UCSBTH-95-9, NSF-ITP-96-06

Scientific Knowledge from the Perspective of Quantum Cosmology*

James B. Hartle†

Department of Physics and Institute for Theoretical Physics,

University of California, Santa Barbara, CA 93106-9530 USA

(January 29, 1996)

Abstract

Existing physical theories do not predict every feature of our experience but only certain regularities of that experience. That difference between what could be observed and what can be predicted is one kind of limit on scientific knowledge. Such limits are inevitable if the world is complex and the laws governing the regularities of that world are simple. Another kind of limit on scientific knowledge arises because even simple theories may require intractable or impossible computations to yield specific predictions. A third kind of limit concerns our ability to know theories through the process of induction and test. Quantum cosmology — that part of science concerned with the quantum origin of the universe and its subsequent evolution — displays all three kinds of limits. This paper briefly describes quantum cosmology and discusses these limits. The place of the other sciences in this most comprehensive of physical frameworks is described.

Typeset using REVTEX

*Contribution to the proceedings of the workshop *Limits to Scientific Knowledge* held at Abisko, Sweden, May 15-19, 1995

†e-mail: hartle@cosmic.physics.ucsb.edu

I. INTRODUCTION

The assignment of the organizers was to speak on the subject of "limits to scientific knowledge". This is not a topic on which I have been forced to reflect a great deal in the course of my efforts in astrophysics, but I shall try to offer a few thoughts on it from the perspective of cosmology. Like any assignment, the first task is to understand what it might mean. I shall say more about this later, but one thing is immediately clear: This is not simply an empirical question, but rather concerns the relationship between what we observe and our theories of what we observe. Limits therefore depend on theories and will vary from one scientific theory to another. The question of what are the *fundamental* limits to scientific knowledge must be examined in the most general theoretical context. In physics this is the subject of quantum cosmology — the quantum mechanics of the universe as a whole and everything inside it. The nature of scientific knowledge in this most comprehensive of theories is the subject of this essay. I shall try to describe a little of what quantum cosmology is about, and address the question of limitations to scientific knowledge in this most general of contexts.

II. THREE KINDS OF LIMITS TO SCIENTIFIC KNOWLEDGE

In this Section three different kinds of limits to scientific knowledge are identified. No claim is made that these are the only kinds of limits, but these three have a general character that is inherent in the nature of the scientific enterprise. Subsequent sections will illustrate these general kinds of limits with examples from quantum cosmology.

A. Limits to What is Predicted

The task of science, as Bohr said, is "to extend the range of our experience and to reduce it to order" [1]. To reduce experience to order is to compress the length of a description of that experience. That compression is achieved when a computer program can be exhibited which, given certain input, outputs a string describing some parts of our experience and the length of that program together with its input are shorter than the length of the output description. Theory supplies the program. For instance, a detailed description of the observations of the positions of the planets over the last 100 years might make up a very long table, but Newton's equations of motion can be used to compress all that information into two much shorter strings: a string stating Newton's theory and another string giving the positions and velocities of the planets at one time.

It is a logical possibility that *every* feature of our experience — the wave function of every quark, the velocity of every molecule, the position of every leaf, the character of each biological species, the action of every human, etc. — is just a very long output of a short computer program with *no* input. However, in the history of scientific inquiry there is no evidence that the universe is so regular. Even the most deterministic classical theories did not claim this. With Newtonian mechanics, Laplace proposed only to predict the future and retrodict the past *given* the present position and velocity of each particle in the universe. That list of initial data would be vastly longer than a few treatises on Newtonian mechanics. Existing theories predict a string describing our experience only given some other, shorter, string as input. Theories do not predict everything that is observed, but only certain regularities in what is observed. Some things are predicted, some are not, and that limit to what is predicted is one kind of limit to scientific knowledge.

Scientific laws must have some degree of simplicity to be discoverable, comprehensible, and effectively applicable by human beings and other complex adaptive systems. If the complexity of the present universe is large, then this necessary simplicity of the laws implies that this kind of limit to scientific knowledge is inevitable. Not everything can be predicted but only those regularities that are summarized in the laws of science. In the following we shall describe what is predicted and what is not predicted in quantum cosmology.[1]

B. Limits to Implementation

To be tested, the predictions of an abstractly represented theory covering a broad class of phenomena must be implemented in particular circumstances. The theory must produce numbers, and that process involves computation. Even if the laws are precisely specified, even if the input to those laws is exactly stated, limitations of our ability to compute may limit our ability to predict. This is another kind of limit to scientific knowledge. The practical limitations of present computing machines are all too familiar. Computing the motion of every particle in a classical gas 10^{22} particles in less than its real evolution time is well beyond the powers of contemporary computers. However, beyond the limitations of contemporary machinery, we may ask whether there are fundamental limitations on what can be computed that are inherent in the form of the laws themselves. The phenomenon of chaos is the source of one kind of limitation. The precision required of initial data to extrapolate a given time into the future increases exponentially with that time for a wide variety of classical systems. Another kind of limit arises in cosmology where resources for computation, both in time and space, are limited. Further, as we shall see, there is some evidence that certain predictions of quantum cosmology may be non-computable numbers.

