Chinese Remainder Theorem: Chinese remainder theorem (CRT) and the Gushov-Solodkin algorithm [1] for phase unwrapping are not stable if the phases (remainders) are noisy. Even for small errors in the measured phase values (remainders), the reconstructed phase can potentially have huge errors.

Recently, there has been some work on developing robust CRT algorithms [2-7]. One family of techniques requires an expensive 2D search [2,3,5]. Some techniques have faster (polynomial-time) algorithms [6, 7] or even closed-form solution [4]. However, these techniques either have very small error tolerance (almost zero) [2,3,4,5], require at least some of the phases (remainders) to be error-free [6], or recover the correct solution probabilistically [7]. In phase-shifting, all the phases may have (potentially) large noise. Because of this, the above mentioned techniques are not practical for shape recovery using phase shifting.

Search Based Solution (our implementation): In our case, the set of solutions is bounded, from 1 to N, where N is the total number of projector columns.

In order to recover the solution, we perform a search in the correspondence space:

Suppose we use F frequencies: $[w_1, w_2, \ldots, w_F]$. Consider a camera pixel whose corresponding projector column is P. Let the correct phase values at P be $[\phi_1, \phi_2, \ldots, \phi_F]$. Using Eqns. 13-16 in the paper, we can recover the following vector for P: $V_P = [\cos(\phi_1), \sin(\phi_1), \cos(\phi_2),\ldots,\cos(\phi_F)]$. Basically, we recover the cos and sin for the first phase of the first frequency, and cos for the phase of every subsequent frequency.

We write V_P concisely as $V_P = [c_1, s_1, c_2,\ldots,c_F]$.

1-D search: We compute $V_x = [c_1, s_1, c_2,\ldots,c_F]$ for every column number x in [1.. N]. Then, P is computed as:

$$P_{estimate} = \operatorname{argmin}_x ||V_P - V_X||^2$$

Computation of the fractional part: The above 1D search recovers the integer component of the correspondence. The fractional part is computed in the second step by using the phase of the first frequency. Since we have both cos and sin of the phase for the first frequency, we can recovery the first phase unambiguously. From this first phase, we compute the fractional component, and add it to the result from the previous step. For more details, see the attached code.

Remarks:

1) Since we are performing a search, we do not explicitly need to recover the phases for any of the frequencies (except the first, which we need to recover the fractional phase component). Unwrapping is done in the intensity space, not the phase space.

2) The search can be performed independently for each camera pixel, and hence can be parallelized. Parallel code is included using MATLAB parallel computation tool-box.

3) We found a better set of 5-frequencies than the one mentioned in the paper (by performing search). The frequency sets along with many others are included. A sample data-set is also included.

References:

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