

BIOMETRICS

Fall 2013

Assignment 1

Due: October 1, 2013 at 5pm in the TA room mailbox

Problem 1 (10 points):

(a) Find the optimal decision regions for a minimum error rate classifier where:

- $\Omega = \{\omega_1, \omega_2\}$
- $p(x | \omega_1) \sim N(2, 0.5)$ (The normal distribution is written as $N(\mu, \sigma^2)$.)
- $p(x | \omega_2) \sim N(1.5, 0.2)$
- ω_1 and ω_2 are equally probable

(b) Now find the optimal decision regions with the following priors:

- $P(\omega_1) = 2/3$
- $P(\omega_2) = 1/3$

Note: Please give the exact solution as well as a numeric approximation

(e.g. $x = \sqrt{4 - \pi} \approx 0.9265$).

Problem 2 (10 points):

Consider a two-class classification problem, where the feature is two-dimensional.

- $p(\mathbf{x} | \omega_1)$ is normal with mean $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and covariance matrix $\begin{bmatrix} 0.7 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$
- $p(\mathbf{x} | \omega_2)$ is normal with mean $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ and covariance $\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.85 \end{bmatrix}$
- The priors are $P(\omega_1) = 3/5$, $P(\omega_2) = 2/5$

Give the most likely class (ω_1 or ω_2) for these 5 samples. Justify your answer.

- $\mathbf{x}_1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 3.7 \\ 3.2 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 3.1 \\ 3.5 \end{bmatrix}$, $\mathbf{x}_4 = \begin{bmatrix} 2 \\ 3.8 \end{bmatrix}$, $\mathbf{x}_5 = \begin{bmatrix} 3.5 \\ 3.2 \end{bmatrix}$

Problem 3 (10 points):

Write down and solve your own probability problem in which Bayes' Rule is needed for the solution.

Problem 4 (required only for 6000 level) (10 points):

Find a scholarly paper in a field outside biometrics that uses Bayes' Rule. Give the citation and write a two- or three- paragraph summary, including an explanation of the author's use of Bayes' Rule.