It is not difficult to display predictions which are computationally intractable but which are measurably inaccessible. Given initial conditions, classical theory predicts the orbits of every molecule of gas in a room. The explicit computation of this prediction at the operating speeds of present computers would take much longer than the age of the universe because of the large number of particles involved. Yet, for the same reason, neither the initial condition nor the predicted orbits are measurably accessible quantities. Merely exhibiting phenomena which are impossible or intractable to compute is not much of a limit if the phenomena are impossible or extraordinarily difficult to measure. The most interesting limits concern phenomena that are easy to measure but difficult to compute.

---

[1]For a lucid discussion in popular language of the notion of complexity and of prediction in quantum cosmology, as well as a summary of some of the author's work with M. Gell-Mann, see [2].

C. Limits to Verification

The above discussion has assumed that we know the laws of physics. However, we arrive at those laws by a process of induction and test. Competing laws consistent with known regularities are winnowed by the process of checking their predictions with new observations. Are there fundamental limits to what we can test, and therefore fundamental limits to how well the theory can be known? Cosmology will provide examples.

D. False Limits

Beware of false limits that arise only from imprecise language or the comparison of a correct theory with an incorrect one. A classic example is provided by the uncertainty principle in quantum mechanics

$$\Delta x \, \Delta p \geq \hbar/2 \ . \tag{2.1}$$

That relation is sometimes described as a limit on our ability to predict (or "measure" or "know") both the position and momentum of a particle at one time to accuracies better than those restricted by (2.1). However, the uncertainty principle is more accurately characterized as a limit on the use of classical language in a quantum mechanical situation.

There is no state of a quantum mechanical particle with a precisely defined position and momentum. That is the content of (2.1). The uncertainty principle, therefore, is not a limit to what observed properties of quantum particle are predicted by the theory. Since there is no quantum state with precisely defined position and momentum, quantum theory predicts that we shall never observe both simultaneously. Thus, as far as the position and momentum of a particle are concerned, there is no disparity within quantum theory between what can be predicted and what is observed arising from (2.1) as there would be in the case of a genuine limit of the type discussed in Section A.

As mentioned earlier, limits to prediction are properties of the theories which specify what can be predicted. Of course, if we *compare* two theories one may predict different phenomena from the other. In classical physics there are states in which the position and momentum of a particle *are* simultaneously specified. In quantum theory there are not. But quantum theory is correct and classical theory incorrect for the domain of phenomena we have in mind. The uncertainty principle (2.1) may be viewed as a kind of limit to how far classical concepts and language can be applied in quantum theory, but, were we to strictly adhere to the language and concepts of quantum theory, it would be no limit at all.

III. DYNAMICAL LAWS AND INITIAL CONDITIONS

As we mentioned above, fundamental limits to what is predicted, to how predictions can be implemented, and to how theory can be verified depend on what the basic theory is. This Section sketches some essential features of basic physical theory today. Of course, we are on dangerous ground here. The most basic laws are often the furthest from definitive experimental test. Nevertheless, it is interesting to see what kinds of limits might exist in the kind of basic theoretical framework that is under active investigation by physicists today.

The most general framework for prediction is quantum cosmology — the quantum theory of the universe as a whole and everything that goes on inside it. In the following I shall describe a little of this theory.

Historically, physics for the most part has been concerned with finding dynamical laws — laws which compress the description of evolution over time to the description of an initial condition. Thus, these dynamical laws require boundary conditions to yield predictions. There are no particular laws governing these boundary conditions. They are specified by our observations of the part of the universe outside the subsystem whose dynamics is of interest. In a room, if we observe no incoming radiation, we solve Maxwell's equations there with no-incoming-radiation boundary conditions. If we prepare an atom in a certain atomic state we solve Schrödinger's equation with that initial condition, etc.

