Mesh Arrangements for Solid Geometry

Qingnan Zhou, Eitan Grinspun, Denis Zorin, Alec Jacobson

Motivation: Fabrication

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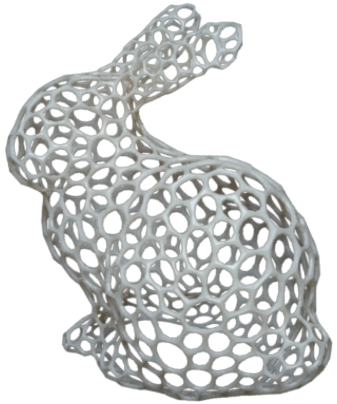

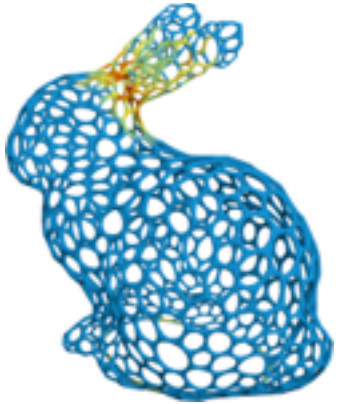

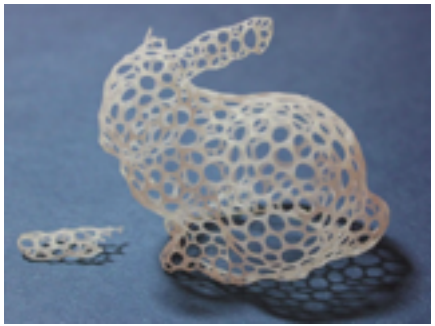

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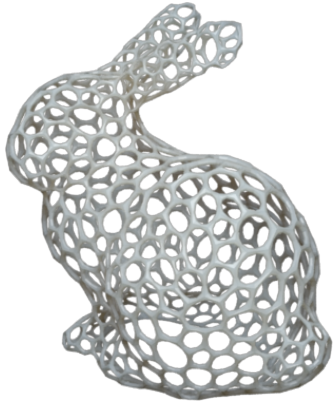
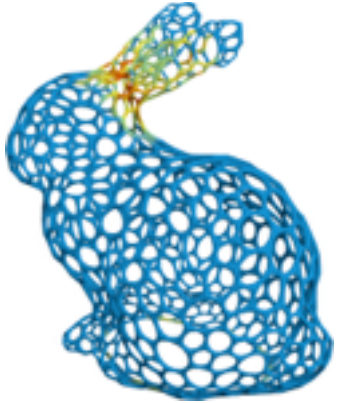
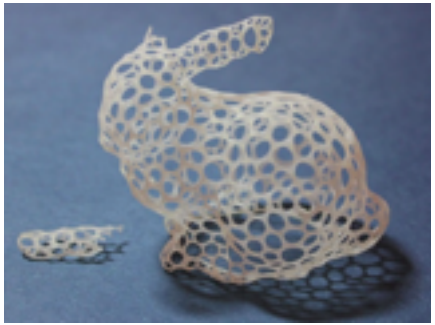
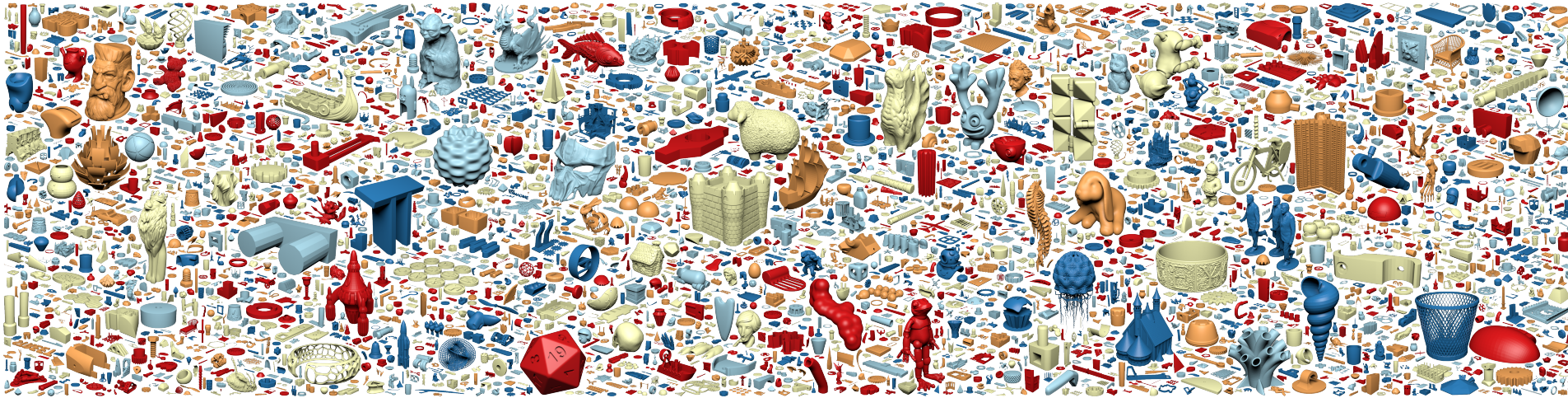




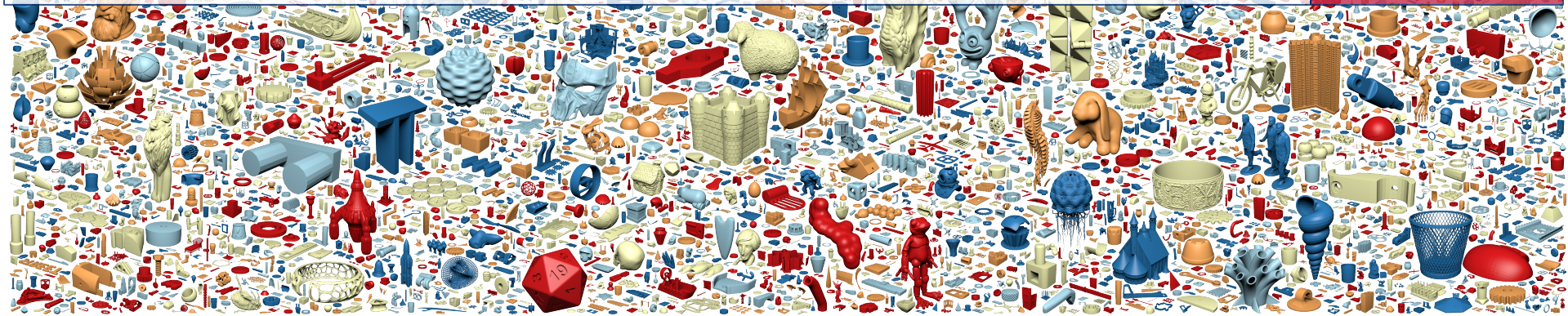


Thingi10K Dataset



Thingi10K Dataset

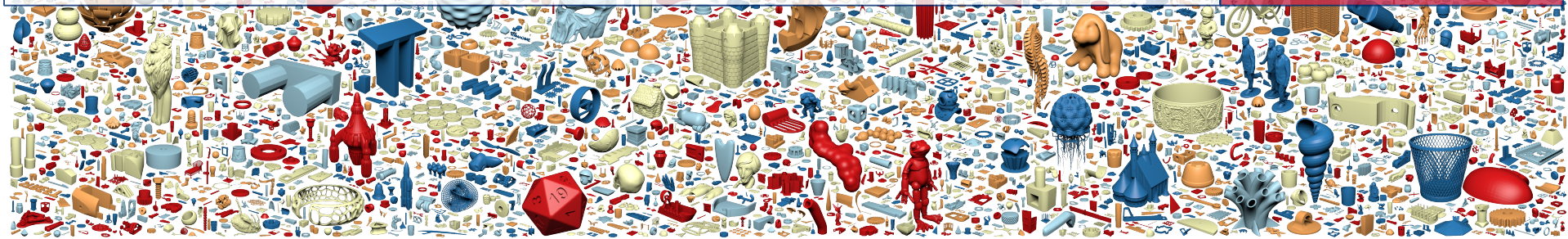
84% Without Degeneracies



Thingi10K Dataset

84% Without Degeneracies

78% Manifold

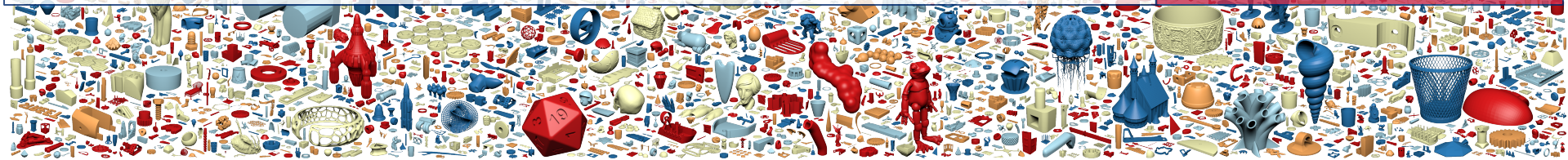


Thingi10K Dataset

84% Without Degeneracies

78% Manifold

74% Single Component



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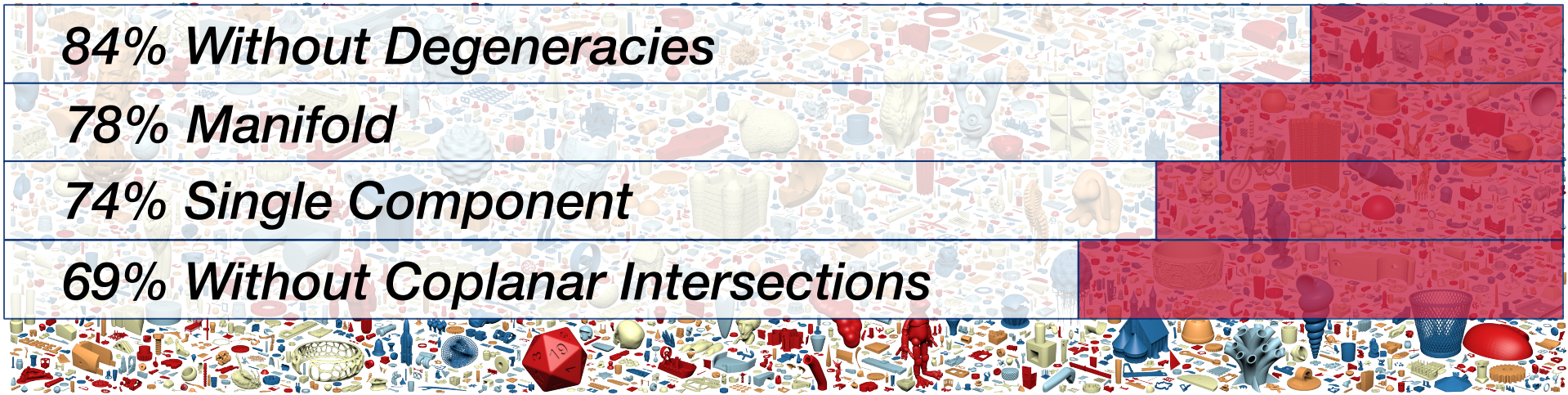

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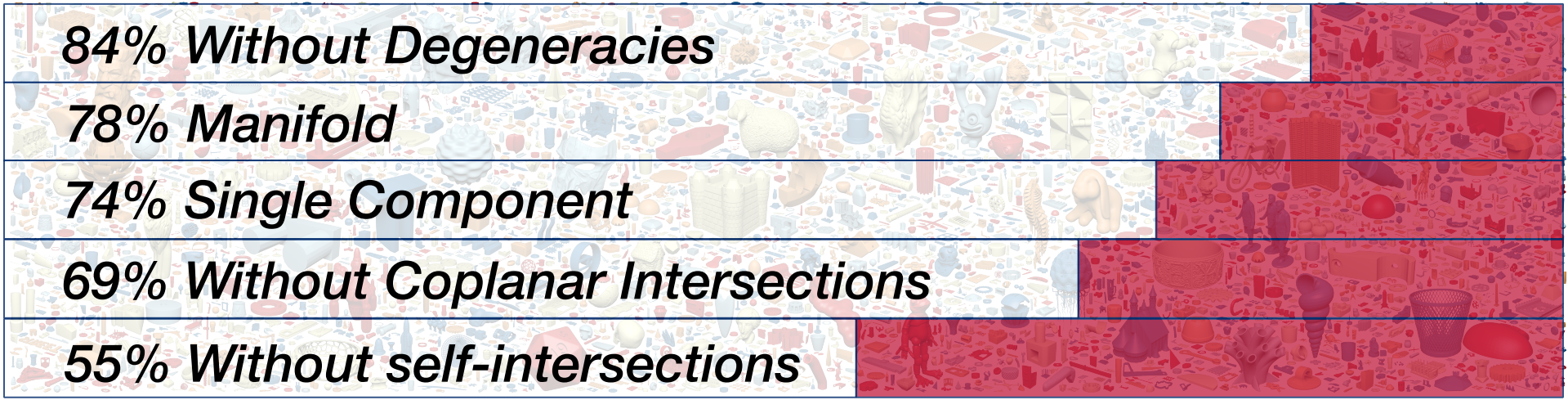
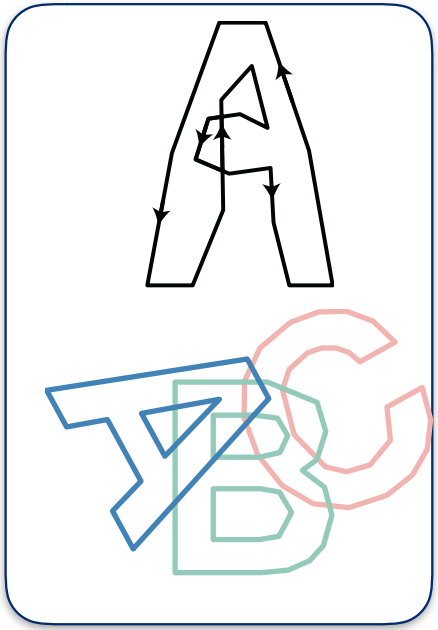


Problem Statement

Input



Problem Statement

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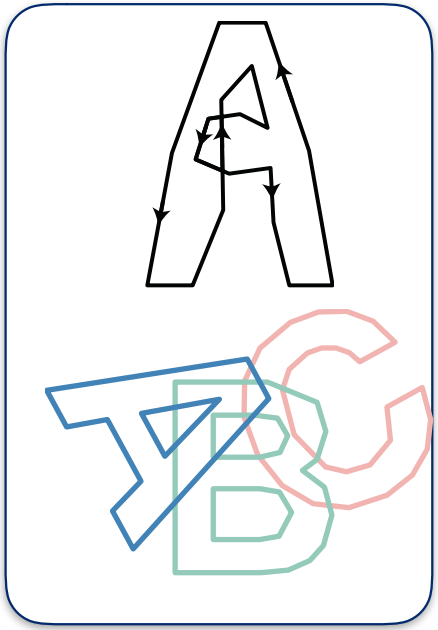
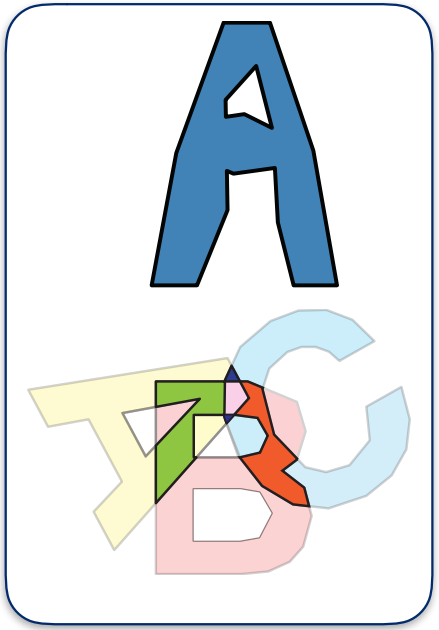


Output Solid



Problem Statement

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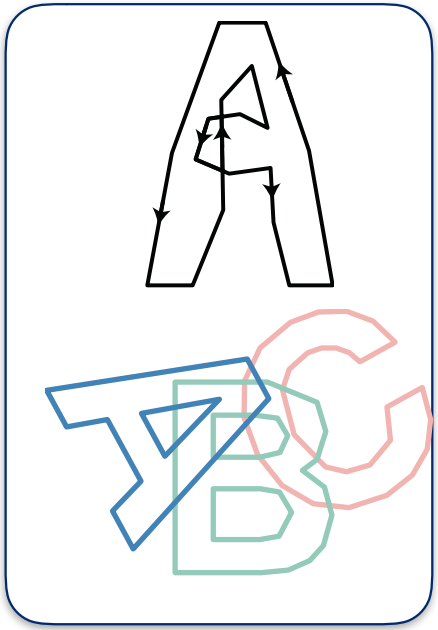
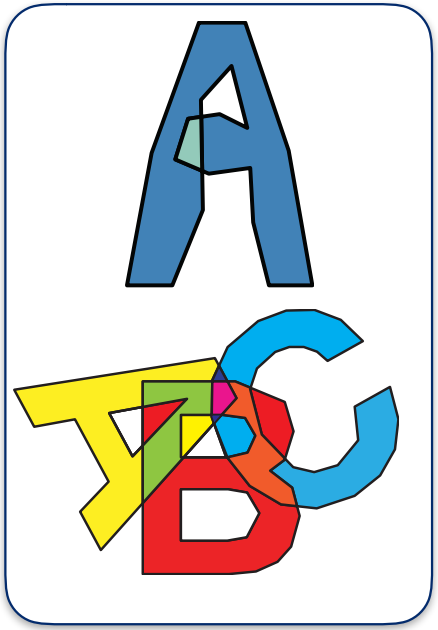
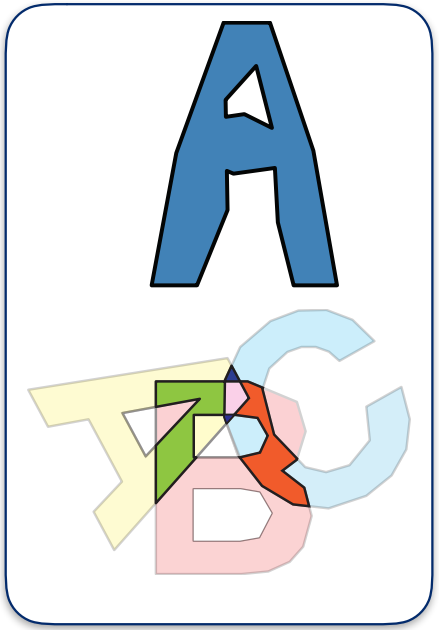


