CS 4733, Class Notes: Forward Kinematics II

1 Stanford Manipulator - First Three Joints

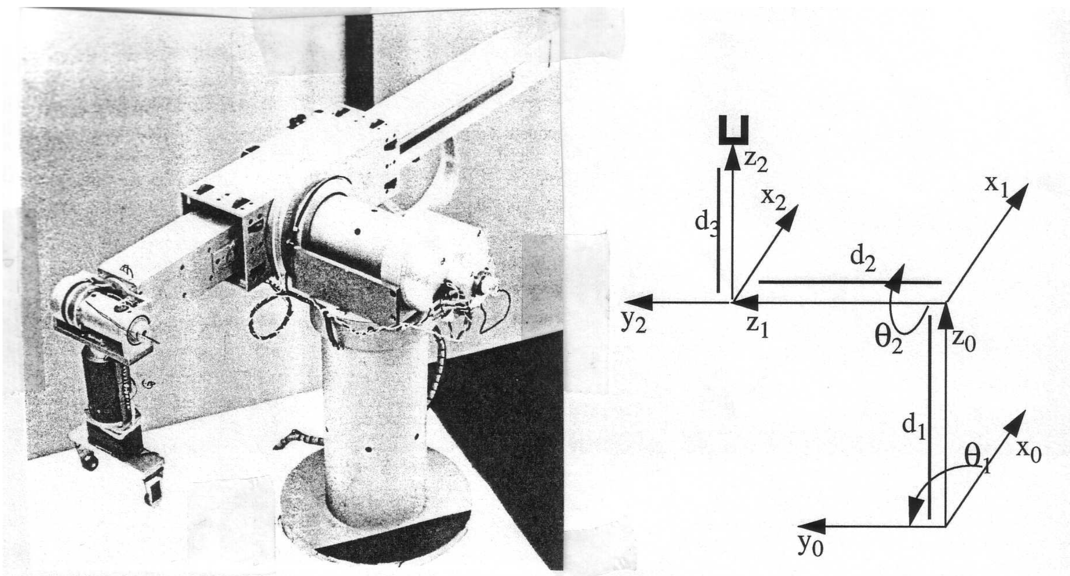

Figure 1: Stanford Robotic Arm. The frame diagram shows the first three joints, which are in a R-R-P configuration (Revolute-Revolute-Prismatic.

joint	θ	d	a	α
nnn1	θ_1	d_1	0	-90
2	θ_2	d_2	0	90
3	0	d_3	0	0
4				
5				
6				

$$T_1^0 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_2^1 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & -S_1d_2 \\ S_1C_2 & C_1 & S_1S_2 & C_1d_2 \\ -S_2 & 0 & C_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_3^0 = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & C_1S_2d_3 - S_1d_2 \\ S_1C_2 & C_1 & S_1S_2 & S_1S_2d_3 + C_1d_2 \\ -S_2 & 0 & C_2 & C_2d_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{if } \theta_1 = \theta_2 = 0, d_3 = 0\text{: } T_3^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{(Zero Position)}$$