But in cosmology we are confronted with a fundamentally different kind of problem. Whether classical or quantum, the dynamical laws governing the evolution of the universe require boundary conditions. But in cosmology there is no "rest of the universe" to pass their specification off to. The boundary conditions must be part of the laws of physics themselves. There is no other place to turn.

A present view, therefore, is that the most general laws of physics involve two elements:

- The laws of dynamics prescribing the evolution of matter and fields and consisting of a unified theory of the strong, electromagnetic, weak, and gravitational forces.

- A law specifying the initial boundary condition of the universe.

There are no predictions of any kind which do not depend on these two laws, even if only very weakly, or even when expressed through phenomenological approximations to these laws (like classical physics) appropriate in particular and limited circumstances with forms that may be only distantly related to those of the basic theory.

The search for a fundamental theory of the dynamics of matter has been seriously under way since the time of Newton. Classical mechanics, Newtonian gravity, electrodynamics, special relativity, general relativity, quantum mechanics, quantum electrodynamics, the theory of the electroweak interactions, quantum chromodynamics, grand unified theories, and superstring theory are but some of the important milestones in this search. The search for a theory of the initial condition of the universe has been seriously under way for not much more than a decade. (See Ref. [3] for a review.) The reason for this difference can be traced to the scales on which the regularities summarized by these two laws emerge. The trajectory of a ball in the air, the flow of water in a pipe, or the motion of a planet in the solar system all exhibit the regularities implied by Newtonian mechanics. The regularities of the dynamical laws of atomic and particle physics can be exhibited in experiments carried out in laboratories or large accelerators. However, characteristic regularities implied by a theory of the initial condition of the universe emerge mostly on much larger, cosmological scales.

On any scale the universe exhibits some regularities in space as distinct from regularities in time. Rocks on one part of the earth are related to rocks on another part. Similarly, there are relations between individual members of biological species, and human history in different locations. These regularities have their origins in the common origin of rocks in the earth, the evolution of biological species, and the facts of human history. On cosmological scales the universe is more regular in space than it is on smaller scales. The progress of

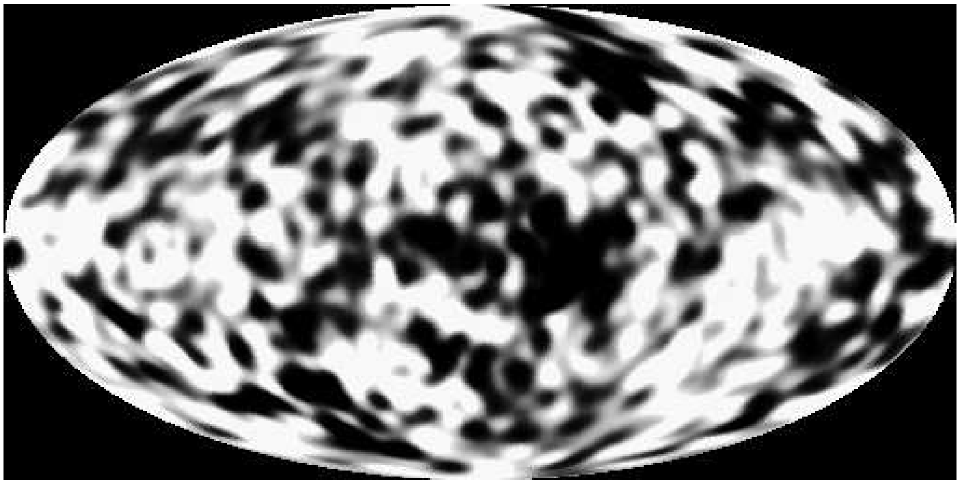

FIG. 1. *A sky map of temperature fluctuations in the cosmic background radiation. This figure may be thought of as a picture of the universe approximately 300,000 years after the big bang. The hot mixture of matter and radiation that exists immediately after the big bang cools as the universe expands. About 300,000 years later the universe has cooled enough that matter and radiation no longer significantly interact. Photons from that time have been traveling freely towards us ever since. Their characteristic temperature now is only 2.7 degrees above absolute zero, yet they can be detected at microwave wavelengths by sensitive instruments. The figure above shows a sky map of the temperature of that radiation based on data taken with the COBE satellite. The dark spots are where the sky is cooler than the mean temperature and the white areas are where it is hotter. The differences in temperature between the darkest black and the whitest while it is only a few hundred* micro *degrees Kelvin. The universe is thus essentially featureless at 300,000 years after the big bang except for these tiny fluctuations. These small fluctuations, however, are the origin of all the complexity in the universe that we see today. [Greyscale adaptation by J. Gundersen of the results of C. Bennett, et. al. Ap. J.,* **436**, *423 (1994)]*