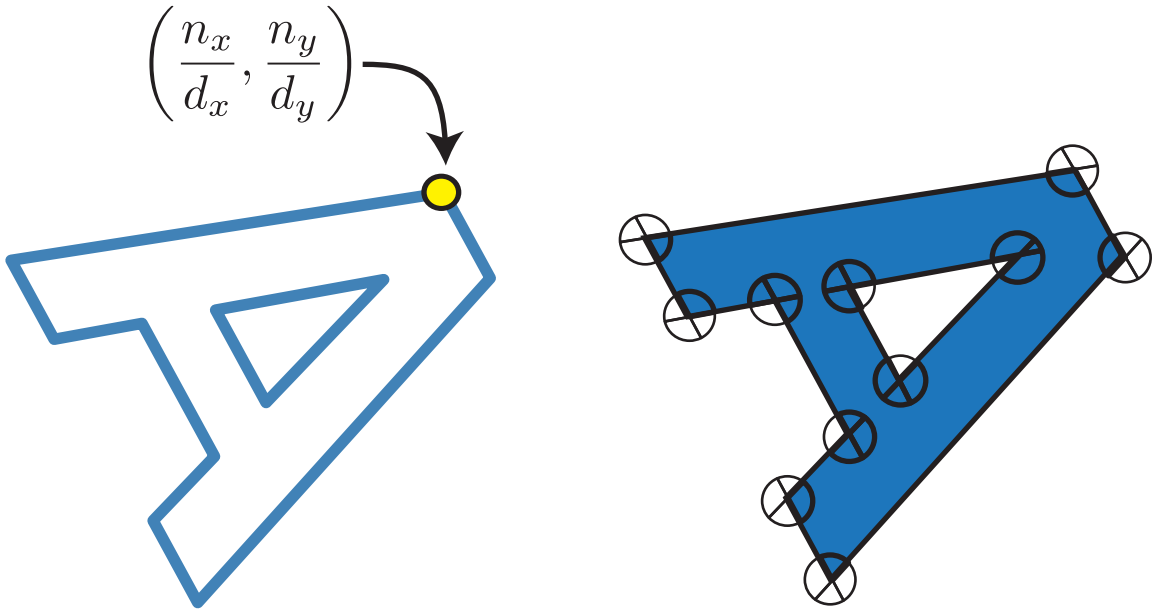
Mesh Arrangement



Output Solid



Previous Works: CGAL



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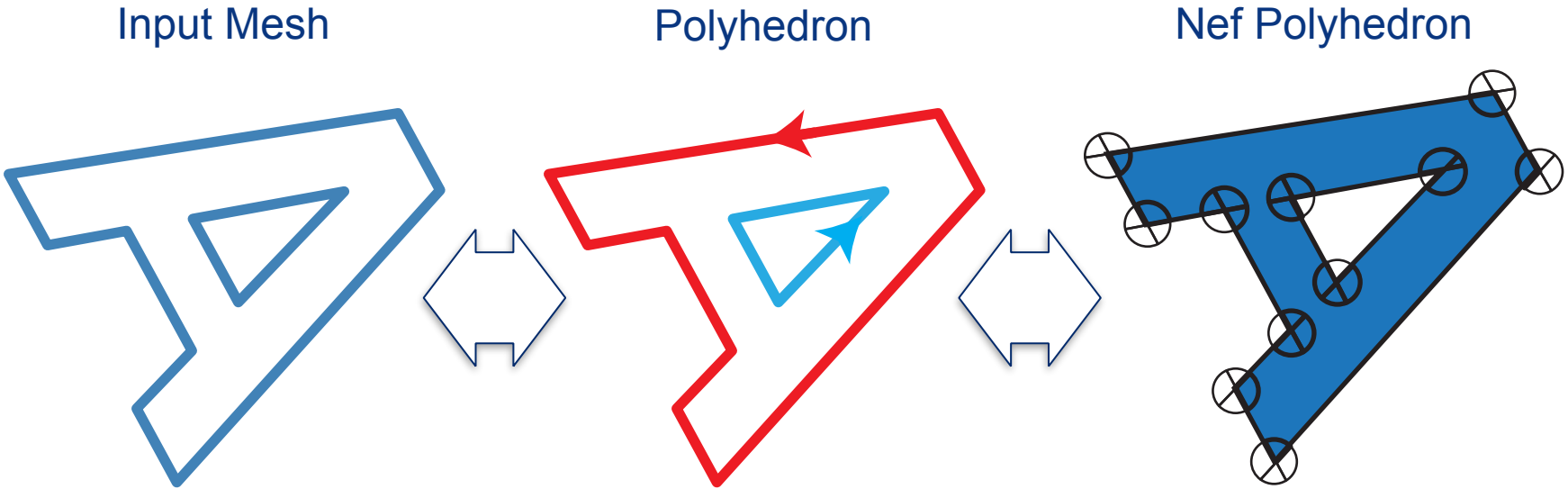
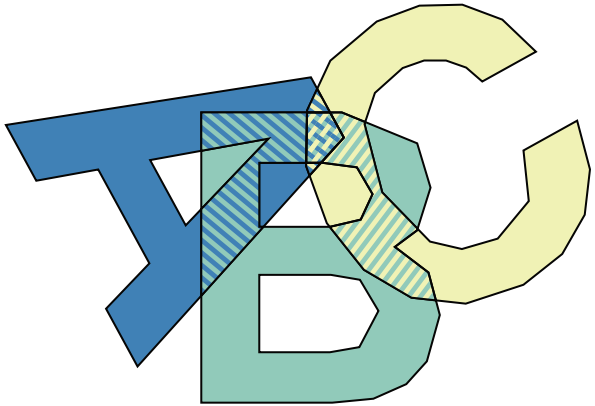


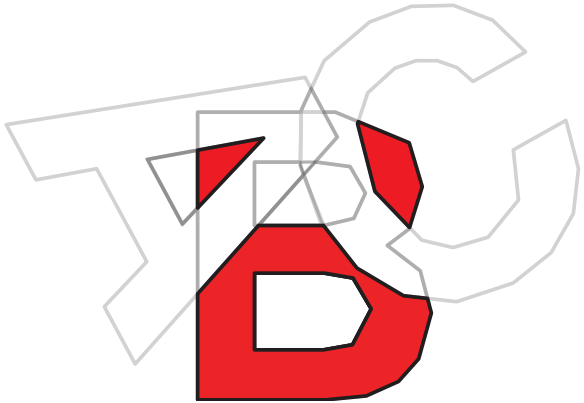
Previous Works: Cork, QuickCSG

Input and Output

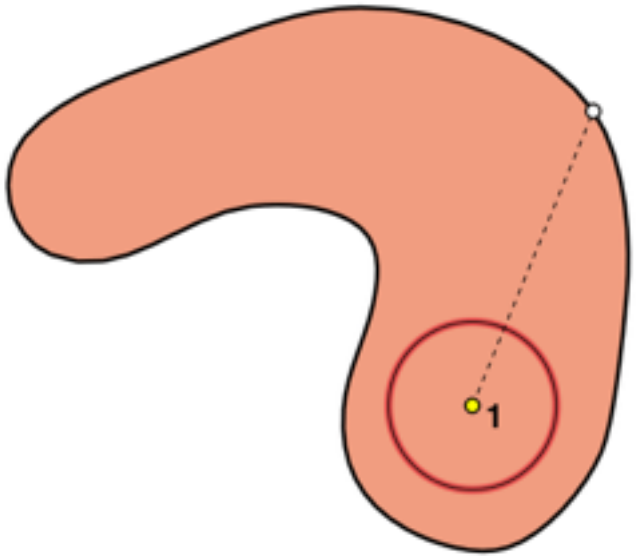
Valid Input



Valid Output

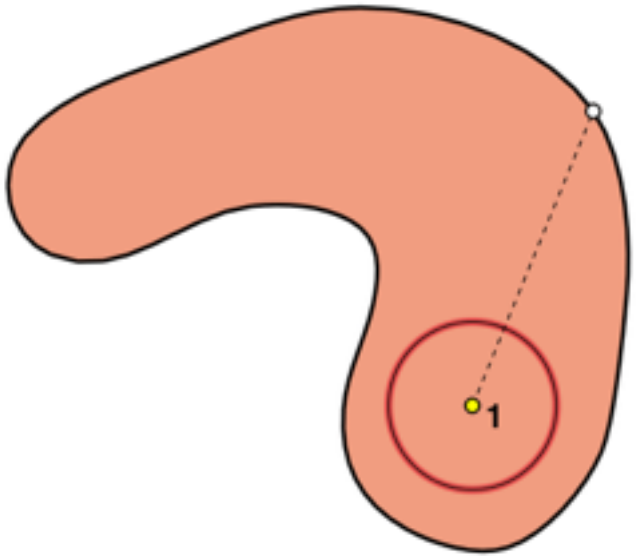


Winding Number



[Jacobson et al. 2013]

Winding Number



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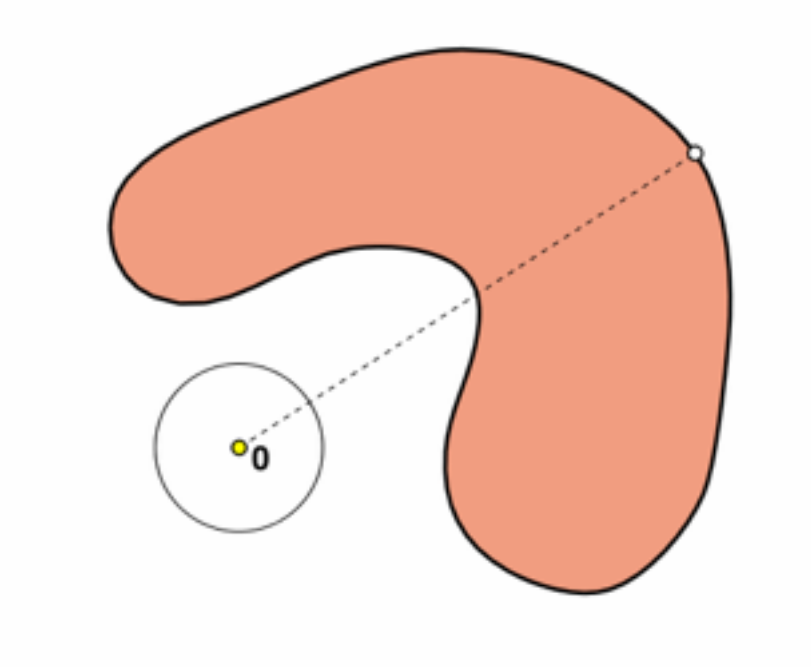
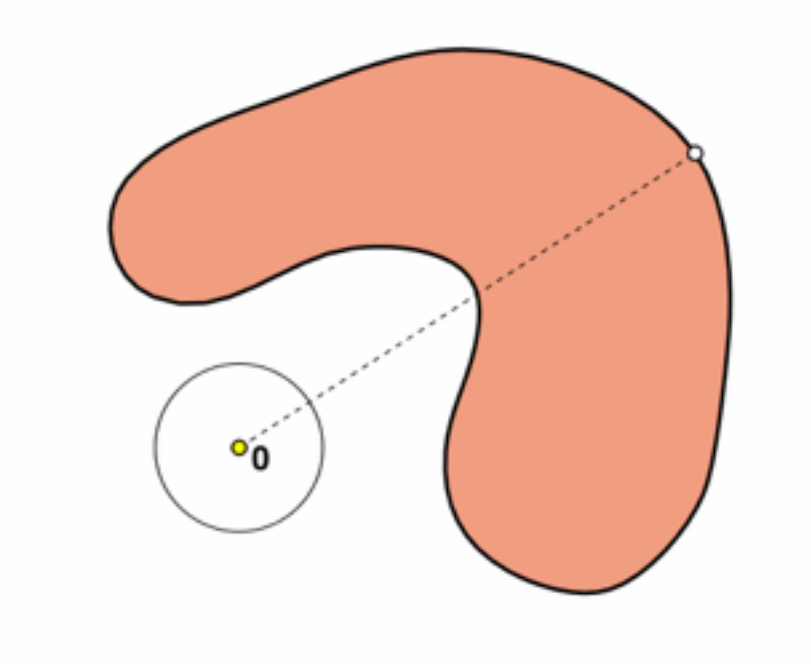


[Jacobson et al. 2013]

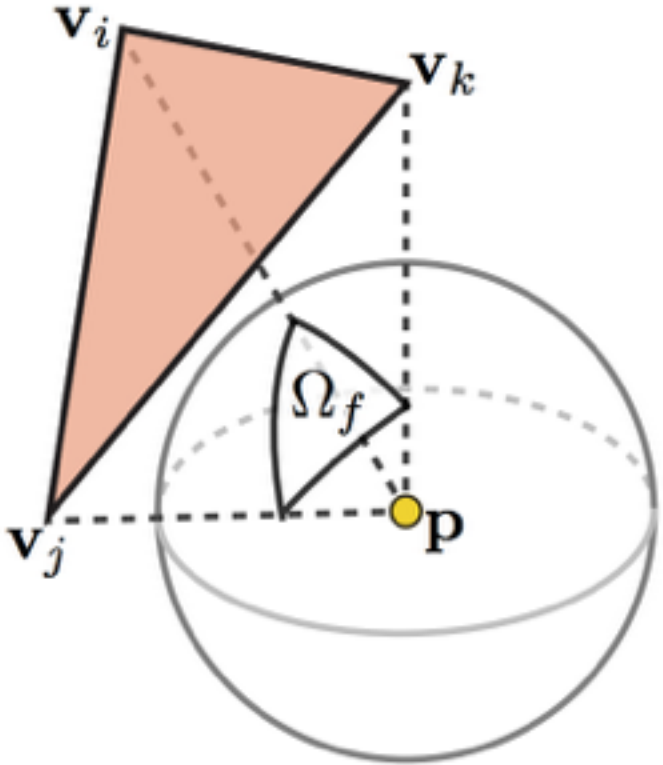
Winding Number



[Jacobson et al. 2013]

Winding Number

$$w(p) = \frac{1}{4\pi} \sum_{f=1}^{m} \Omega_f(p)$$



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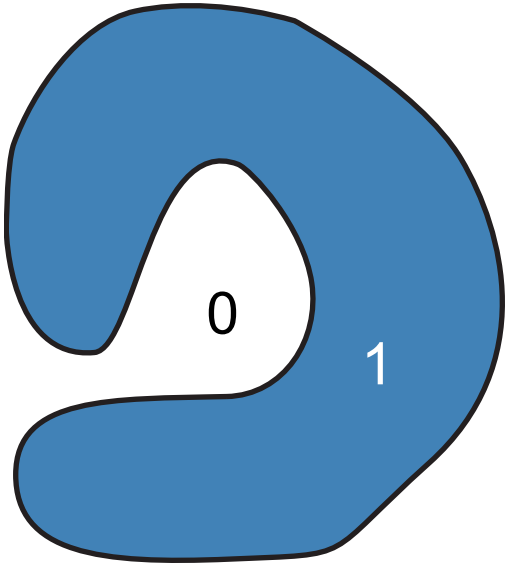

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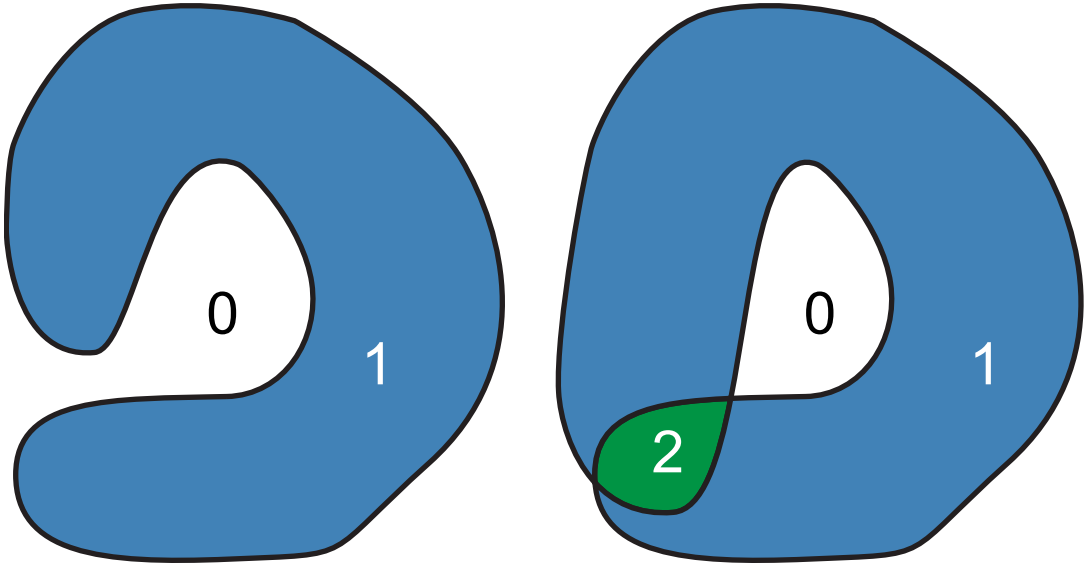

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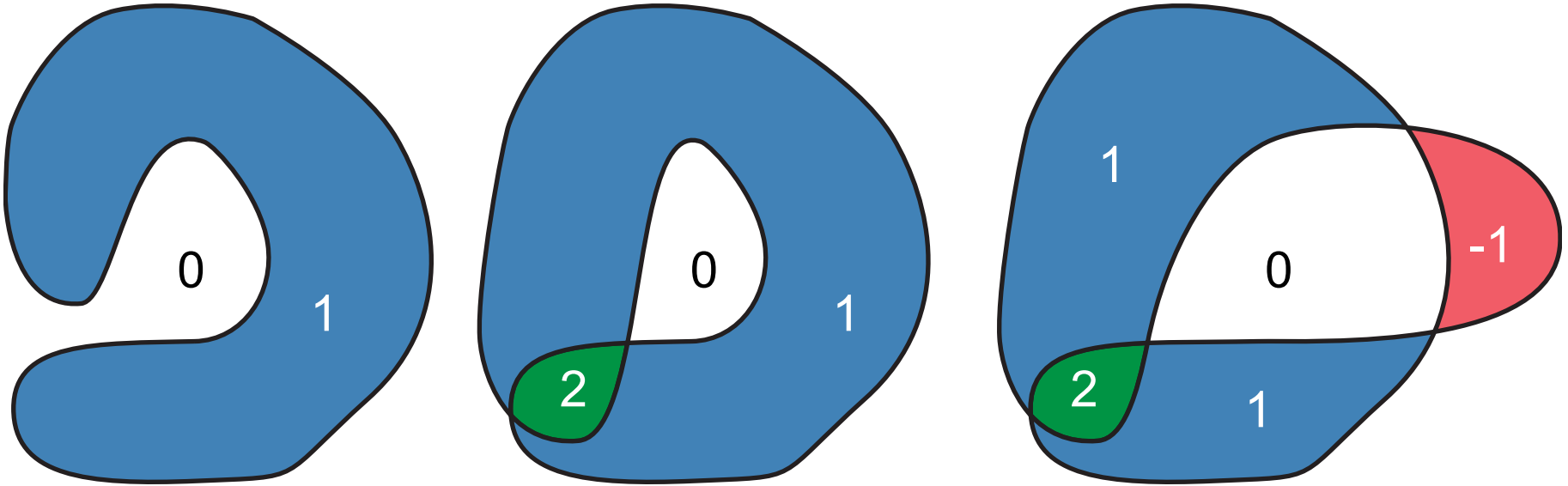

Definitions

PWN Mesh:

A mesh that induces **P**iecewise-constant **W**inding **N**umbers.

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A mesh that induces **P**iecewise-constant **W**inding **N**umbers.

