2.3. Build up compatible sets starting from the rightmost column of the pair chart.

	1	2	3	4	5	6
2						
3	X					
4						
5	X	X	X			
6			X	X		
7		X			X	

Columns 6, 5, and 4 result in 67, 56, 45 47. In third column, since 3~4 and 3~7, we get 347, so the set is now 67, 56, 45, 347. Next we have 2~/3, 4, 6, so intersecting 346 with 347 yields 34 and so the set now becomes 67, 56, 45, 347, 234, 26.
Finally, for the first column 1~/2, 4, 6, 7 results in the final set, which are the maximum compatibles (MC's)
{167, 56, 45, 347, 234, 126, 147, 124}

2.15. We can say that two classes are "compatible", in that their exams can be simultaneous, if their membership does not overlap, i.e., no student is in both classes. This definition of compatibility has the same property of our definition for flow table rows in that if A~B, A~C, AND B~C, then, since no student can be in more than one of A, B, and C, exams can be held simultaneously for all 3. So we can generate MC's as in the case of flow table rows, and then, since there are no constraints corresponding to implications among the compatibles, we can choose the minimal set of MC's that covers all classes and assign an exam date for each set. Note that we might, after this is done, find that some of the chosen MC's include common members. These should be eliminated, since we don't want to have MORE than one exam for any class.

The pair chart is shown below (instead of X's for the INcompatible pairs, checks are shown for the COMPATIBLE pairs).

	A	B	C	D	E	F	G	H	J
B									
C									
D	√								
E		√		√					
F	√			√					
G	√								
H			√				√		
J			√						
K		√			√				

The MC's are: GH, BEK, DE, ADF, CH, CJ, AG
A minimal covering set is easily found by inspection to be: BEK, ADF, GH, CJ. In more complicated cases, the technique used as part of the Quine-McCluskey method can be used to find the minimal covering from the MC's.

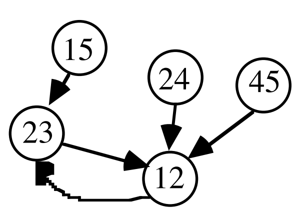
There are no overlaps, so we can assign one of these to each of the 4 exam time slots. (I would re-order these to ADF, BEK, CJ, GH.)

2.5. The pair chart is:

	1	2	3	4
2	23 45			
3	XX	12		
4	XX	12	XX	
5	23 34 XX	35 XX	XX	12

The MC's are 45, 23, 24, 12.

The pairwise implication graph is:



A minimal closed covering is clearly {12, 23, 45} which leads to the table below:

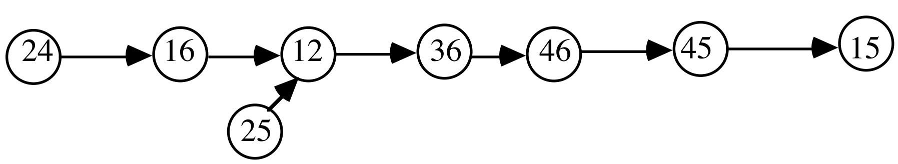
		X = 0	X = 1
(12)	1	2,00	3,--
(23	2	1,01	3,10
(45)	3	1,11	2,11

2.6. The final pair chart is:

	1	2	3	4	5
2	36				
3	XX	14 XX			
4	13 XX	16	XX		
5		12	XX	15	
6	12	XX	46	45	14 25 26 XX

The MC's are {45, 46, 36, 125, 16}

The implication graph is:



T o find a minimal closed covering, we work backward from the right end of the implication graph, since clearly any compatible we choose will imply 15, 45, etc. WE thus choose the pairs, 15, 45, 46, 36, and then 12, which would give us a 5-row solution, but, looking at the MC's we can see that 15 and 12 can be covered by 125, with all implications included. So now we have {125, 36, 45, 46} a minimal closed covering with 4 members.