observation in astronomy in recent decades has given us an increasingly detailed picture of the universe on ever larger scales of space and time. The remarkable inference from these observations is that the universe becomes increasingly simple as we move to larger scales in space and more distant times in the past. Galaxies are not very complicated objects but still exhibit a variety of types and considerable individuality. On the larger scale of a tenth of the radius of the universe the galaxies are no longer individual objects, but there is considerable structure in their distribution. Pictures of the distribution of galaxies on the sky, which probe out to greater distances show less structure. On the largest scales, the distribution of the cosmic background radiation temperature, which is as close as we can come to a picture of the universe three hundred thousand years after the big bang, reveals almost no structure at all. (See Figure 1.) The deviations in this temperature from exact smoothness (exact isotropy) are measured in tens of *millionths* of a degree. However, those deviations are important! They are the origin of all the complexity in the universe we see today. As the universe evolves, these fluctuations grow, collapse, and fragment through gravitational attraction to become the galaxies, stars, and planets which characterize the universe today. Initially very close to equilibrium, the matter in the universe is thereby driven further from equilibrium. That disequilibrium is necessary for chemistry, geology, life, biology, and human history.

The evidence of the observations then is that the universe was a simpler place earlier than it is now — more homogeneous, more isotropic, with matter more nearly in thermal equilibrium. The aim of quantum cosmology is a quantum theory of this simple initial condition.

IV. CLASSICAL AND QUANTUM INITIAL CONDITIONS

It is an inescapable inference from the physics of the last sixty years that we live in a quantum mechanical universe — a world in which the basic laws of physics conform to that general framework for prediction we call quantum mechanics. We perhaps have little evidence for peculiarly quantum mechanical phenomena on large and even familiar scales, but there is no evidence that the phenomena that we do see cannot be described in quantum mechanical terms and explained by quantum mechanical laws. This is the first reason that the search for a theory of the initial condition is carried out in the framework of *quantum* cosmology. There is, however, another reason: quantum indeterminacy is probably necessary for a comprehensible basic, scientific theory of the initial condition.

To explain this necessity and also to understand a bit of the machinery of quantum cosmology, consider a model universe. Suppose the universe consists of a box the size of the visible universe containing a large number N of particles interacting by fixed potentials. To simulate the expansion of the universe we could let the box expand. That's actually not a bad model for what goes on in more recent epochs of the universe.

Classically a history of this model universe is a curve in a $6N$ dimensional phase space of the positions and momenta of all the particles in the box. Classical evolution is deterministic — if the point in phase space specifying the system's configuration is known at one time, the location at all other times is determined by the equations of motion. A classical theory of the initial condition of the model universe thus might specify the initial point in phase space at $t = 0$. However, such a theory would necessarily be hopelessly complex because it would have to encode all the complexity we see today. Its description would be too long to be comprehensible.

A *statistical* classical initial condition could be simpler. Such an initial condition would only give a *probability* for the initial point in phase space and therefore only a probability for the subsequent evolution. Present predictions of the future would then be probabilistic. For example, observers at any time in the history of the universe can only see galaxies within a distance close enough that their light could have reached them in the time since the big bang. This cosmological horizon expands as the universe ages. One new galaxy comes over this cosmological horizon approximately every 10 minutes. A statistical initial condition might not predict with near certainty, say, the specific locations of the individual new galaxies, but rather their statistical distribution on the sky. Similarly, with a classical initial condition in which matter was initially in thermal equilibrium, one might predict the overall intensity of the background radiation on the sky, but not the location of any particular fluctuation in its intensity.

Probabilities in classical physics reflect ignorance. A classical statistical law of the initial condition would mean that we have some information about how the universe started out, but not all. However, we learn from observation. With every observation we could refine our theory of the initial condition which would therefore become increasingly complex, reflecting the complexity of the present, and thus become increasingly less comprehensible.

Quantum mechanics is inherently indeterministic and probabilities are basic. The most complete specification of the initial state of our model box of particles would be a wave function on the configuration space of all their positions

$$\Psi\left(\vec{x}_1, \cdots, \vec{x}_N\right) \tag{4.1}$$

— a wave function of the universe for this model. Unlike classical physics, subsequent observation will not improve this initial condition, although the results of observation can be used to improve future predictions. Thus, in quantum mechanics, it is natural to have a simple, comprehensible law of the initial condition which is consistent with the complexity observed today.[2]

V. WHAT IS PREDICTED IN QUANTUM COSMOLOGY?

My colleague, Murray Gell-Mann, once asked me, "If you know the wave function of the universe, why aren't you rich?" The answer is that very little is predicted with certainty by such a quantum initial condition of the universe and certainly not of much use in generating wealth. What might be predicted by an initial condition for cosmology is the subject of this Section.