Solid Mesh:

A PWN mesh that

- induces 0/1 winding number field
- without self-intersection or degeneracies

PWN Mesh

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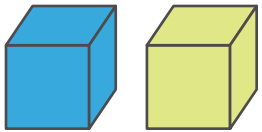

Multiple Components

PWN Mesh

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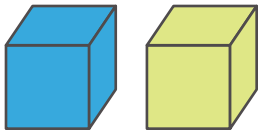

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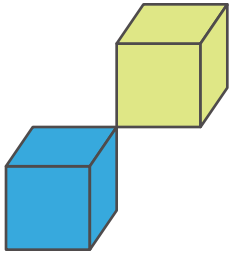

Vertex Non-manifold

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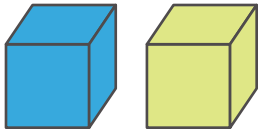

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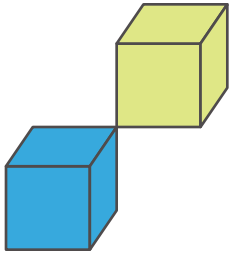

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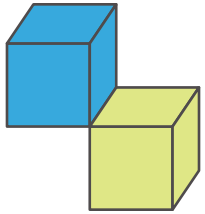

Edge Non-manifold

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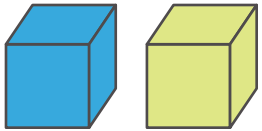

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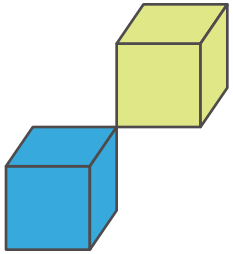

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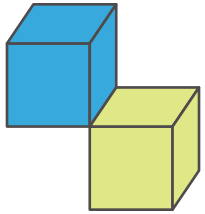

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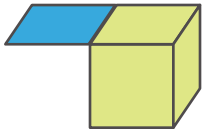

Degenerated Faces

PWN Mesh

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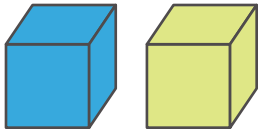

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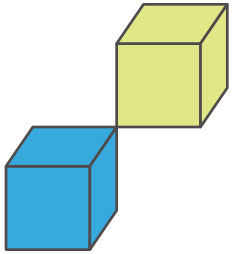

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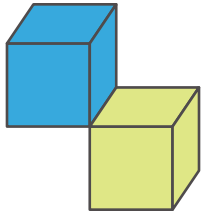

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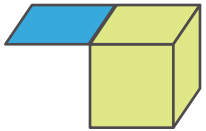
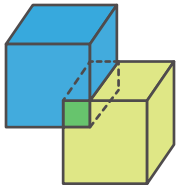


Degenerated Faces



Self-intersection

PWN Mesh

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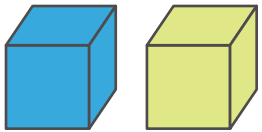

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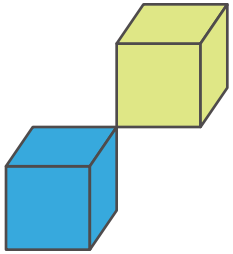
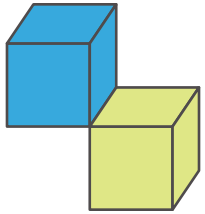


Vertex Non-manifold



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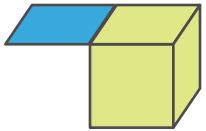

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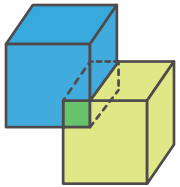

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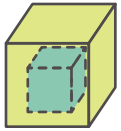

Nested Components

PWN Mesh

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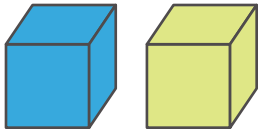
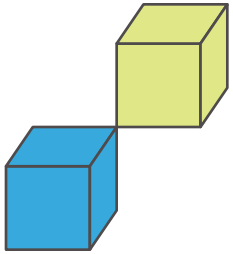
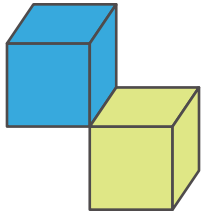


Multiple Components



Vertex Non-manifold



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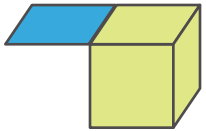
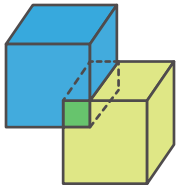


Degenerated Faces



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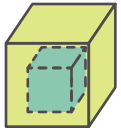

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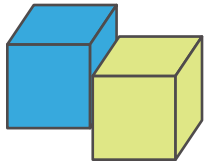
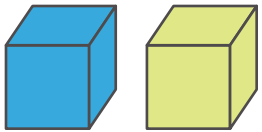


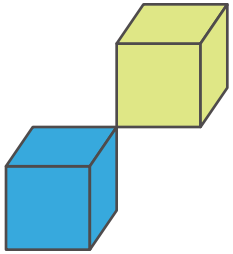
Overlapping Faces

PWN Mesh

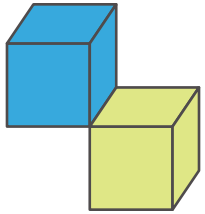
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



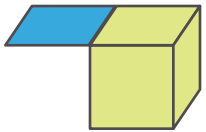
Multiple Components



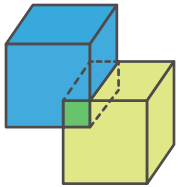
Vertex Non-manifold



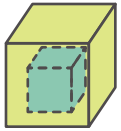
Edge Non-manifold



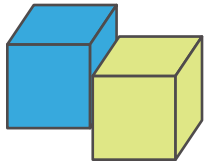
Degenerated Faces



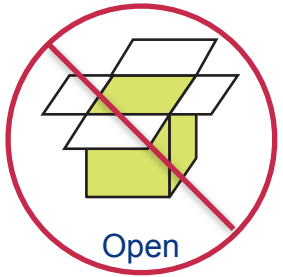
Self-intersection



Nested Components

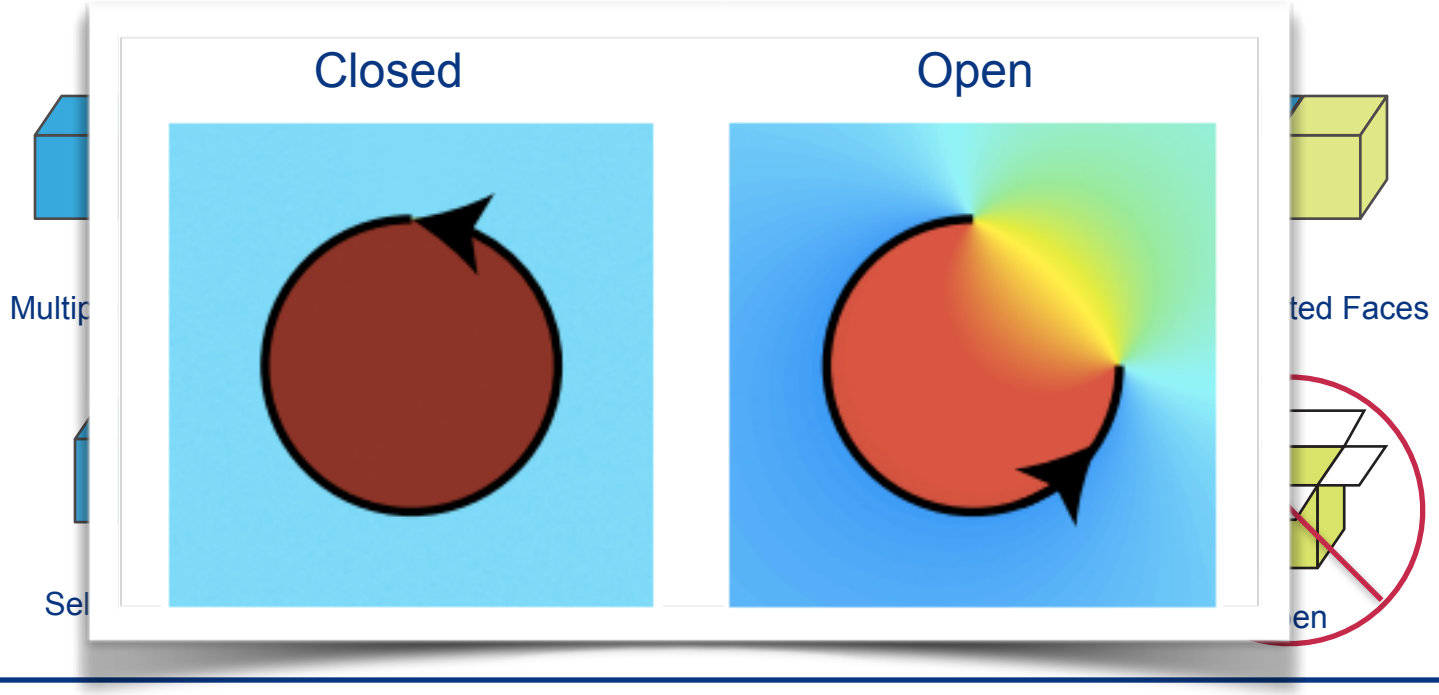


Overlapping Faces



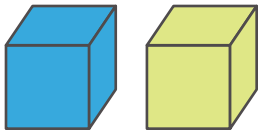
PWN Mesh

PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.

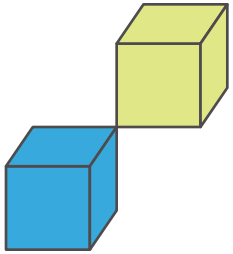


PWN Mesh

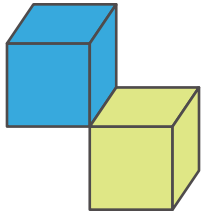
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



