Discrete Math W3203, Spring 2015
Homework 1 Problem Set

1.1, 12f) Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination

r: You pass the course

Express each of these propositions as an English sentence.

f) $(p \wedge q) \vee (\neg q \wedge r)$

1.1, 14f) Let p, q, and r be the propositions

p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in this class.

Write these propositions using p, q , and r and logical connectives (including negations).

f) You get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

1.1, 28c) State the converse, contrapositive and inverse of each of these conditional statements.

c) When I stay up late, it is necessary that I sleep until noon.

1.1, 32d) Construct a truth table for each of these compound propositions.

d) $(p \wedge q) \rightarrow (p \vee q)$

1.1, 36) Construct a truth table for each of these compound propositions.

e) $(p \vee q) \wedge \neg r$

f) $(p \wedge q) \vee \neg r$

1.3, 4b) Use truth tables to verify the associative laws

b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

1.3, 24) Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

1.4, 16d) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

d) $\forall x\, (x^2 \neq x)$

1.4, 50) Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x\, (P(x) \vee Q(x))$ are not logically equivalent.

1.5, 10f) Let F(x, y) be the statement "x can fool y" where the domain consists of all people in the world. Use quantifiers to express each of these statements.

f) No one can fool both Fred and Jerry.

1.5, 38c) Express the negations of these propositions using quantifiers, and in English.

c) There is a student in this class who has taken every mathematics course offered at this school.

1.6, 10c) For each of these sets of premises, what relevant conclusion or conclusions can be draw? Explain the rules of inference used to obtain each conclusion from the premises.

c) "All insects have six legs." … "Dragonflies are insects." … "Spiders do not have six legs." … "Spiders eat dragonflies."

1.6, 34) The logic problem has these two assumptions:

1. "Logic is difficult or not many students like logic."

2. "If mathematics is easy, then logic is not difficult."

By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

a) That mathematics is not easy, if many students like logic.

c) That mathematics is not easy, or logic is difficult.

1.7, 42) Prove that these four statements about the integer n are equivalent:

(i) n^2 is odd

(ii) $1-n$ is even

(iii) n^3 is odd

(iv) n^2+1 is even

1.8, 26) Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert a 1 to produce nine new bits. Then you erase the nine original bits. Show that when you iterate this procedure, you can never get nine zeros. [Hint: work backward, assuming that you did end up with non zeros].

1.8, 30) Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.