Quantum mechanics predicts probabilities for sets of alternatives. In our model universe in a box, for example, it might predict the probabilities for alternative ranges of the position of a particle at a particular time, or the probabilities for alternative distributions of energy density in the box, and many other sets of alternatives. These are the probabilities for alternatives which are *single* events in a *single* closed system — the universe as a whole.

What do such probabilities of single events mean? Some may find it helpful to think of these probabilities as predictions of relative frequencies in an imaginary infinite ensemble of universes, but they are not frequencies in any accessible sense. Rather, to understand what the probabilities of single events mean it is best to understand how they are used. Probabilities of single events can be useful guides to behavior even when they are distributed over a set of alternatives so that none is very close to zero or one. Examples are the probability that it will rain today or the probability of a successful marriage. However, because the probabilities are distributed, the event which occurs — rain or no rain, divorce or death before parting — does not test the theory that produced the probabilities. Tests of the theory occur when the probabilities are *near certain*, by which I mean sufficiently close to zero or one that the theory would be falsified if an event with probability sufficiently close to zero occurred, or an event with a probability sufficiently close to one did not occur.[3] Various strategies can be used to identify sets of alternatives for which probabilities are near zero or

[2] For an early statement of this, see [4].

[3] *How* close to zero or one probabilities must be for near certain predictions depends on the circumstances in which they are used as I have discussed elsewhere [5].

one. The most familiar is to study the frequencies of outcomes of repeated observations in an ensemble of a large number of identical situations. Such frequencies would be predicted with certainty in an infinite ensemble. However, since there are no genuinely infinite ensembles in the world, we are necessarily concerned with the probability for the deviations of the frequency in a finite ensemble from the expected behavior of an infinite one. Those are probabilities for single properties (the deviations) of a single system (the whole ensemble) that become closer and closer to zero or one as the ensemble is made larger.

Another strategy to identify alternatives with probabilities near zero and one is to consider probabilities conditioned on other information besides that given in the theory of dynamics and the initial condition of the universe. Present theories of the initial condition do not predict the observed orbit of Mars about the sun with any significant probability. But they do predict that the *conditional* probability for the observed orbit is near one *given* a few previous observations of Mars' position. Such conditional probabilities are what are used in the rest of the sciences when they are viewed from the perspective of quantum cosmology as we shall discuss in more detail in the subsequent Sections.

In the following discussion it will be helpful to use just a little of the mathematics of quantum mechanics to discuss quantum cosmology.[4] For simplicity and definiteness let us continue to discuss the model universe of N particles in a box. The quantum initial state of this model universe is represented by a state vector $|\Psi\rangle$ in a Hilbert space, or equivalently by a wave function of the coördinates of all the particles in the box:

$$\Psi\left(\vec{x}_1, \cdots, \vec{x}_N\right) \ . \tag{5.1}$$

General alternatives at a moment of time whose probabilities we might want to consider can always be reduced to a set of "yes-no" alternatives. For instance, questions about the position of a particle can be reduced to questions of the form: "Is the particle in this region — yes or no?", "Is the particle in that region — yes or no?", etc. A set of "yes-no" alternatives at one moment of time, say $t = 0$, is represented by a set of orthogonal projection operators $\{P_\alpha\}$, $\alpha = 1, 2, \cdots$ — one projection operator for each alternative. (A projection operator is one whose square is equal to itself.) The projection operators satisfy

$$\textstyle\sum_\alpha P_\alpha = I \ , \qquad \text{and} \qquad P_\alpha P_\beta = 0 \ , \ \alpha \neq \beta \ , \tag{5.2}$$

showing mathematically that they represent an exhaustive set of exclusive alternatives. The same set of alternatives at a later time t is represented by a set of (Heisenberg picture) projection operators $\{P_\alpha(t)\}$. The time dependence of each $P_\alpha(t)$ is given by

$$P_\alpha(t) = e^{iHt} P_\alpha e^{-iHt} \tag{5.3}$$

where H is the Hamiltonian encapsulating the basic dynamical theory. The probability predicted for alternative α at time t is

$$p(\alpha) = \left\| P_\alpha(t) |\Psi\rangle \right\|^2 \ , \tag{5.4}$$

---

[4]For more details at an elementary level see [6] and in greater depth see [7].

where $\|\cdot\|$ means the length of the Hilbert space vector inside. For this model, the Hamiltonian H specifies the first of the two elements of a basic physical theory described in Section III — the fundamental theory of dynamics. The state vector $|\Psi\rangle$ or equivalently the wave function $\Psi(\vec{x}_1, \cdots \vec{x}_n)$ specifies the second element — the initial condition.

