Instructions

- Problems 1-5 are each worth 10 points.
- Submit your solutions in pdf format. Late submissions will not be accepted.
- You can discuss with TAs or other students but you must acknowledge them at the beginning of each problem and your solutions must be written in your own words.

Problems

1. Consider the following Turing machine:

\[ M = \left( \{q_0, q_1, q_2, q_{\text{acc}}, q_{\text{rej}}\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, q_{\text{acc}}, q_{\text{rej}} \right) \]

Describe the language \( L(M) \) if \( \delta \) consists of the following sets of rules (if an entry \( \delta(q, a) \) is missing, by default it means that the Turing machine enters \( q_{\text{rej}} \)). No justification is needed.

a) \( \delta(q_0, 0) = (q_1, 1, R); \delta(q_1, 1) = (q_0, 0, R); \delta(q_1, B) = (q_{\text{acc}}, B, R). \)
b) \( \delta(q_0, 0) = (q_1, 1, R); \delta(q_1, 1) = (q_2, 0, L); \delta(q_2, 1) = (q_0, 1, R); \delta(q_1, B) = (q_{\text{acc}}, B, R). \)

2. Show that the class of Turing-recognizable languages is closed under concatenation.

3. Read pages 180–181 of the Sipser book on Enumerators. Show that a language \( L \) is Turing-decidable if and only if there is a Turing machine \( M \) that enumerates \( L \) and such that the strings in \( L \) are output by \( M \) in length-increasing fashion. (Hint: The if direction is trivial when \( L \) is finite so focus on the case when \( L \) has infinitely many strings.)

4. (Wait for class on Nov 10) Prove that the following language is not recognizable:

\[ L = \left\{ (M) : M \text{ is a TM that does not halt on the empty string } \epsilon \right\}. \]

5. (Wait for class on Nov 10) Let \( L \) be the language consisting of pairs \( \langle M, k \rangle \), where \( M \) is a Turing machine and \( k \) is an integer, such that \( |L(M)| \geq k \).

a) Prove that \( L \) is recognizable.
b) Prove that \( L \) is not decidable.

Hint: In the a) part, using a nondeterministic TM may simplify the proof a little bit.