The corresponding reduced flow table is:

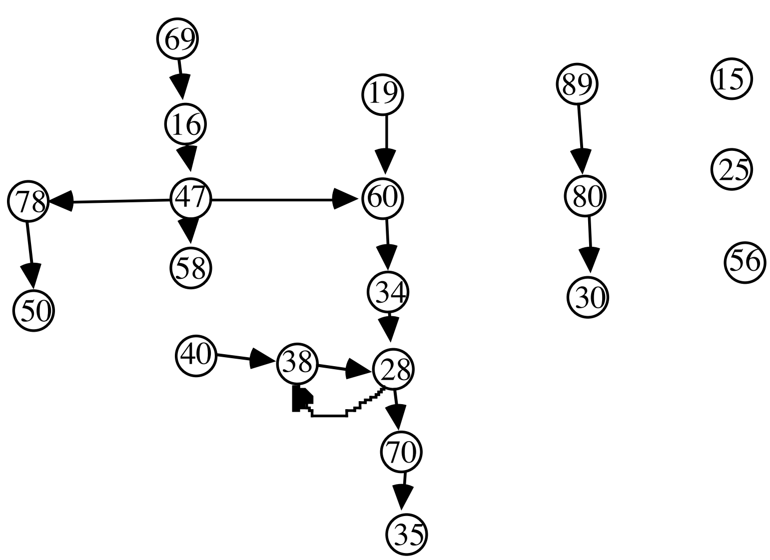
		X1X2 00	01	11	10
(125)	1	2,0	1,0	1,1	1,1
(36)	2	-,1	1,1	4,0	3/4,1
(45)	3	1,0	1/3.-	1,1	1,1
(46)	4	1,0	1,1	2/6,1	3,1

2.7. The final pair chart follows: (10 is abbreviated by 0)

	1	2	3	4	5	6	7	8	9
2	36 XX								
3	26 XX	27 XX							
4	68 78 XX	20 78 38XX	28 70						
5				49 XX					
6	47	XX	67 XX	48 60 XX					
7	57 67 XX	26 57 37XX	27 67 XX	78 58 60	49 XX	XX			
8	68 70 XX	38 70	28	XX		XX	50		
9	60	12 30 XX	17 20 XX	10 80 XX	49 XX	16	XX	80	
10	37 XX	37 XX		38		34	35	30	XX

The MC's are: {89, 780, 169, 560, 3580, 470, 340, 258, 156}

The implication graph is:



Finding a minimum closed set of compatibles for this table is not easy. A key decision, which can really only be made by trial and error, is whether to include the subgraph that is implied by the choice of 28, this would mean including the pairs, 38, 28, 70 and 35. It makes possible additional choices such as 40, 19, 60, and 34. Note that if the set is chosen, we can use the larger compatible, 358, so the starting set is {28, 358, 70}. It is closed. Now we need to cover the additional rows 1, 4, 6, 9. If we simply add them as degenerate, 1-member compatibles, then we have a 7-member set. We could combine 1 and 9 (i.e., use 19) if we add 60 and 34 to our set (which also cover 4 and 6). This gives us a 6-member closed covering {28, 358, 70, 19, 60, 34}.

Can we do better? It is tempting to back up one step and use the larger compatible 169. But, since 69 is at the top of a chain of implications, we would then have to add 47, 78, and 50 to the pairs covered in the previous solution. We would have the set { 28, 358, 70, 169, 60, 34, 47, 78, 50}. Inspection of the MC's suggest merging 47 and 70--both covered by 470, and the necessary implications are OK. So we can reduce the set to { 28, 358, 470, 169, 60, 34, 78, 50}. But no other reductions in the set size are possible, so this is a dead end with an 8-member set.

Establishing that our 6-member covering is the best that can be done involves some further arguments that are somewhat tedious. The general idea is to focus on how best to cover row-2, which is included in only one MC, 258. It is easy to see that there is no argument for using 2 rather than 25, or 28 rather than 258. So we can say that a minimal solution will entail using 258, or 25. We have already seen the best that can be done with 258 (we could expand the 28 in our 6-member solution to 258, but that doesn't help). NOT using 28 (or 258) greatly restricts the choices that can be made. (Look at the implication graph. 69, 16, 47, 19, 10, 38, and 34 all imply 28.) A careful examination shows we can't do better than 7-member set under these conditions.
So the minimal closed set of compatibles is indeed {19, 28, 34, 358, 60, 70}. The resulting reduced table is shown below.

		X1X2			
		00	01	11	10
(19)	1	5,1	1,-	3,-	6,-
(258)	2	4,	6,1	1,1	5,-
(34)	3	2,-	6,-	3,0	4,-
(358)	4	2,-	6,1	1,1	6,-
(60)	5	-,0	5,0	-,-	3,1
(70)	6	6,0	5,1	3,-	4,-