Probabilities for alternatives at a moment of time are not the most general predictions of quantum mechanics. More generally, one can ask for the probabilities of sequences of sets of alternatives at a series of different times $t_1 < t_2 < \cdots < t_n$ making up a set of alternative *histories* for the universe. Each history corresponds to a particular sequence of alternatives $(\alpha_1, \cdots, \alpha_n)$ and is represented by an operator that is the chain of projections corresponding to the sequence of alternatives

$$C_\alpha = P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1) \ . \tag{5.5}$$

Here, the index α is shorthand for the whole sequence $(\alpha_n, \cdots, \alpha_1)$ and the superscripts on the P's indicate that different sets of alternatives can be considered at different times. When the operator C_α is applied to the initial state vector $|\Psi\rangle$, one obtains the *branch state vector* $C_\alpha|\Psi\rangle$ corresponding to the history α. The probability of the history α is the length of the history's branch state vector:

$$p(\alpha) = \|C_\alpha|\Psi\rangle\|^2 \ . \tag{5.6}$$

Probabilities of histories are essential for predicting such everyday things as the orbit of the moon, which is a sequence of positions at a series of times.

We can now begin to analyze the question of what is predicted in quantum cosmology and what is not predicted. The most characteristically quantum mechanical limitation on what can be predicted is that not every set of alternative histories that may be described can be assigned probabilities by the theory because of quantum mechanical interference. That is very clearly exemplified in the two-slit thought experiment illustrated in Figure 2. Electrons proceed from an electron gun through a barrier with two slits on their way to detection at a screen. Passing through slit A or slit B defines two alternative histories for the electrons arriving at a fixed point y on the screen. In the usual story if we have not measured which slit an electron passed through, then it would be inconsistent to predict probabilities for these alternative histories. It would be inconsistent because the probability to arrive at y would not be the sum of the probability to pass through A to y and the probability to pass through B to y:

$$p(y) \neq p_A(y) + p_B(y) \ . \tag{5.7}$$

That is because in quantum mechanics probabilities are the squares of amplitudes and

$$|\psi_A(y) + \psi_B(y)|^2 \neq |\psi_A(y)|^2 + |\psi_B(y)|^2 \ . \tag{5.8}$$

It is not that we are ignorant of which slit an electron passes through, so that the probabilities are 50–50. It is inconsistent to discuss probabilities at all.

Quantum mechanics, in any of its various levels of formulation, therefore contains a rule specifying which sets of alternative histories may be assigned probabilities and which may

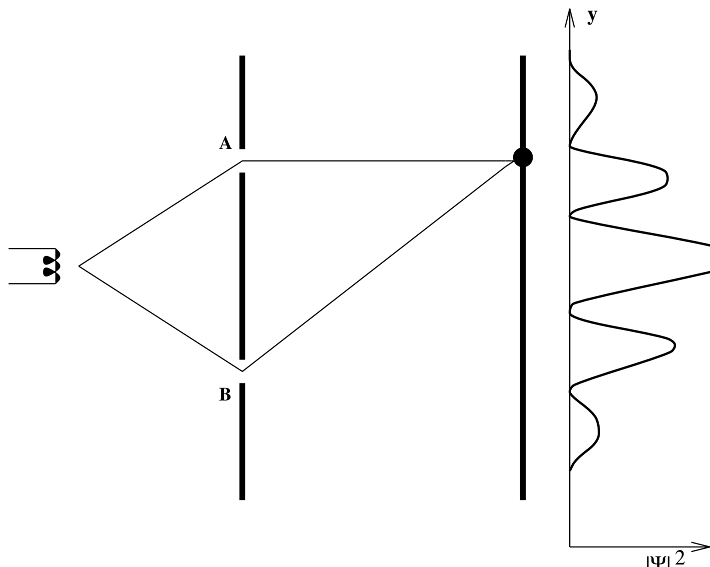

FIG. 2. *The two-slit experiment. An electron gun at left emits an electron traveling towards detection at a screen at right, its progress in space recapitulating its evolution in time. In between there is a barrier with two slits. Two possible histories of an electron arriving at a particular point on the screen are defined by whether it went through slit A or slit B. In quantum mechanics, probabilities cannot be consistently assigned to this set of two alternative histories because of quantum mechanical interference between them. However, if the electron interacts with apparatus that measures which of the slits it passed through, then interference is destroyed, the alternative histories decohere, and probabilities can be assigned to the alternative histories.*

