

Calculators, Interpreters, and Compilers: Theory and Practice

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November 21, 2025



Lee Chae Yeon (이채연), South Korean pop singer, on
Jiwhaza 작렬! 정신통일 Burst! Mental Concentration,
SBS, 2007, episode 20

$$2 + 2 \times 2$$

$$2 + 2 \times 2$$

$$2 + (2 \times 2)$$

$$2 + 2 \times 2$$

$$2 + (2 \times 2)$$

$$2 + (4)$$

$$6$$

$$2_+_2_*_2$$

$$2_+_2_*_2$$

2	+	2	*	2
---	---	---	---	---

$$2_+_2_*_2$$

2	+	2	*	2
---	---	---	---	---

number plus number times number

token $\rightarrow$ $\sqcup$ *token*

token $\rightarrow$ $\sqcup$ *token*
| *plus*
| *times*
| *number*

token $\rightarrow$ $\sqcup$ *token*
| *plus*
| *times*
| *number*

plus $\rightarrow$ +

times $\rightarrow$ *

token $\rightarrow$ $_$ *token*

| *plus*

| *times*

| *number*

plus $\rightarrow$ +

times $\rightarrow$ *

number $\rightarrow$ *digit* *digits*

$token \rightarrow _ \quad token$
 $\quad \quad | \quad plus$
 $\quad \quad | \quad times$
 $\quad \quad | \quad number$

$plus \rightarrow +$

$times \rightarrow *$

$number \rightarrow digit \quad digits$

$digits \rightarrow digit \quad digits$
 $\quad \quad | \quad \epsilon$

$digit \rightarrow 0 \quad | \quad 1 \quad | \quad 2 \quad | \quad 3 \quad | \quad 4$
 $\quad \quad | \quad 5 \quad | \quad 6 \quad | \quad 7 \quad | \quad 8 \quad | \quad 9$

```
void next_token()
{
    char c;
    while ((c = *nextc) == '_') nextc++;
    switch (c) {
        case '\0': token = end;                break;
        case '+': token = plus; nextc++; break;
        case '*': token = times; nextc++; break;
        case '0': case '1': case '2': case '3': case '4':
        case '5': case '6': case '7': case '8': case '9':
            token = number; token_value = 0;
            do
                token_value = token_value * 10 + c - '0';
            while ((c = *(++nextc)) >= '0' && c <= '9');
            break;
        default:
            fprintf(stderr, "unexpected_character_ '%c'\n", c);
            exit(1);
            break;
    }
}
```

$$2 + 2 \times 2$$

$expr \rightarrow$ $expr \ plus \ expr$
 | $expr \ times \ expr$
 | $number$

$$2 + 2 \times 2$$

sum → *sum plus sum*
 | *product*

product → *product times product*
 | *number*

$$2 + 2 \times 2$$

sum → *sum plus product*
 | *product*

product → *product times number*
 | *number*

$$2 + 2 \times 2$$

sum $\rightarrow$ *product sums*

sums $\rightarrow$ *plus product sums*

| ϵ

product $\rightarrow$ *number products*

products $\rightarrow$ *times number products*

| ϵ

product $\rightarrow$ *number products*
products $\rightarrow$ *times number products*
| ϵ

```
int product() {  
    if (token != number) syntax_error();  
    int product = token_value;  
    next_token();  
    while (token == times) {  
        next_token();  
        if (token != number) syntax_error();  
        product *= token_value;  
        next_token();  
    }  
    return product;  
}
```

$sum \rightarrow product\ sums$
 $sums \rightarrow plus\ product\ sums$
 $\quad \quad \quad | \quad \epsilon$

```
int sum() {  
    if (token != number) syntax_error();  
    int sum = product();  
    while (token == plus) {  
        next_token();  
        sum += product();  
    }  
    return sum;  
}
```

```
$ cc -Wall -o calc calc.c
```

```
$ ./calc "2+2*2"  
6
```

```
$ ./calc "32*3"  
96
```

```
$ ./calc "32 * 3"  
96
```

```
$ ./calc "2*2 + 10*10"  
104
```