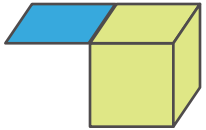
Multiple Components



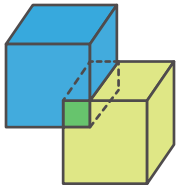
Vertex Non-manifold



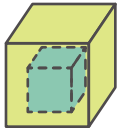
Edge Non-manifold



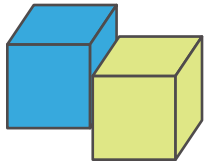
Degenerated Faces



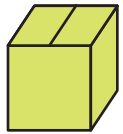
Self-intersection



Nested Components



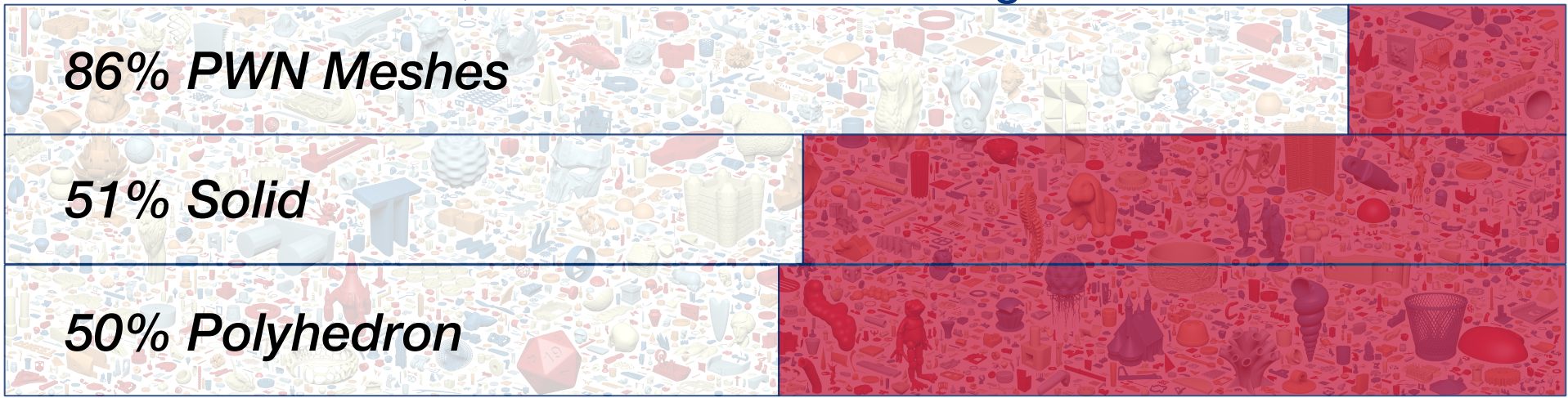
Overlapping Faces



Topologically Open

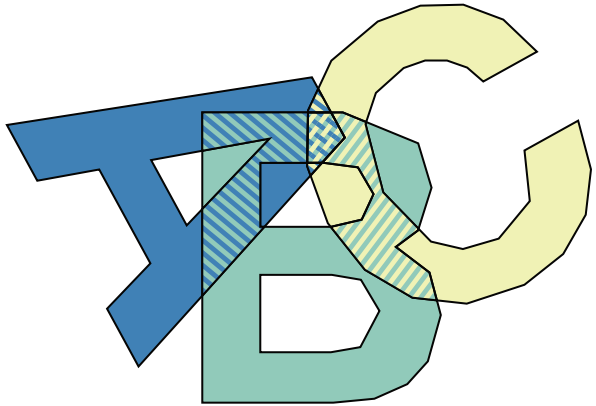
Valid Input

10,000 Meshes from Thingiverse

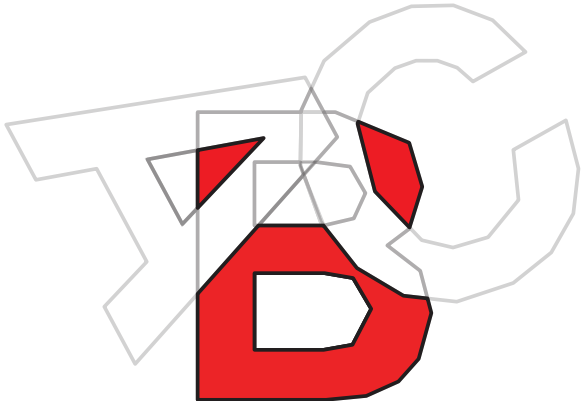


Input and Output

Input: PWN Meshes



Output: Solid Mesh

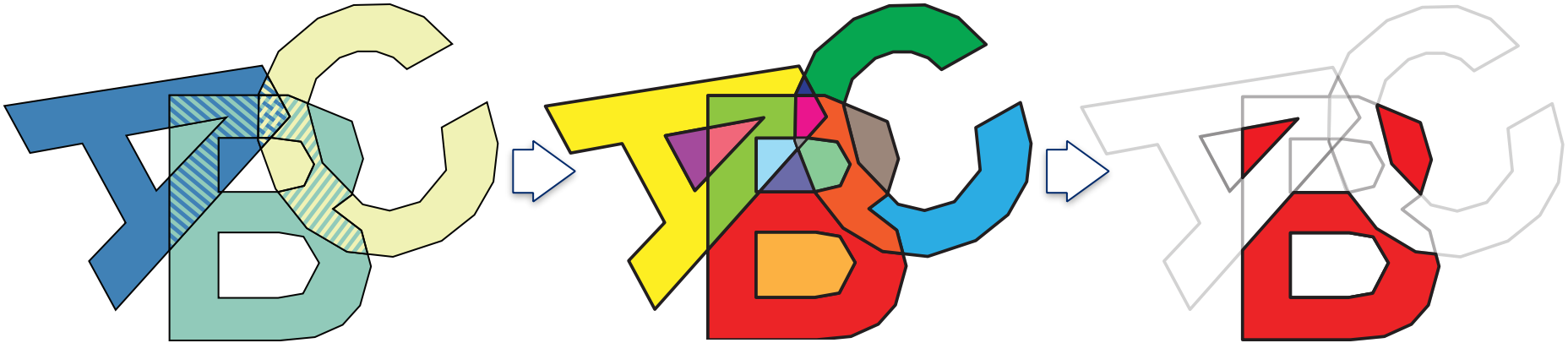


Input and Output

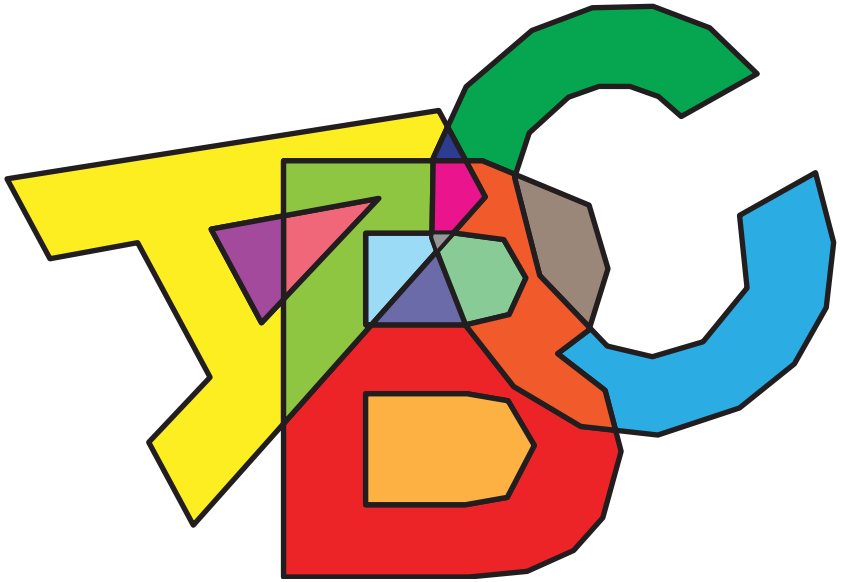
Input: PWN Meshes

Mesh Arrangment

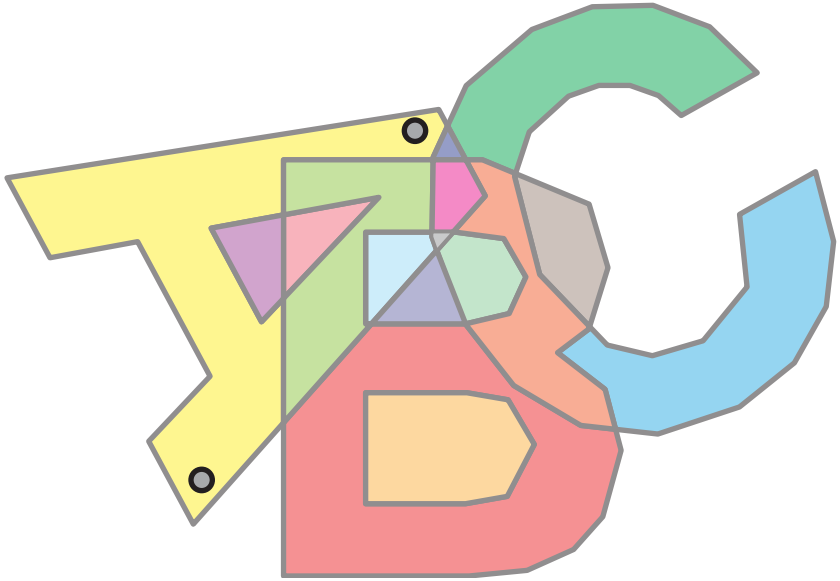
Output: Solid Mesh



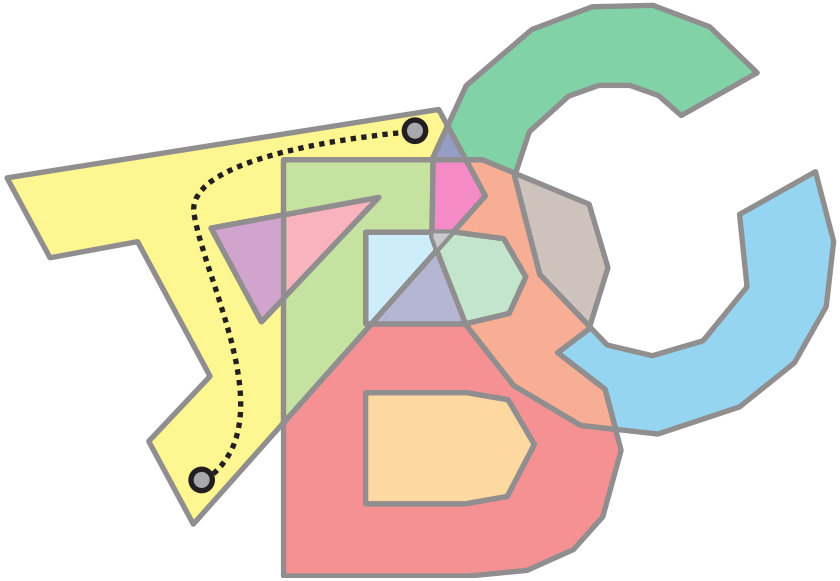
Mesh Arrangement



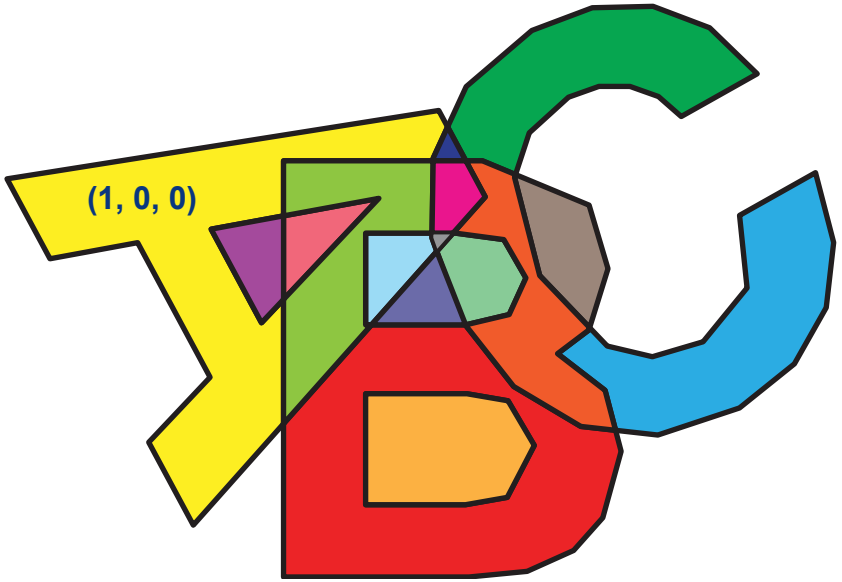
Mesh Arrangement



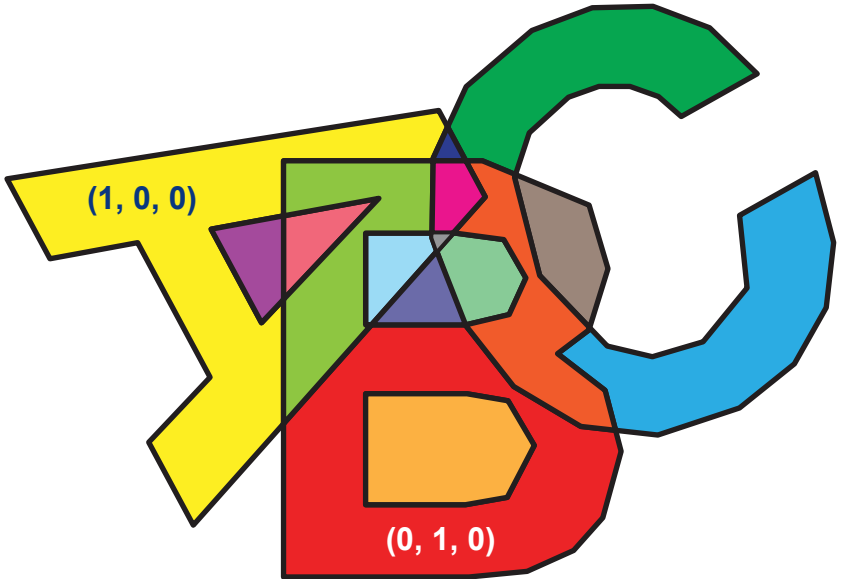
Mesh Arrangement



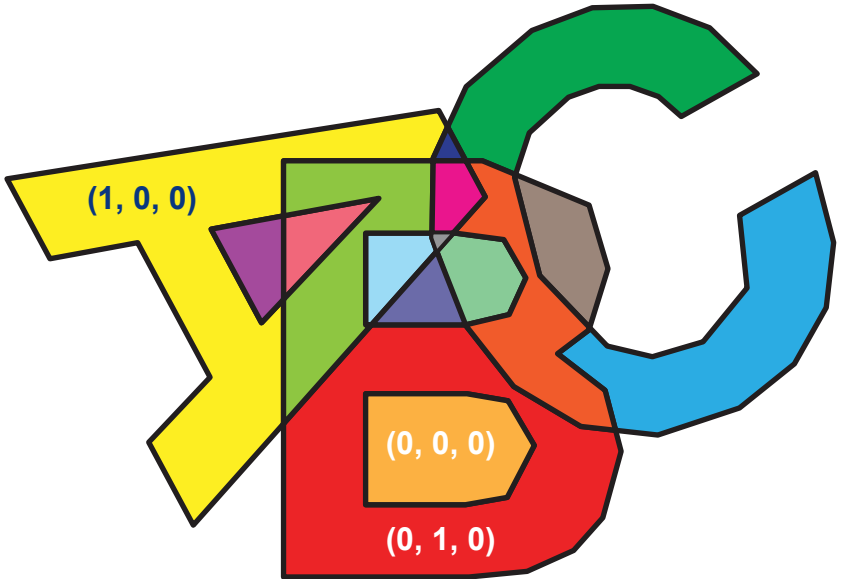
Mesh Arrangement



Mesh Arrangement



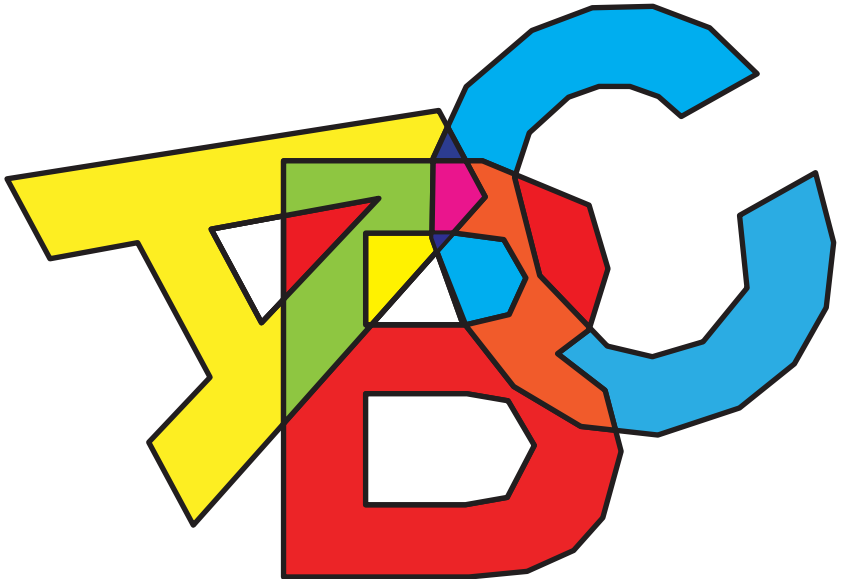
Mesh Arrangement



Mesh Arrangement

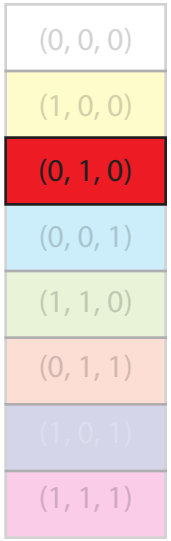
(w_A, w_B, w_C)

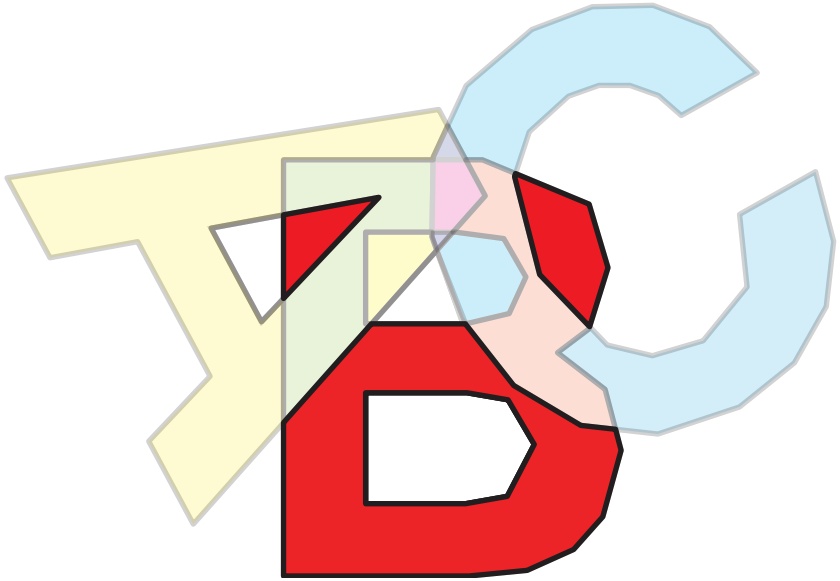
(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)



Mesh Arrangement

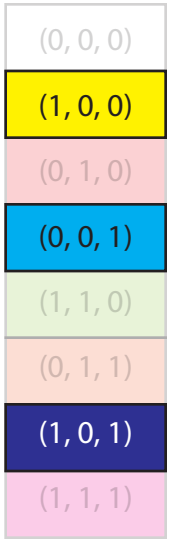
(w_A, w_B, w_C)

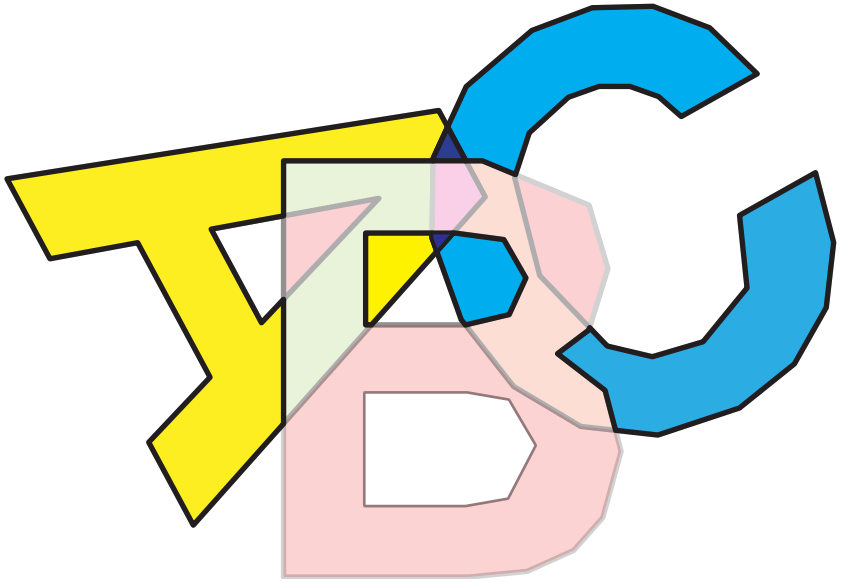




Mesh Arrangement

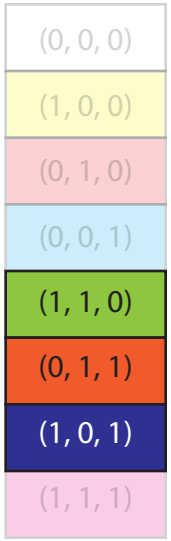
(w_A, w_B, w_C)

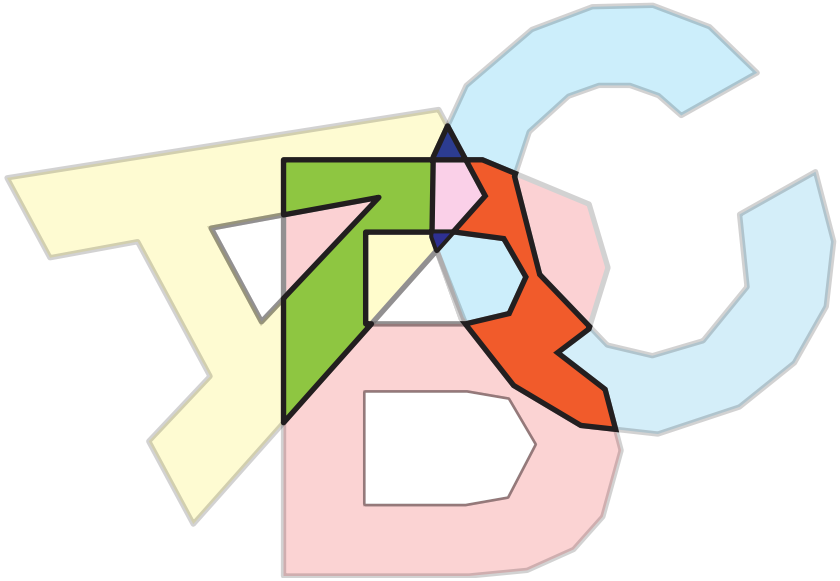




Mesh Arrangement

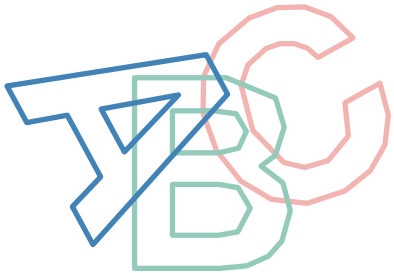
(w_A, w_B, w_C)