not. In the most general context of the quantum mechanics of the universe that rule is as follows [8–10]: Probabilities may be consistently assigned to just those sets of histories for which there is vanishing interference between the individual members of the set as a consequence of the universe's initial state $|\Psi\rangle$. Such sets of histories are said to decohere. The condition for a decoherent set of histories is that the branches of the initial state $C_\alpha|\Psi\rangle$ corresponding to individual histories be mutually orthogonal:

$$\langle\Psi|C_\alpha^\dagger \cdot C_\beta|\Psi\rangle \approx 0 \ , \quad \alpha \neq \beta \ . \tag{5.9}$$

The most general probability sum rules are satisfied as a consequence. Consistency limits the predictions of quantum theory to the probabilities of *decoherent* sets of alternative histories.

As an example of how the decoherence of a set of histories comes about, think about a single millimeter-sized dust grain in a quantum state that is a superposition of two positions about a millimeter apart located deep in intergalactic space. Consider alternative histories of the position of this particle at a sequence of a few times. (The P's in (5.5) would then be projections onto ranges of this position.) Were the particle isolated, this situation would be analogous to the two-slit experiment, and histories of differing positions would not decohere. However, even deep in space this particle is not isolated. The all-pervasive light from the big bang illuminates the particle, and about 10^{11} cosmic background photons scatter from it every second. Through these interactions, this seemingly isolated dust grain becomes correlated with radiation in a part of the universe whose size is growing at the speed of light. The two states with different positions become correlated with two different, nearly orthogonal states of the radiation after a time of about a nanosecond. By this means, a branch of the initial state in which the grain is initially at one position becomes orthogonal to a branch in which the grain is a millimeter away. Decoherence of alternative histories of position has been achieved because the relative phase between states of different position has been dissipated by feeble interactions with the background radiation. Mechanisms such this are widespread in the universe and typical of those effecting the decoherence of histories of the kinds of classical variables we like to follow. (See, *e.g.* [11,12])

In the above example, decoherence of alternative histories of the position of the dust grain is achieved at the cost of ignoring the photons that are effecting the decoherence. That is an example of *coarse-graining*. Were we to consider a set of alternative histories of states of the cosmic background radiation as well as the position of the grain we would be, in effect, following all possible phase information. Such a set of alternative histories would generally not decohere. Except for trivial cases, sets of histories must describe *coarse-grained* alternatives in order for probabilities to be predicted at all. This necessary imprecision is a genuine limit to what can be predicted in quantum cosmology, in contrast to the limits of the kind associated with the uncertainty principle which are merely limits to the applicability of classical modes of description.[5]

We thus have the picture of a vast class of all possible sets of alternative histories and a smaller subclass of decoherent sets of histories for which quantum theory predicts probabilities. For almost none of these decoherent sets is there a history predicted with certainty on the basis of the initial state alone. If one history has probability one, then all alternatives to it must have probability zero. Suppose we have such a set, and let α_c be the label of the certain history, then from (5.6)

$$\|C_\alpha|\Psi\rangle\|^2 = 0 \ , \quad \alpha \neq \alpha_c \ , \tag{5.10}$$

which implies

$$C_\alpha|\Psi\rangle = 0 \ , \quad \alpha \neq \alpha_c \ . \tag{5.11a}$$

Then, since $\Sigma_\alpha C_\alpha = I$ as a consequence of (5.2), we also have

$$C_\alpha|\Psi\rangle = |\Psi\rangle \ , \quad \alpha = \alpha_c \ . \tag{5.11b}$$

Decoherence , eq. (5.9), is then automatic for such sets of histories in which one is certain.

---

[5]Decoherence also implies another kind of limit to classical predictability which should be mentioned although we cannot discuss it in any depth here. As described, realistic mechanisms of decoherence involve the dispersal of phase information concerning a subsystem into an environment that interacts feebly with it. Those interactions produce noise which limits the classical predictability of the subsystem. Thus, for classical predictability appropriate and sufficient coarse-graining is needed for the decoherence necessary to predict probabilities at all. But further coarse-graining is needed for the subsystem to have sufficient inertia to resist the noise that those mechanisms of decoherence produce and thereby become classically predictable. (For an introductory discussions see [13]. For a more detailed one see [7].)

