IN SEARCH OF THE UNKNOWABLE

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Some years ago I mentioned to a very senior European scientist that I was interested in understanding the intrinsic limits to scientific knowledge. He replied that such limits had been established by Kurt Goedel. That interpretation of Goedel's fundamental contribution to logic seems to be a commonly held belief, but it is simply not so. I want to tell you why Goedel's theory does not limit scientific knowledge and then discuss how we might establish intrinsic limits.

Many of you are familiar with the place of Goedel's theorem in twentieth century thought. For those of you who are not, here's a quick summary.

The great mathematician David Hilbert posed the following decision problem: Devise a mechanical procedure that could determine whether or not any given mathematical statement was true. Hilbert believed that such procedures existed and for decades mathematicians tried to solve Hilbert's decision problem. Then in 1931 Goedel shocked the mathematical world by showing that if a system as rich as arithmetic is consistent it cannot be complete. That is, there are statements within arithmetic that cannot be proved true or false within the system; arithmetic is undecidable. Starting with Goedel's seminal paper there followed a stream of impossibility results on undecidability, non-computability, and intractability.

Now what does this have to do with science? Goedel's theorem and the other impossibility results are about the formal manipulation of symbols; that is, about mathematics. Science certainly uses mathematics, but science is also very different from mathematics. (Of course, you'd hardly know that if you looked at some of the papers in Physical Review.)

Science is about understanding the universe and everything in it. A scientific question does not come equipped with a mathematical model. Here are some examples of scientific questions:

- How do children acquire language?
- Will human activities cause major global changes, and what will be the effects of those changes on ocean levels, on agriculture, and on biodiversity?
- How do physical processes in the brain give rise to subjective experience - that is, how do such processes explain consciousness?
- Will the universe expand forever, or will it collapse?
- Can the healthy , active lives of people be prolonged by, say, a factor of two or three? What would be the effects on society?

There are limits in various disciplines. We heard about some of them yesterday. My question is: Can we prove discipline-independent limits? For example, can we apply suitably modified ideas from theoretical computer science to obtaining limits to scientific knowledge? In particular, can we use suitably modified ideas from my own field of computational complexity?

The computational complexity of a mathematically posed problem is the minimal computational resource required to solve the problem. Often the resource that's studied is time. Then the computational complexity is the minimal time required to solve a problem. Establishing the computational complexity can be very difficult. Indeed, for many problems we have no idea of their computational complexity. For other problems the computational complexity has been proven to be exponential in the size of the input. Such problems are said to be computationally intractable. Solving a computationally intractable problem on even the fastest computers could take longer than the time since the big bang. (I should mention parenthetically that there is much interest in quantum computation because it might enable us to solve problems which are believed to be computationally intractable on a classical computer.)

Note that computational complexity is a property of the problem. That's because the computational complexity is the minimal cost of any possible algorithm for solving the problem. We consider all possible algorithms and take the minimal cost. That'll be important when I talk about limits to knowledge.

Now I want to up the ante.

Can we use this mode of thought, which is for mathematically posed problems, to prove a limit to scientific knowledge? Here's one gedankenattack (that's a neologism using two languages like eigenvalue).

Imagine considering all the mathematical models that capture the essence of a certain scientific question. That's analogous to considering all possible algorithms for a fixed mathematical problem when we study complexity. But I've gone up a level. I fix the scientific question and consider all mathematical models. For each model I consider all algorithms. Perhaps we can prove impossibility theorems for each of the mathematical models. Perhaps we could prove that every one of these models is undecidable, non-computable, or intractable. That would do it. - - It would show that the scientific question is unanswerable.

Here's another idea: In establishing the computational complexity of a mathematical problem the key part is to prove a lower bound theorem which states that no algorithm, whether currently known or not, can solve the problem with lower computational cost. To prove a lower bound theorem we do not, of course, list all possible algorithms. The attack is indirect -- we use certain properties of the mathematical problem to prove the lower bound. Can we use this mode of thought to show that any mathematical model that captures the essence of a scientific question must fail.

To summarize:

1. Do impossibility theorems on formal models imply limits to knowledge?

   NO

2. Will the two gedankenattacks I've outlined succeed in establishing limits to scientific questions?

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Where does that leave us? I certainly believe there are limits to knowledge. Perhaps general attacks such as I've outlined above will succeed for at least some questions. For other questions, broad principles such as data limitations will indicate the answer to the scientific question is unknowable. The limits to knowledge may be discipline dependent, being very different in, say, physics than in prehistoric linguistics.

As you all know, often the most fun in research is when we have a tree of ideas but don't yet know the answer or how to get to the answer. That's where we stand today as we search for the unknowable.