Algorithm

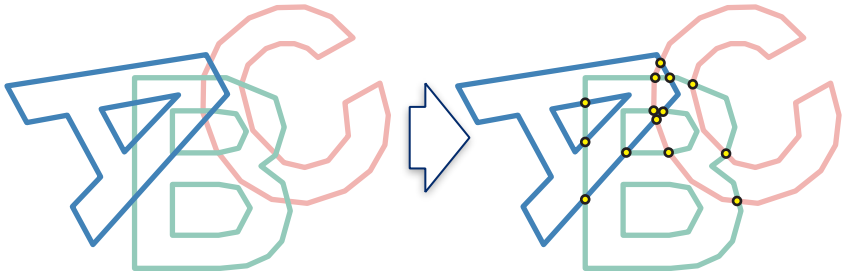
Input Meshes



Algorithm

Input Meshes

Resolving
(Self-)Intersections

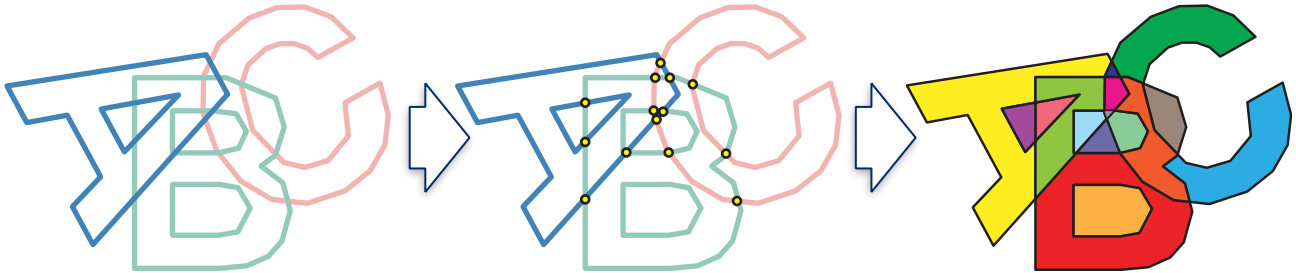


Algorithm

Input Meshes

Resolving (Self-)Intersections

Extracting Cells



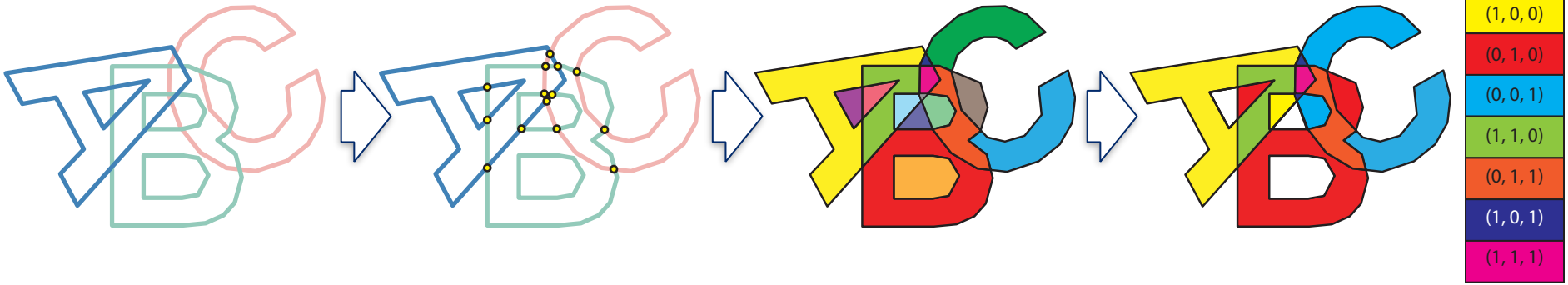
Algorithm

Input Meshes

Resolving (Self-)Intersections

Extracting Cells

Propagating Winding Numbers

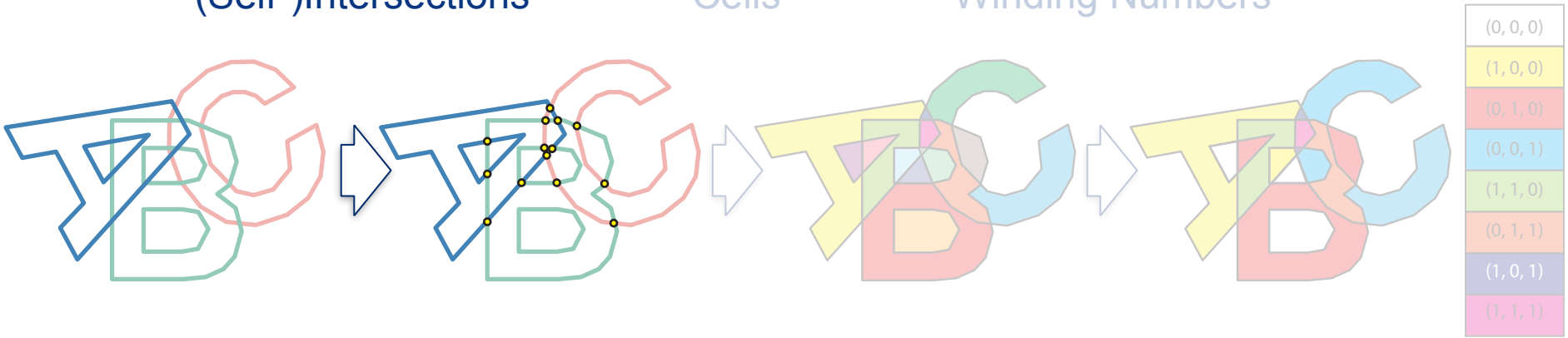


Algorithm

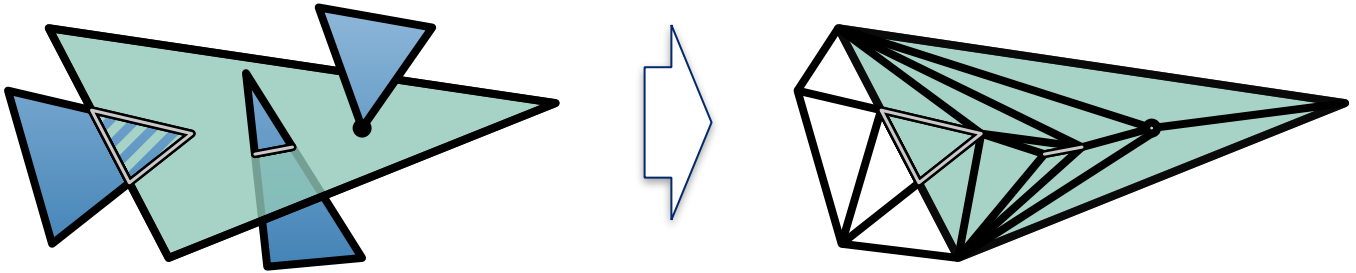
Resolving (Self-)Intersections

Extracting Cells

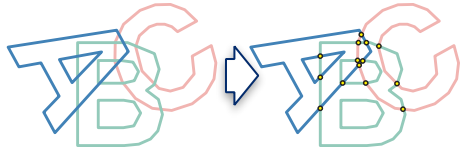
Propagating Winding Numbers



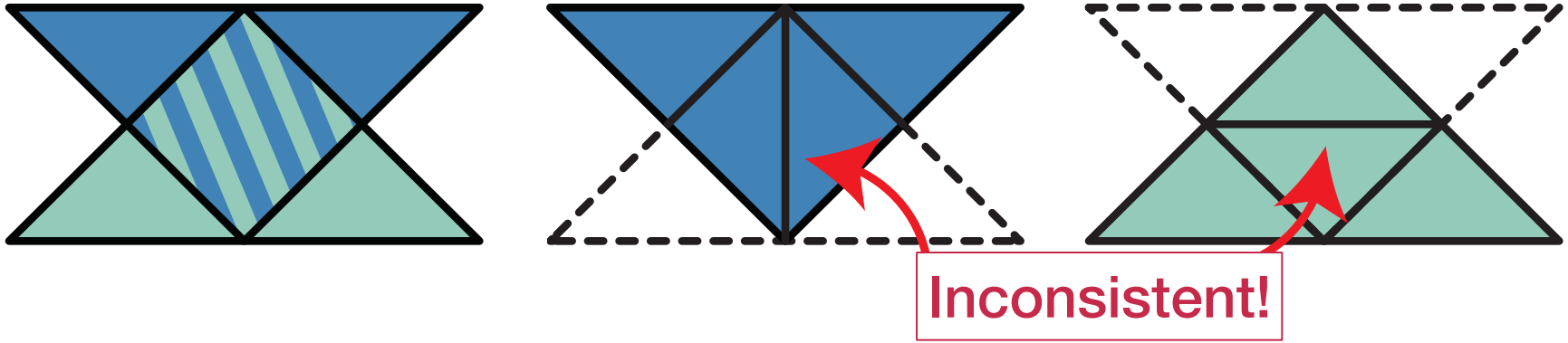
Resolving (Self-)Intersections



Resolving (Self-)Intersections



Pitfall: CDT is not unique, overlapping triangle needs to be consistently resolved!