2 4-DOF Gantry Robot

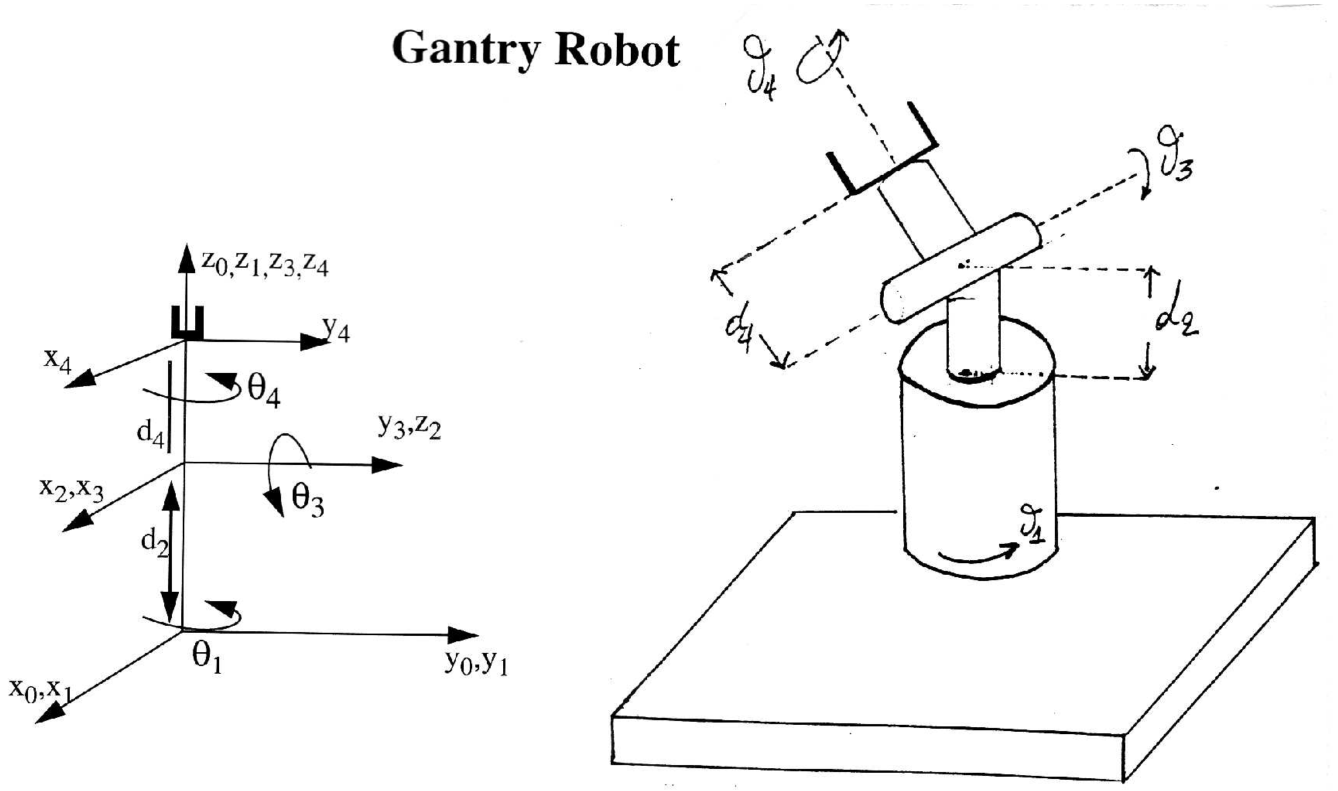

Figure 2: Gantry Robot Arm. This arm is in a R-P-R-R configuration. θ_1, θ_3 , θ_4 are the revolute joint angle variables and d_2 is the prismatic joint variable. d_4 is a constant.

joint	θ	d	a	α
1	θ_1	0	0	0
2	0	d_2	0	-90
3	θ_3	0	0	90
4	θ_4	d_4	0	0

$$A_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} C_3 & 0 & S_3 & 0 \\ S_3 & 0 & -C_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_4^3 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^0 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^2 = \begin{bmatrix} C_3 & 0 & S_3 & 0 \\ S_3 & 0 & -C_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_3C_4 & -C_3S_4 & S_3 & S_3d_4 \\ S_3C_4 & -S_3S_4 & -C_3 & -C_3d_4 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^0 = A_2^0 A_4^2 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3C_4 & -C_3S_4 & S_3 & S_3d_4 \\ S_3C_4 & -S_3S_4 & -C_3 & -C_3d_4 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_3C_4 - S_1S_4 & -C_1C_3S_4 - C_4S_1 & C_1S_3 & C_1S_3d_4 \\ C_3C_4S_1 + C_1S_4 & -C_3S_1S_4 + C_1C_4 & S_1S_3 & S_1S_3d_4 \\ -S_3C_4 & S_3S_4 & C_3 & C_3d_4 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

if $\theta_1 = \theta_3 = \theta_4 = 0, d_2 = 0$: $A_4^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (Zero Position)

if $\theta_1 = \theta_3 = \theta_4 = 90, d_2 = D$: $A_4^0 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 1 & 0 & D \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3 Scara Arm

A SCARA arm (Selective Compliant Articulated Robot Arm) is a commonly found nrobotic manipulator. It is well suited for pick-and-place operations where an object is approached from above, grasped and then transported to another location where the object is deposited.

