Single Variable Calculus Review
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Limit Definitions:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}; \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Differentiation Rules

- Constants: $\frac{d}{dx} c = 0$
- Constant factor: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$
- Sum: $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$
- Power Rule: $\frac{d}{dx} x^n = nx^{n-1}$

Trigonometric differentiation

- $\frac{d}{dx} \sin x = \cos x; \quad \frac{d}{dx} \cos x = -\sin x$
- $\frac{d}{dx} \tan x = \sec^2 x; \quad \frac{d}{dx} \cot x = -\csc^2 x$
- $\frac{d}{dx} \sec x = \sec x \tan x; \quad \frac{d}{dx} \csc x = -\csc x \cot x$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1; \quad \lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

Integration Rules

- Constant factor: $\int [c \cdot f(x)]\, dx = c \int f(x)\, dx$
- Sum: $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$
- Power Rule: $\int x^n\, dx = \frac{1}{n+1} x^{n+1} + c$, except when $n = -1$

- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Integration Tricks

- U-substitution: convert the entire integral to be in terms of a new variable
- Integration by parts: $\int u\,dv = uv - \int v\,du$
- Trig substitution:
  - For $\sqrt{a^2 - x^2}$: Let $x = a\sin\theta \Rightarrow \sqrt{a^2 - x^2} = a\cos\theta$
  - For $\sqrt{x^2 + a^2}$: Let $x = a\tan\theta \Rightarrow \sqrt{x^2 + a^2} = a\sec\theta$
  - For $\sqrt{x^2 - a^2}$: Let $x = a\sec\theta \Rightarrow \sqrt{x^2 - a^2} = a\tan\theta$

Parametric Functions

- Derivatives of parametric functions: $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \dfrac{d^2y}{dx^2} = \dfrac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}}$
- Arc lenth: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Some practice problems

Solve $\lim_{x \to 0} \frac{x^3 - 7x}{x^3}$

Solve $\lim_{x \to 0} \frac{\sin(5x)}{3x}$

Differentiate $y = (7x^2 + 2x + 1)^2$

Differentiate $y = \csc x \cot x$

Integrate $\int_3^5 3x^2 dx$

Integrate $\int \frac{\sin(\ln x)}{x} dx$

Answers

- $-\inf$
- $\frac{5}{3}$
- $2(14x + 2)(7x^2 + 2x + 1)$
- $-\csc x(\csc^2 x + \cot^2 x)$
- 98
- $-\cos(\ln x) + C$ (Hint: use U substitution)