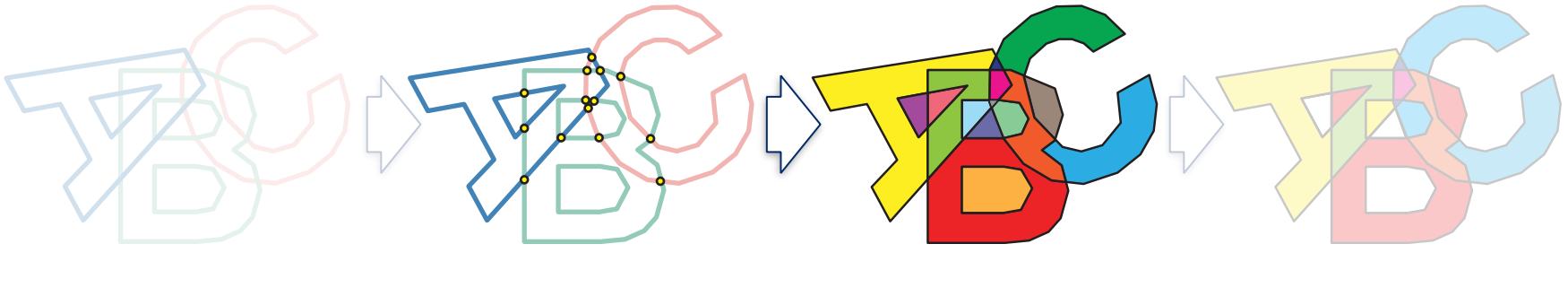


Algorithm

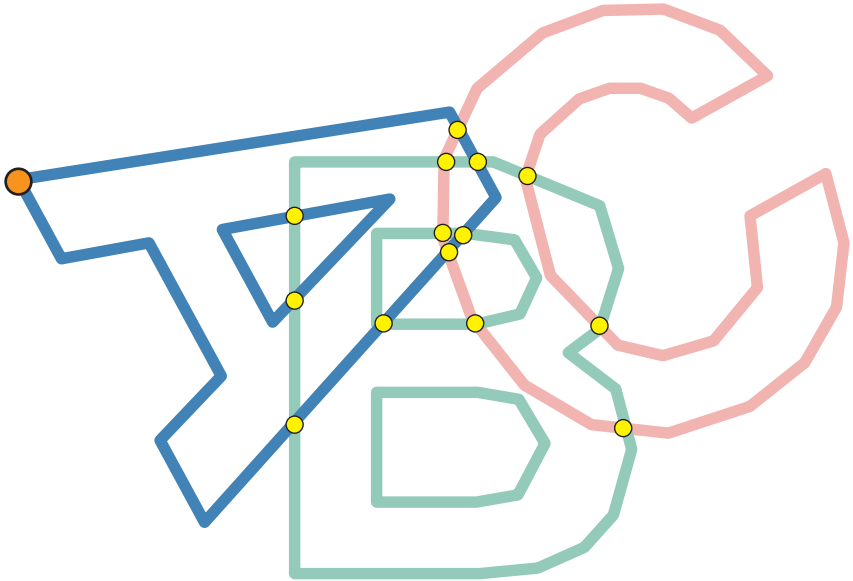
Resolving (Self-)Intersections

Extracting Cells

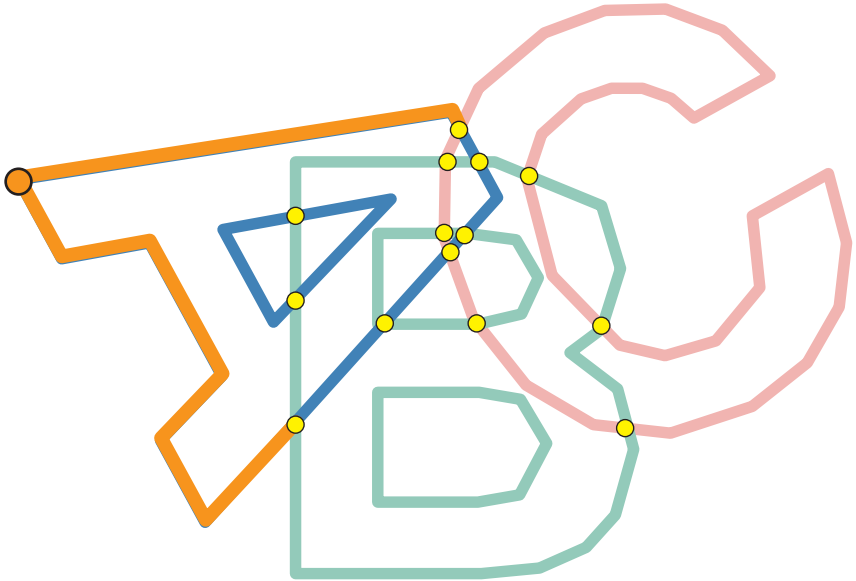
Propagating Winding Numbers



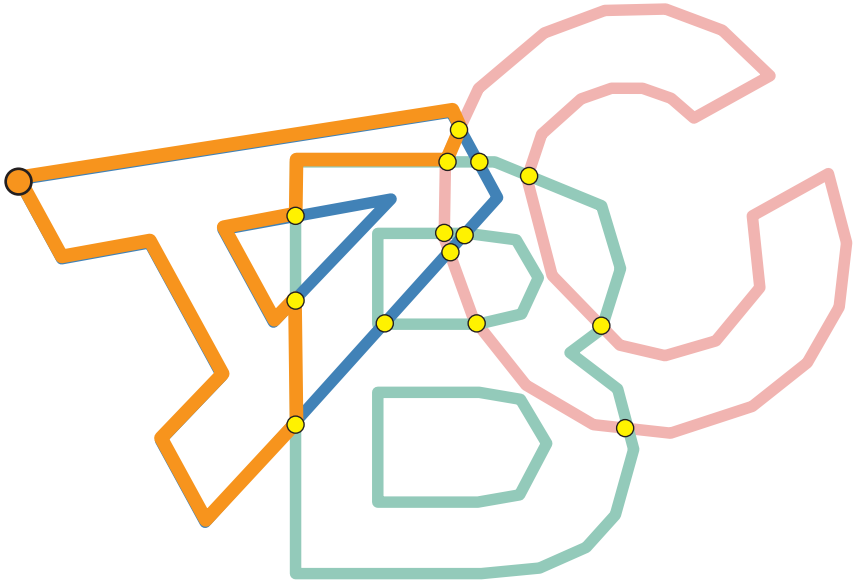
Extracting Cells

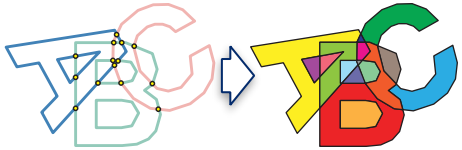


Extracting Cells

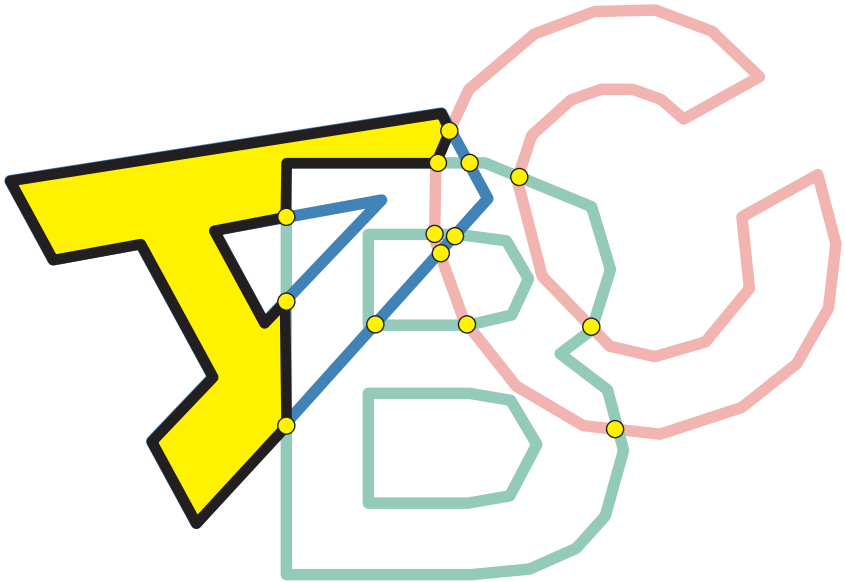


Extracting Cells

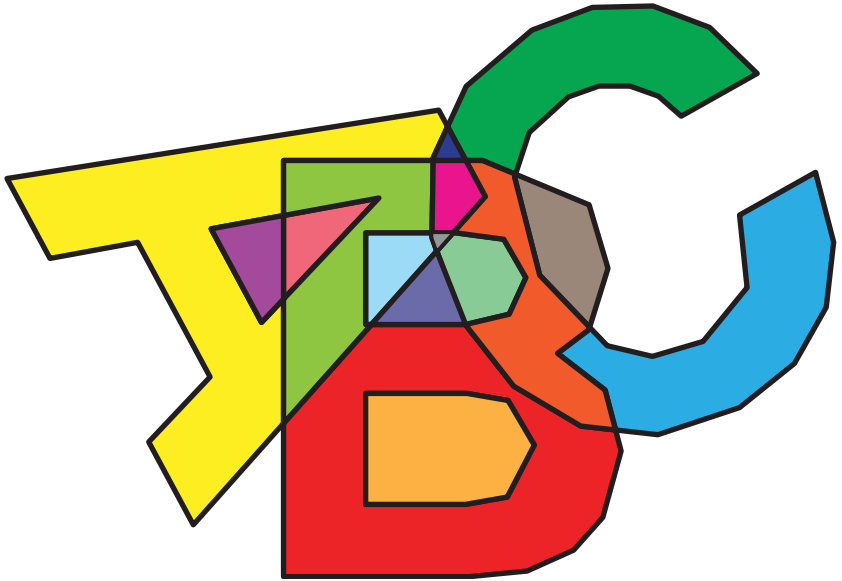




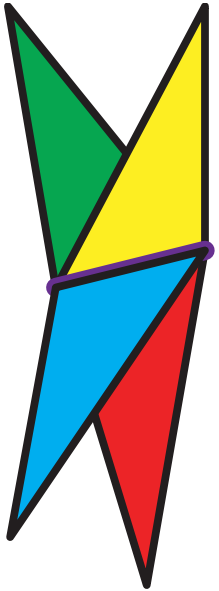
Extracting Cells



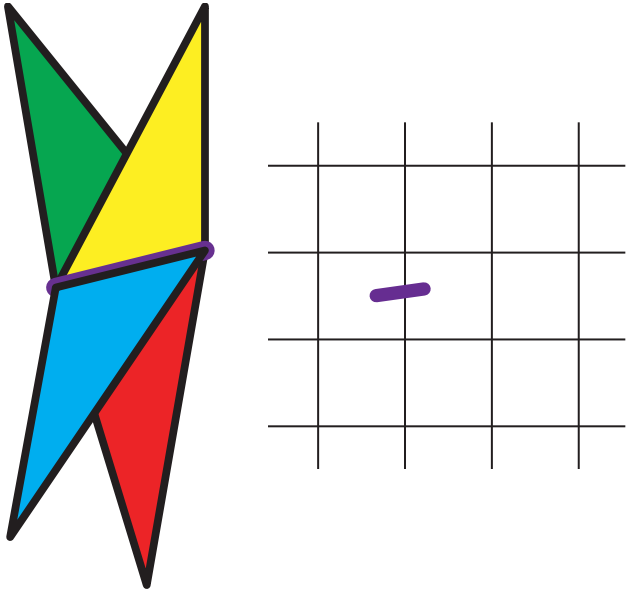
Extracting Cells



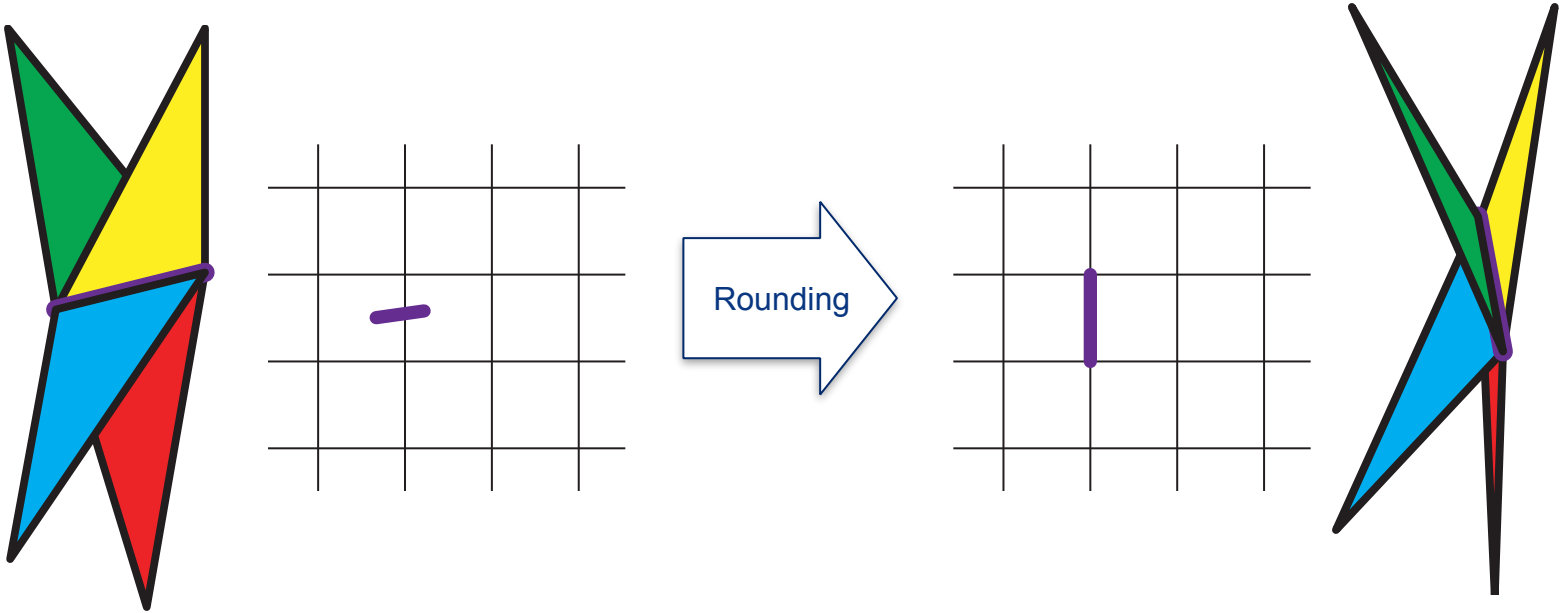
Extracting Cells



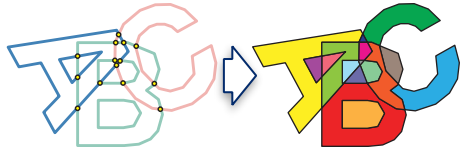
Extracting Cells

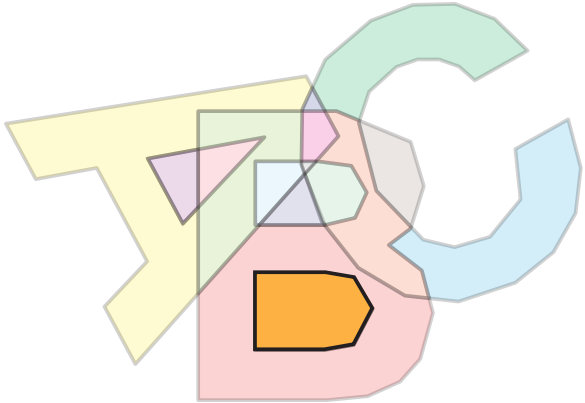


Extracting Cells

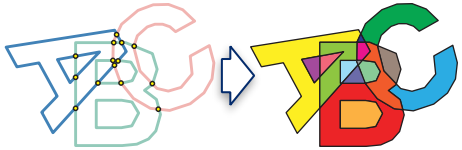


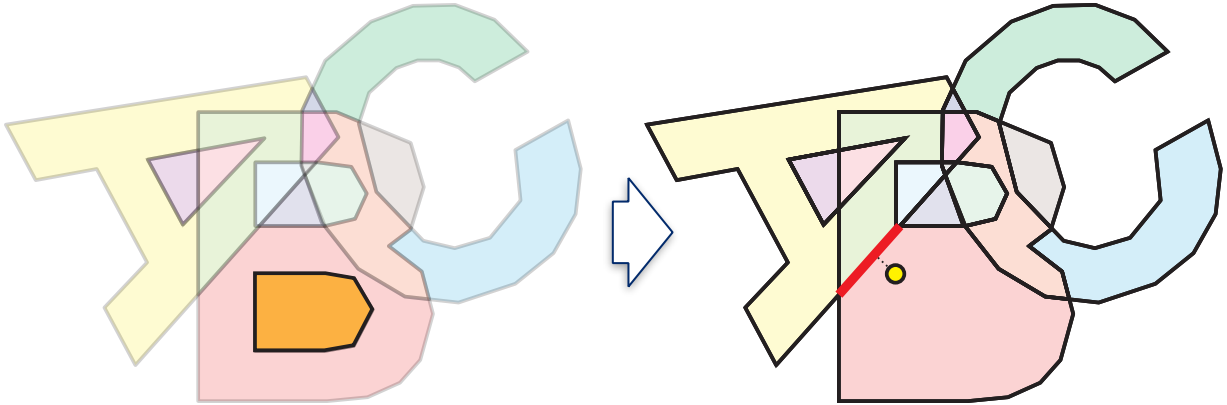
Extracting Cells



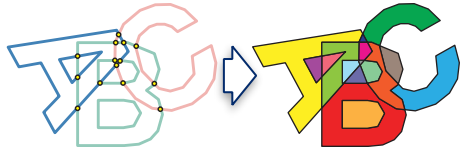


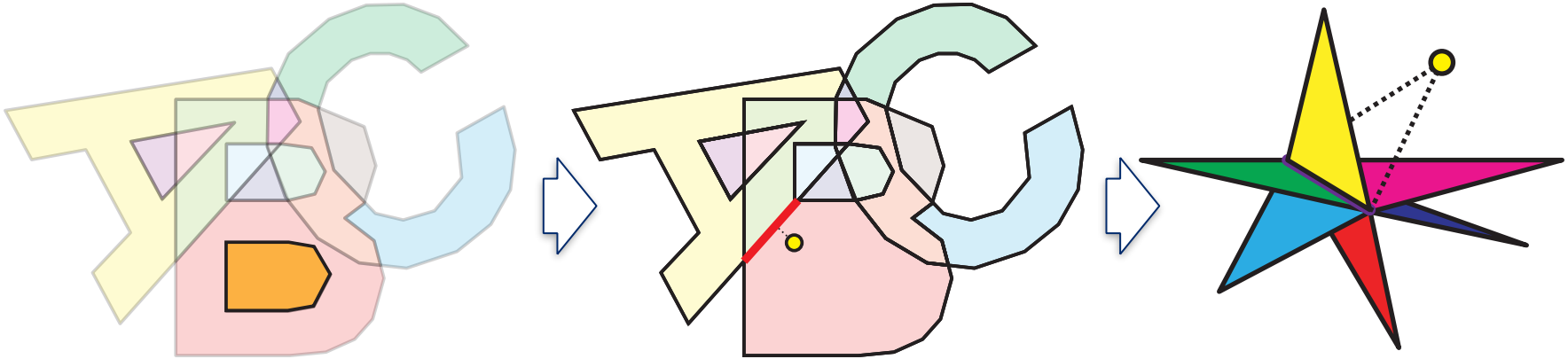
Extracting Cells



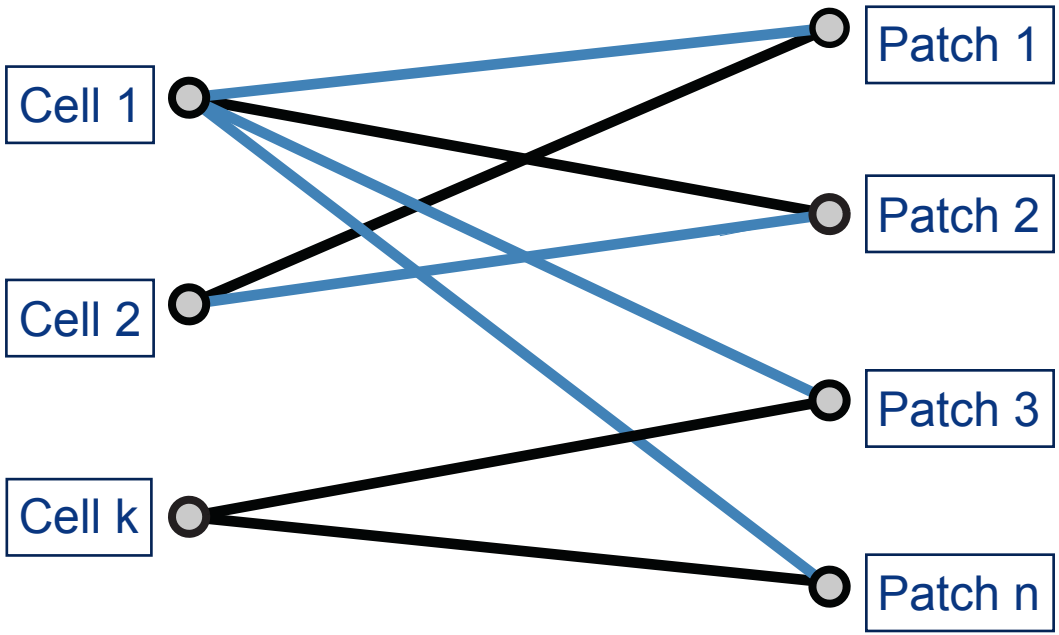


Extracting Cells





Extracting Cells

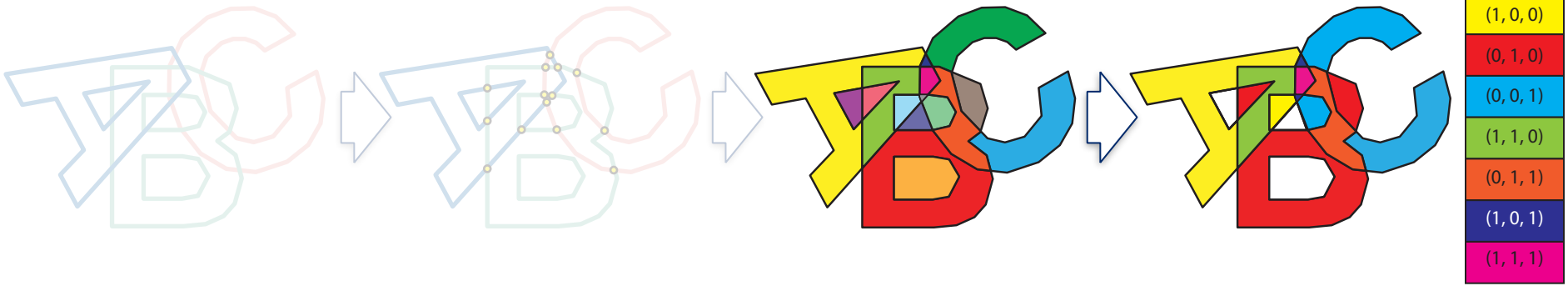


Algorithm

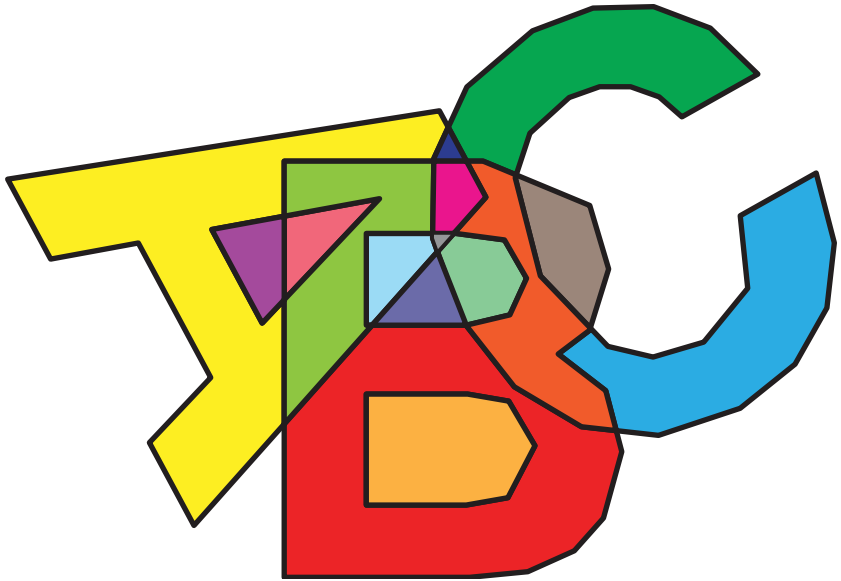
Resolving (Self-)Intersections

Extracting Cells

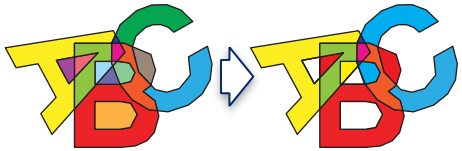
Propagating Winding Numbers



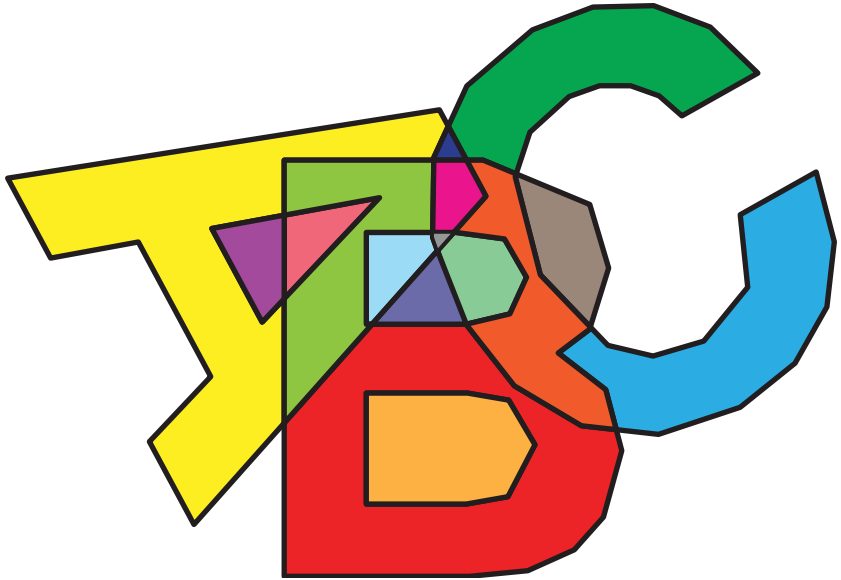
Propagating Winding Numbers



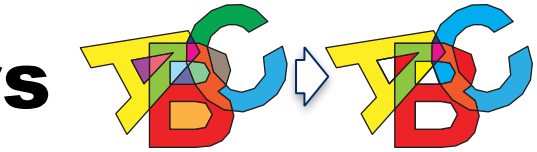
Propagating Winding Numbers



(0, 0, 0)



Propagating Winding Numbers



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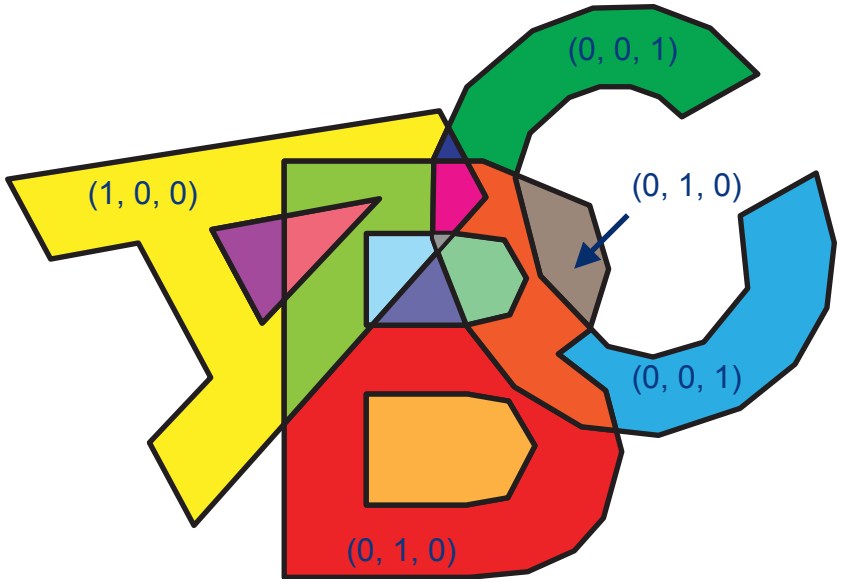
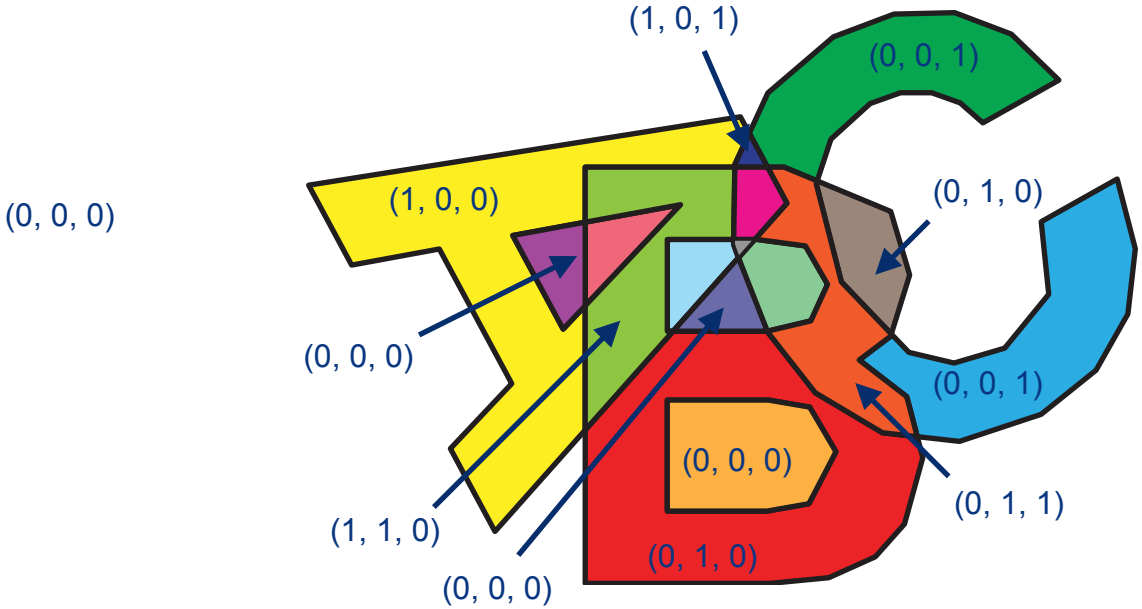
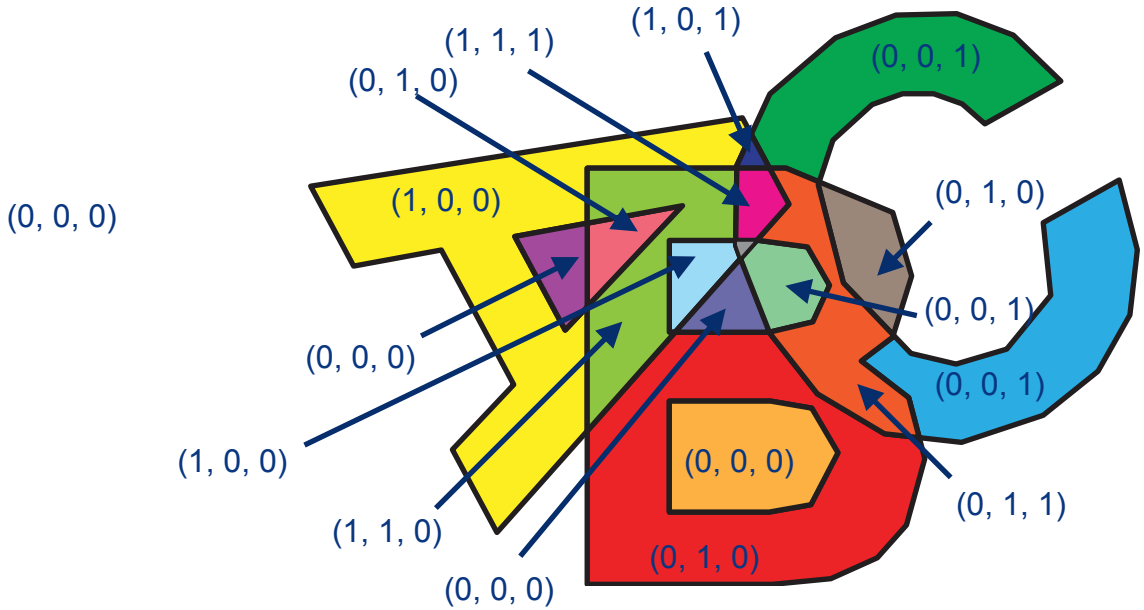


