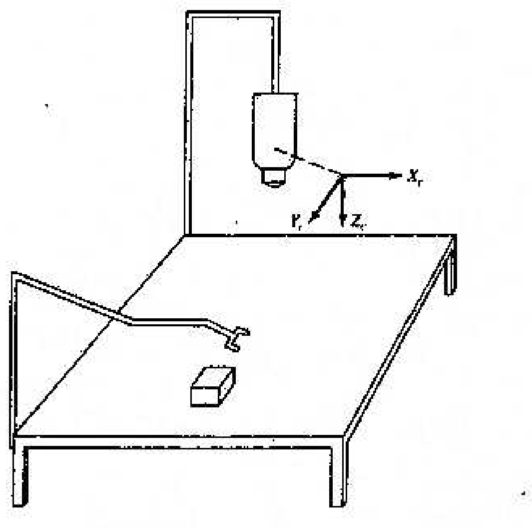

Figure 4: Robot work station

$$\begin{aligned} T_{obj}^{cam} &= T_{base}^{cam}\, T_{obj}^{base} \\ (T_{base}^{cam})^{-1}\, T_{obj}^{cam} &= T_{obj}^{base} \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & -1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 11 \\ -1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_{obj}^{base}$$

The position of the center of the cube with respect to the base coordinate system is $(11, 10, 1)^T$, which is the translation vector of the origin in the transform T_{obj}^{base}.