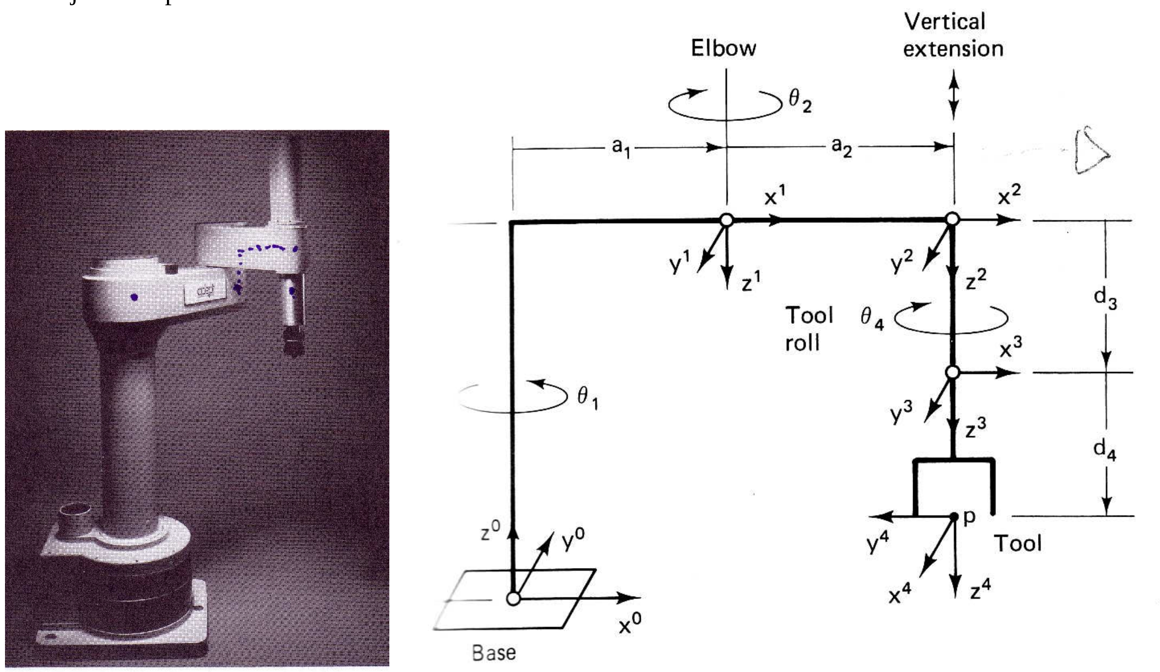

Figure 3: Adept 1 Scara Robot arm. This arm is in a R-R-P-R configuration. θ_1, θ_2 , θ_4 are the revolute joint angle variables and q_3 is the prismatic joint variable. The robot is pictured in the $Home$ position in the frame diagram using the values of the joint variables listed in the table below.

axis	θ	d	a	α	Home
1	θ_1	d_1	a_1	π	0
2	θ_2	0	a_2	0	0
3	0	q_3	0	0	100
4	θ_4	d_4	0	0	$\pi/2$

$$T_{tool}^{base} = T_1^0 T_2^1 T_3^2 T_4^3 =$$

$$\begin{bmatrix} C_1 & S_1 & 0 & a_1C_1 \\ S_1 & -C_1 & 0 & a_1S_1 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{tool}^{base} = \begin{bmatrix} C_{1-2-4} & S_{1-2-4} & 0 & a_1C_1 + a_2C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_1S_1 + a_2S_{1-2} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $S_{1-2-4} = Sin(\theta_1 - \theta_2 - \theta_4)$ (same for C_{1-2-4}).