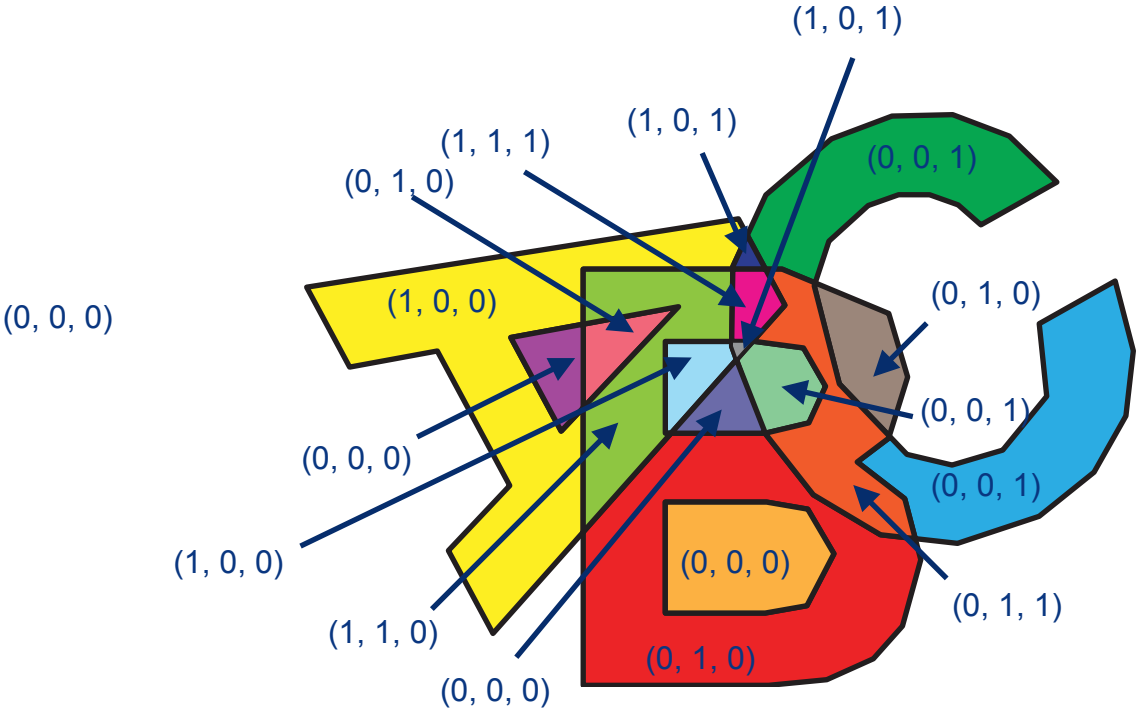
Propagating Winding Numbers



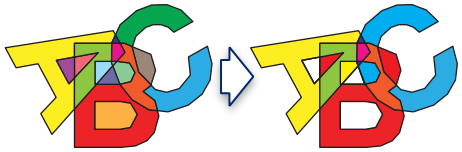
Propagating Winding Numbers



Propagating Winding Numbers

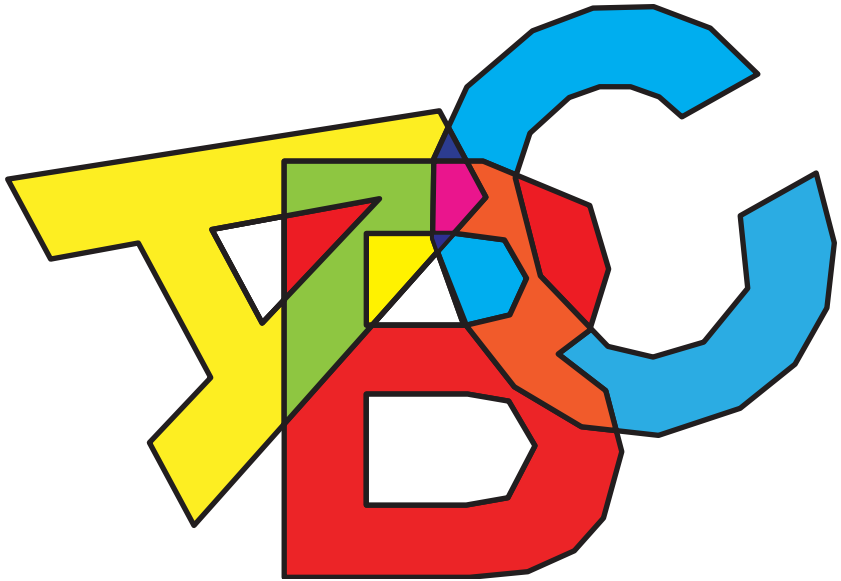


Propagating Winding Numbers



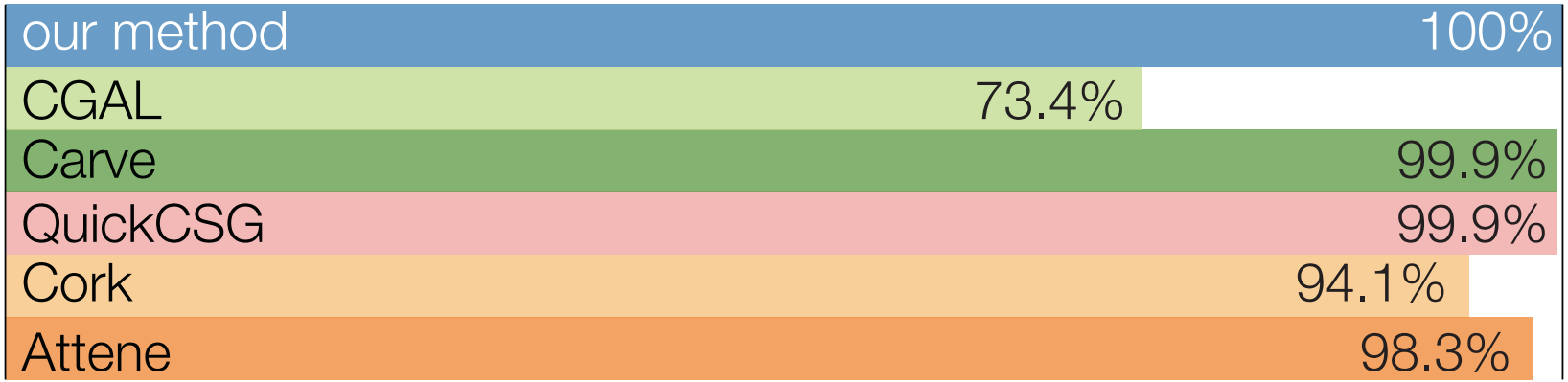
(w_A, w_B, w_C)

(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)

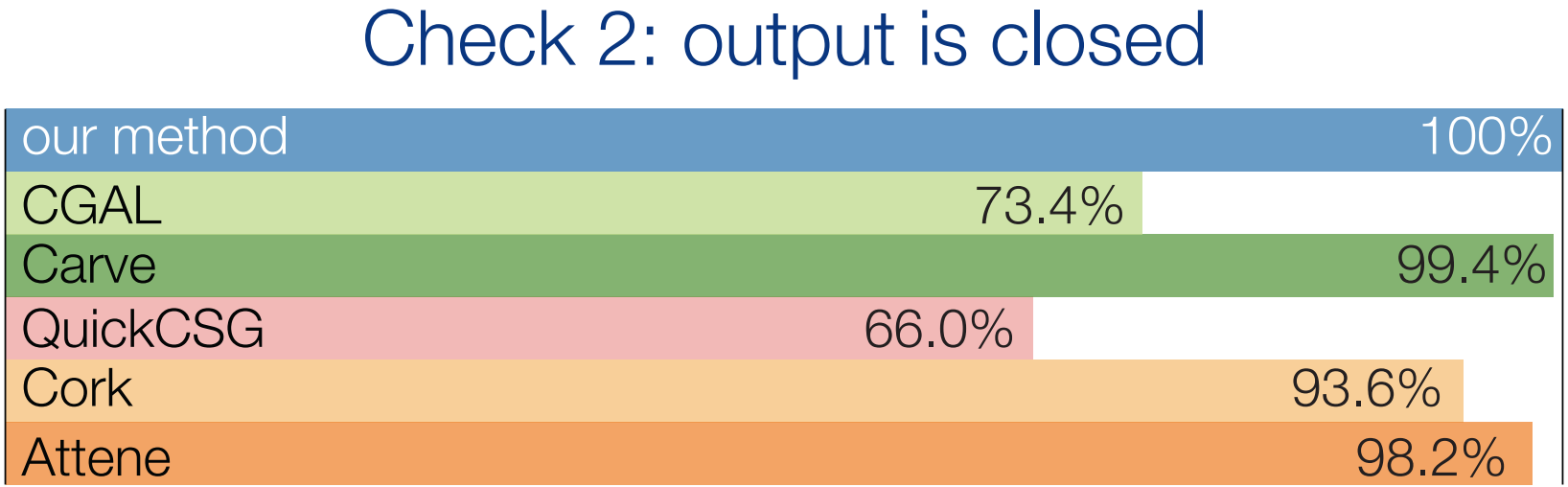


Results: Self Union

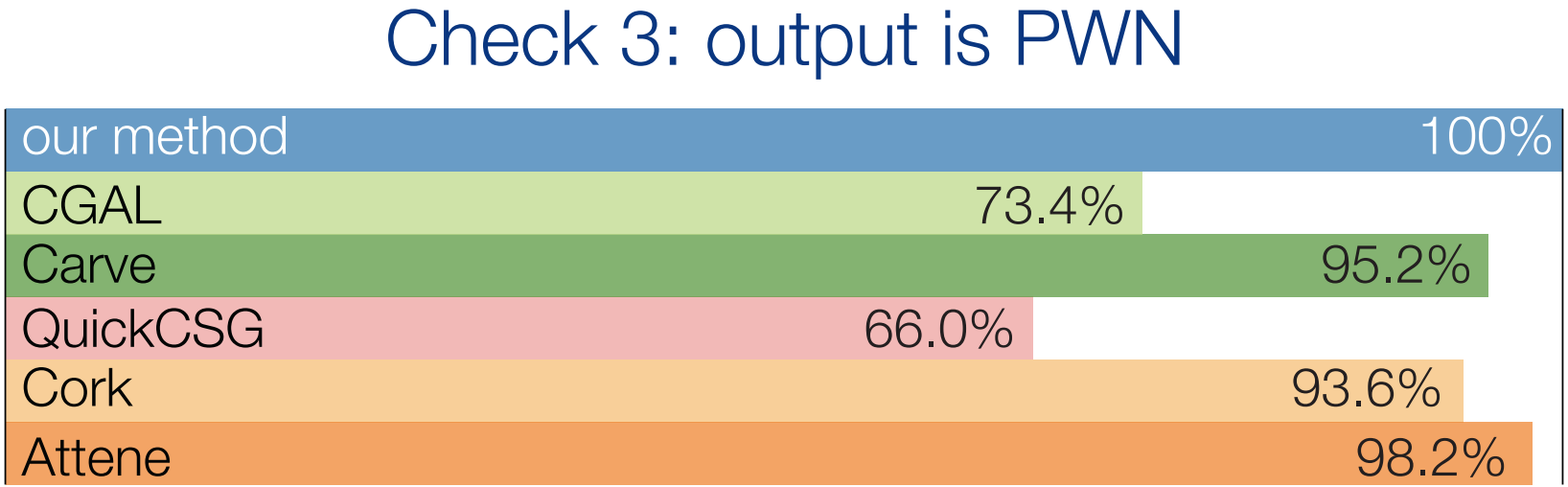
Check 1: produced an output



Results: Self Union

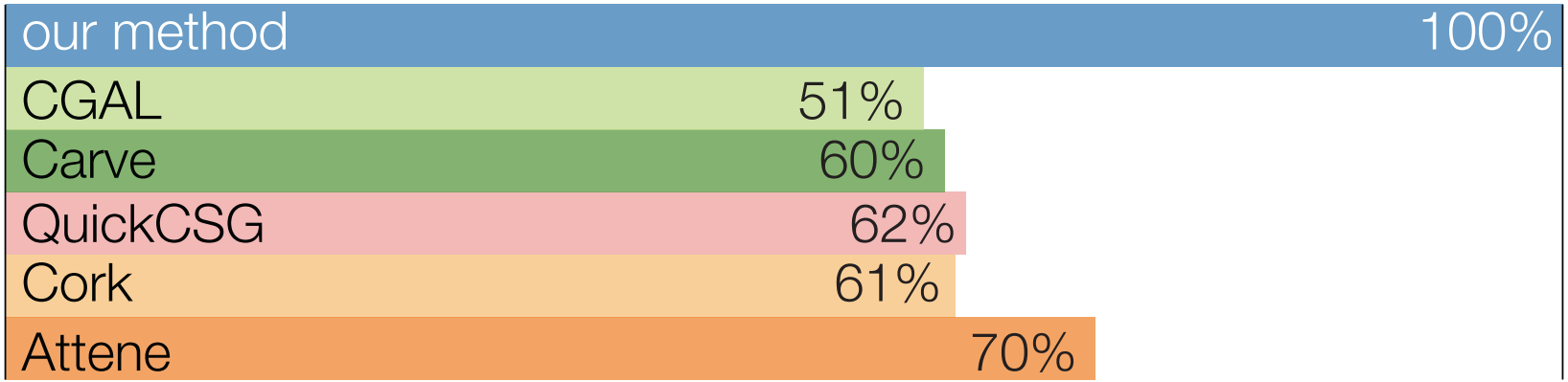


Results: Self Union

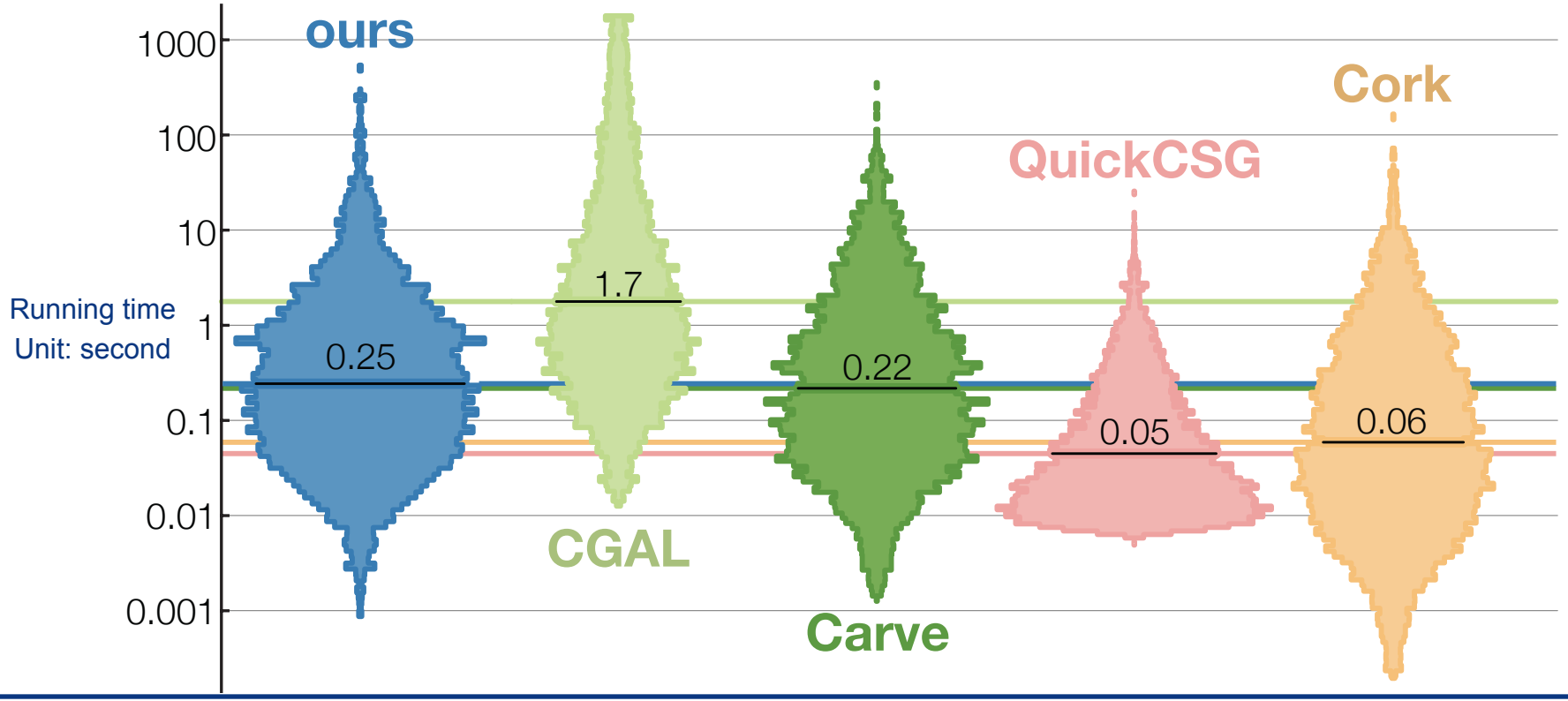


Results: Self Union

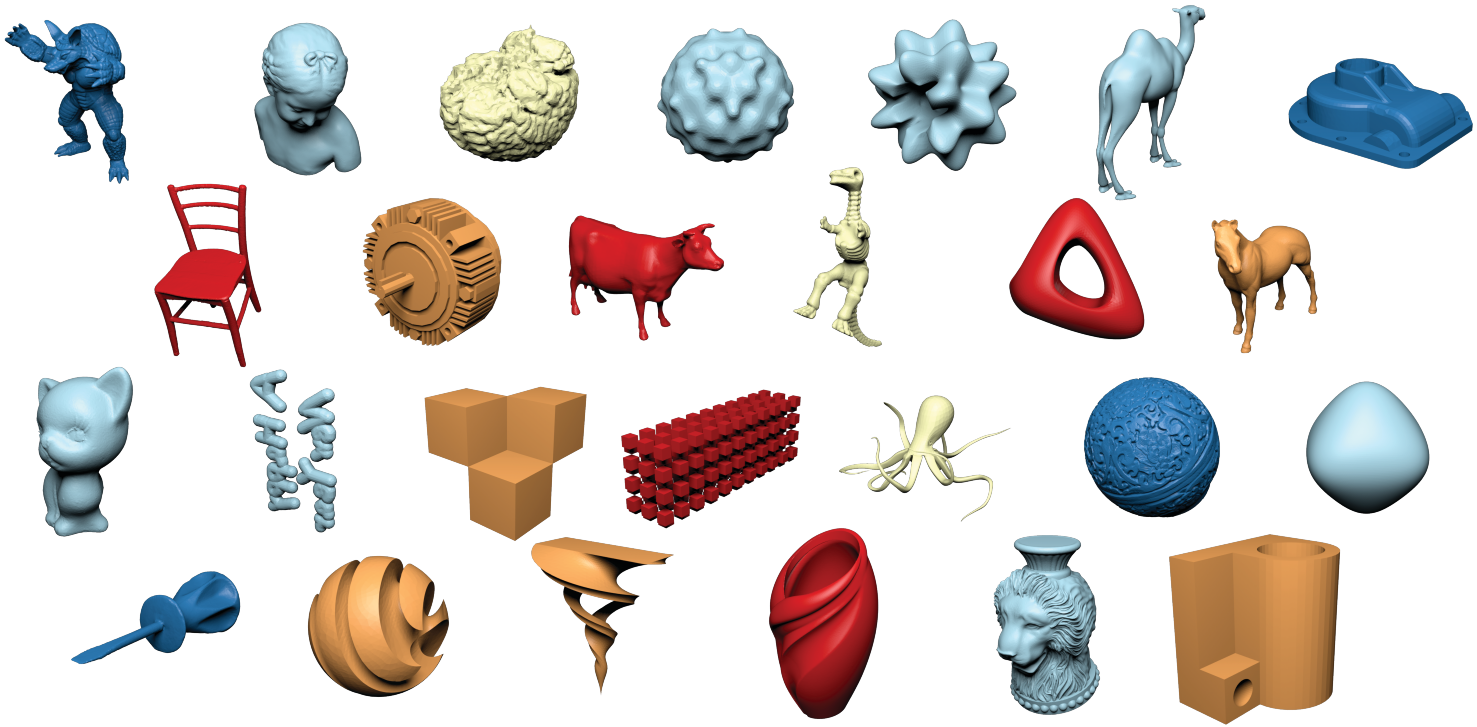
Check 4: output is self-intersection free



Results: Self Union

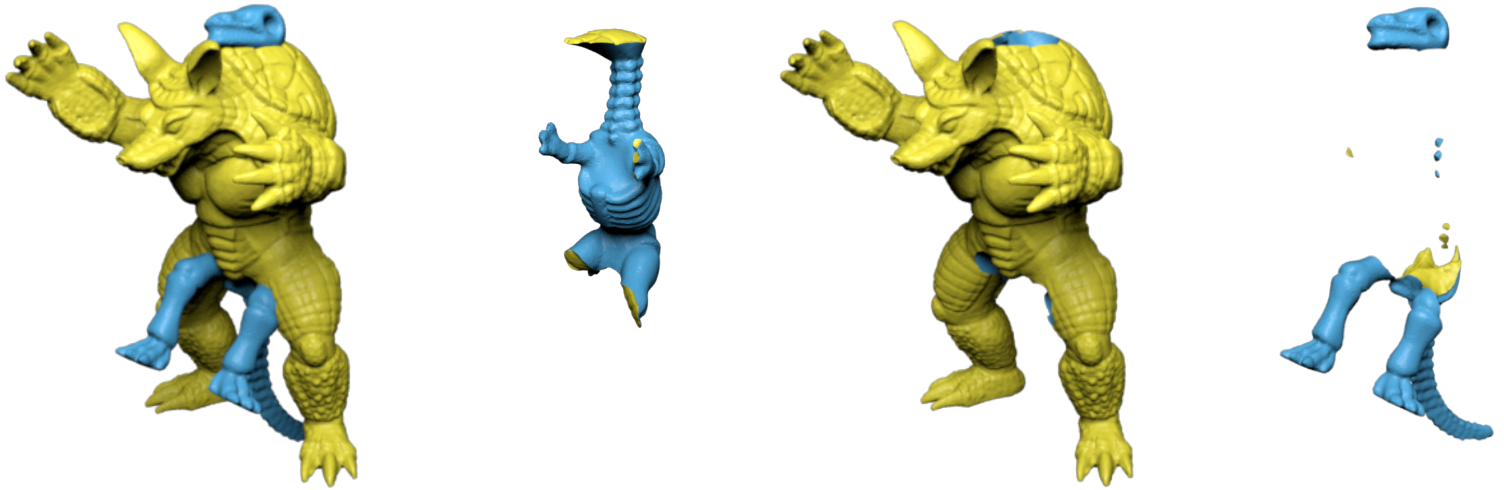


Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset

$A \cup B$ $A \cap B$ $A \setminus B$ $B \setminus A$



1404 results for all possible pairs.

