A Celtic Framework for Knots and Links

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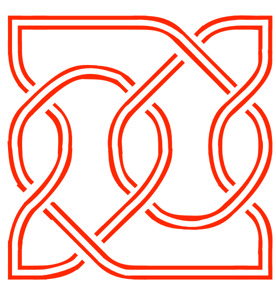
Abstract

We describe a method used by modern graphic artists to design what are traditionally called *Celtic knots*. It is easily proved that every such design specifies an alternating projection of a link. We use medial graphs and graph minors to prove, conversely, that every alternating projection of a link is topologically equivalent to some Celtic link. If the requirement of alternation is dropped, we obtain a *Celtistic* framework for organizing the study of all knots and links. The Celtistic framework suggests some new geometric invariants of links and some new recursively specifiable sequences of links. It also leads us to explore new variations of problems regarding such sequences, including the derivation of formulae for infinite sequences of knot polynomials. By way of contrast, the general problem of calculating Jones polynomials of knots is known to be $\#P$-hard. This talk involves a confluence of ideas from knot theory, topological graph theory, and the theory of orthogonal graph drawings.



A Celtic knot.