Results: [Barki et al. 2015] Dataset

Results: [Barki et al. 2015] Dataset

Results: [Barki et al. 2015] Dataset

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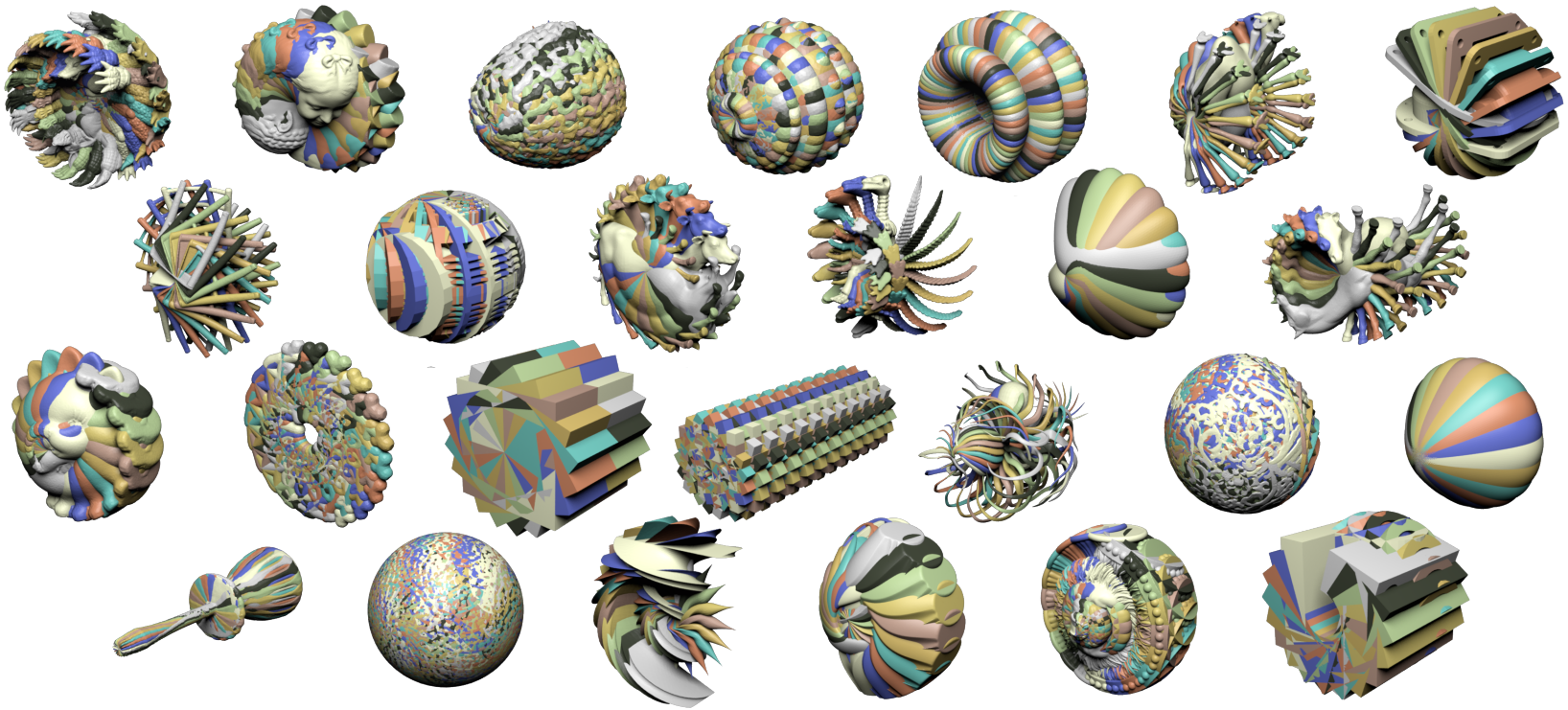
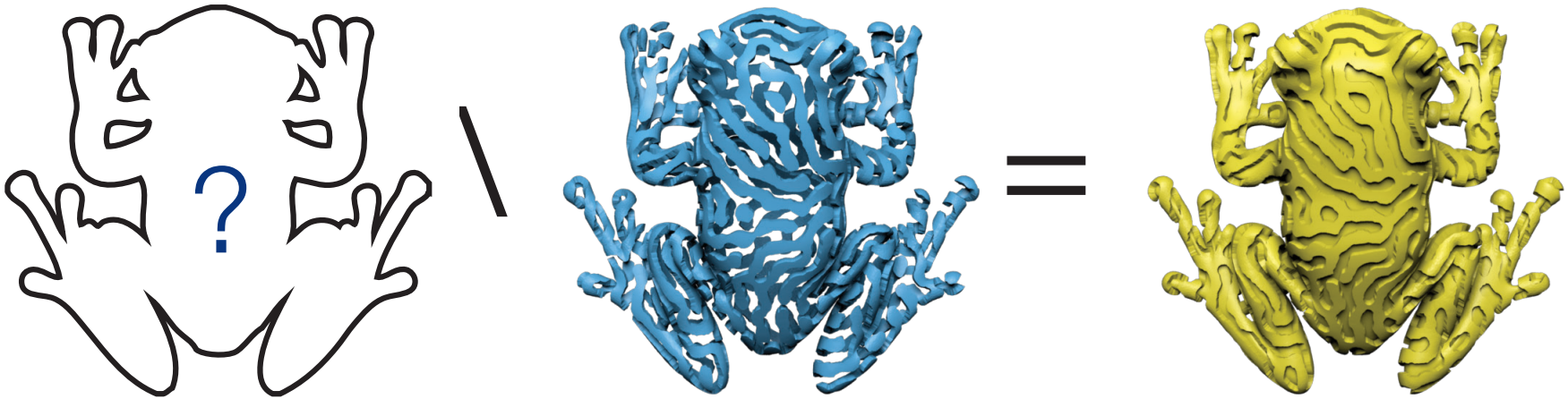
Results: [Barki et al.] Dataset

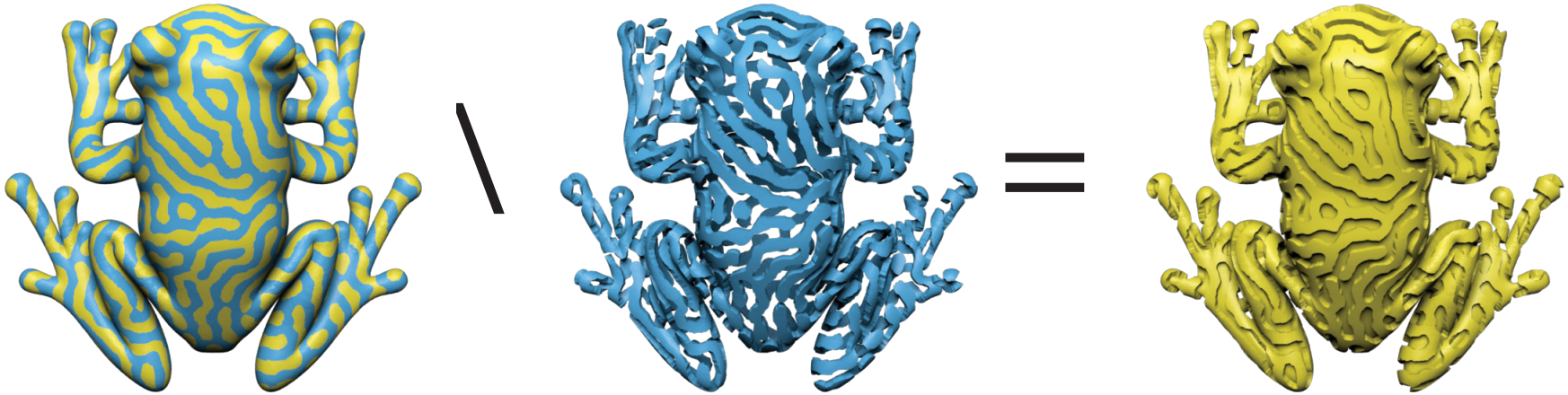


Result: Reverse Boolean



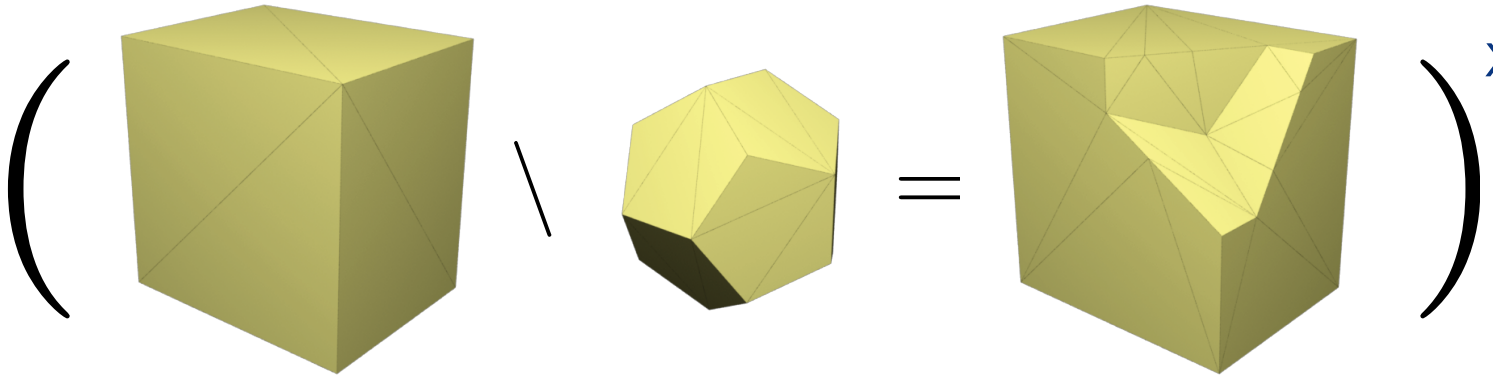
Credit: nervoussystem, 2014

Result: Reverse Boolean

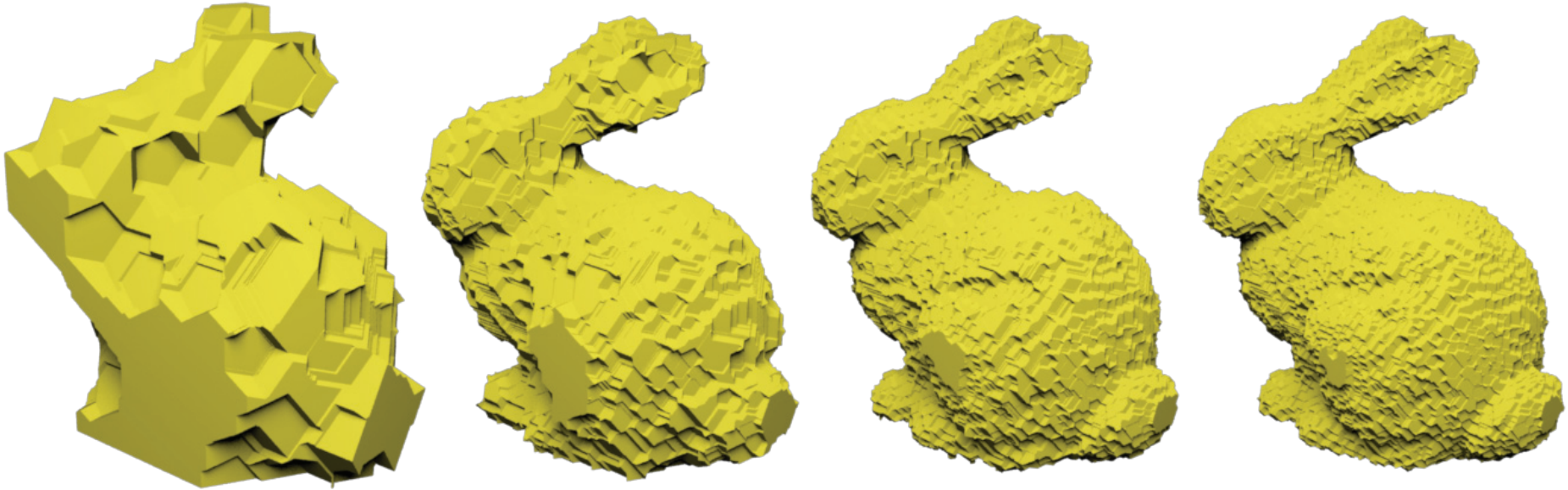


Credit: nervoussystem, 2014

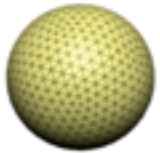
Results: Bunny Carve



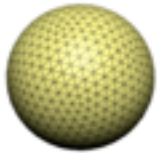
Results: Bunny Carve



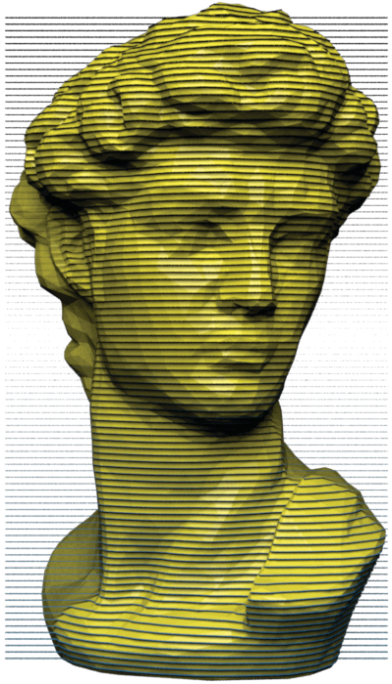
Results: Swept Volume



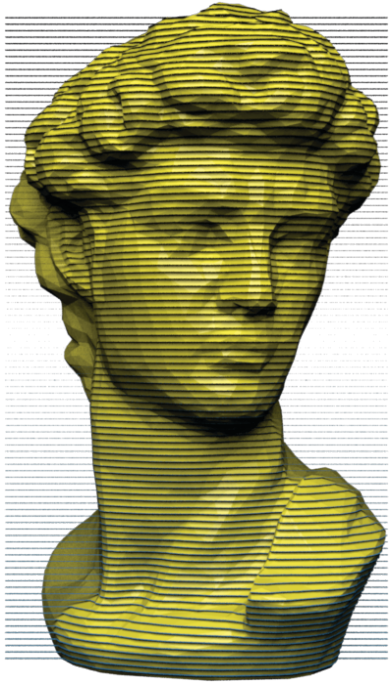
Results: Swept Volume



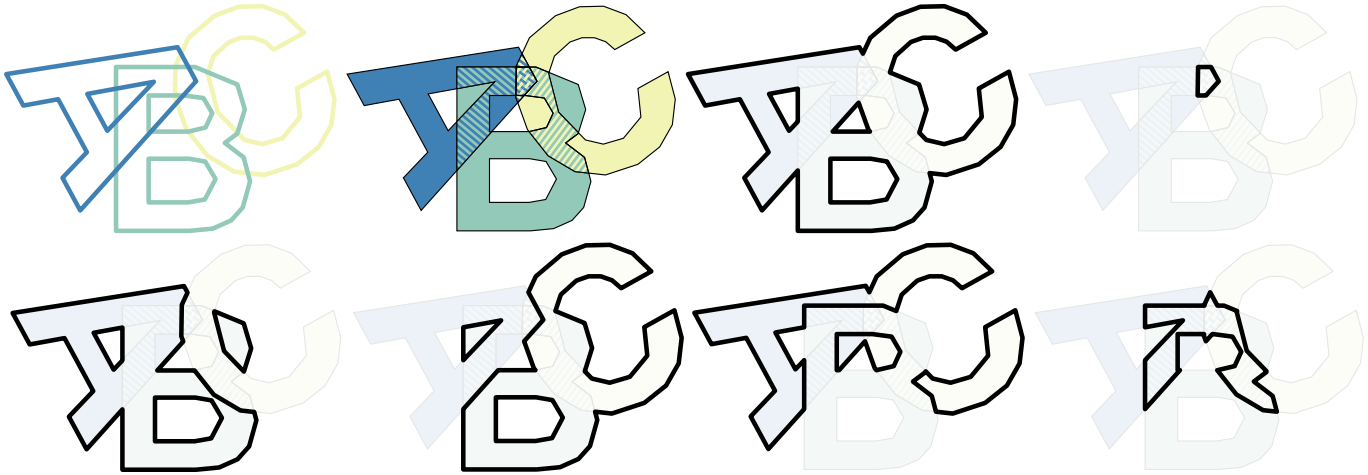
Results: CNC Milling



Results: CNC Milling



Conclusion



Future Works

- Boost performance with adaptive techniques.

- Repairing non-PWN meshes

Resources

Source Code

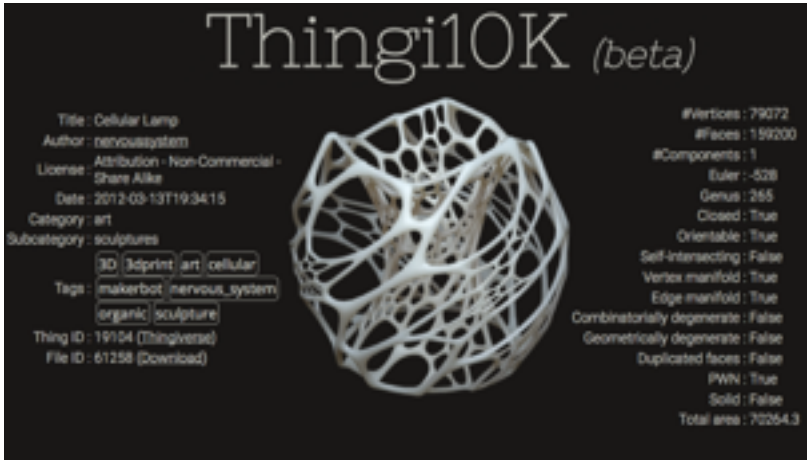
libigl

https://github.com/libigl/libigl

PyMesh

https://github.com/qnzhou/PyMesh

Dataset: Thingi10K



https://ten-thousand-models.appspot.com/