Eq. (5.11b) shows that operators of histories that are predicted with certainty act as projection operators on the initial state. An alternative predicted with probability one is thus mathematically equivalent to the alternative corresponding to the question "Is the universe in state $|\Psi\rangle$"? These are very special questions. Out of the class of sets of decoherent histories almost none correspond to sets in which one history is a certain prediction of the initial condition and the theory of dynamics alone.

In quantum cosmology we might hope that some gross features of the universe might be among those that are predicted with near certainty from the initial condition and dynamics alone. These include features such as the approximate homogeneity and isotropy of the universe on scales above several hundred megaparsecs[6], its vast age after the big bang when measured on elementary particle time scales, and certain features of the spectrum of density fluctuations that grew to produce the galaxies. On more familiar scales we may hope that the laws of the initial condition and dynamics would predict the homogeneity of the thermodynamic arrow of time and the wide range of scale and epoch on which the regularities of classical physics are exhibited. There has even been speculation that phenomena on very small scales, such as the dimensionality of spacetime or certain effective interactions of the elementary particles at accessible energy scales, may be near certain predictions of the initial condition and dynamics. But there is little reason to suspect that a simple theories of the initial condition and fundamental dynamics will predict anything about the behavior of the New York stock market with near certainty and a great many other interesting phenomena as well. That is why you can't get rich knowing the wave function of the universe!

The situation is very different if information beyond laws of dynamics and the initial condition is supplied and probabilities conditioned on that information are considered. There are many sets of *conditional* probabilities in which one member of the set is near certain. These conditional probabilities are the basis of prediction in all the other sciences when viewed from the perspective of quantum cosmology as will be described in the next Section.

I have described various limitations on what can be predicted in quantum cosmology. Yet there is a sense in which we, as information gathering and utilizing physical systems, make use of only a small part of the possible predictions of quantum cosmology. That is because of our almost exclusive focus on alternatives defined in terms of the variables of classical physics — averages over suitable volumes of densities of energy and momentum, densities of nuclear and chemical species, average field strengths, etc. Such classical quantities are represented by quantum operators called quasiclassical operators. (They are termed *quasi*classical because they do not behave classically in all circumstances.) Certainly our immediate experience can be described in terms of quasiclassical variables even when — as in the clicks of a Geiger counter — these variables do not obey deterministic classical laws.

Even in our theorizing about regions of space or epochs in time that are very distant from us, we often focus on histories of alternatives of quasiclassical operators. Only in the microscopic arena do we consider non-quasiclassical alternatives such as election spin and coherent superpositions of position. Even then we typically consider such alternatives only when they are tightly correlated with a quasiclassical variable as in a measurement situation.

---

[6] A megaparsec (Mpc) is a convenient unit for cosmology. One megaparsec = 3.3 million light years = 3.1×10^{24}cm. The size of the universe visible today is of order several thousand megaparsecs.

However, quantum field theory exhibits many more kinds of variables than the small set of quasiclassical ones. Decohering sets of histories can be constructed from alternative values of non-quasiclassical operators as well as from quasiclassical ones. Indeed, the quasiclassical sets of histories are but a small subset of the whole class of decohering histories. Quantum theory does not privilege one set of decohering histories over another. Probabilities are predicted for all such sets of alternatives. Histories of non-quasiclassical alternatives are not beyond reach. Suppose we were to make measurements of peculiarly quantum mechanical variables involving large numbers of particles in regions of macroscopic dimensions. The histories that would be relevant for the explanation of the outcomes of these measurements would not be histories of quasiclassical variables in these regions, but rather histories of the non-quasiclassical alternatives that were measured. The reason for our preference for quasiclassical sets of alternative histories, like all other questions concerning ourselves as particular physical systems, probably lies in our evolutionary history — not in the framework of quantum theory itself.

VI. DIFFERENCES BETWEEN THE SCIENCES

Using the conditional probabilities of quantum cosmology, a particular orbit of the earth about the sun could be predicted with near certainty given a few previous positions of the earth and a description of the earth and solar system in terms of the fundamental fields which are the language of quantum cosmology. The probabilities for the outcome of chemical reactions become near certain predictions of quantum cosmology given a description in terms of fundamental fields of the molecules involved and the conditions under which they interact. The probabilities for the behavior of sea turtles in particular environments could, in principle, become predictions of quantum cosmology given a description of sea turtles and their environments in the language of quantum cosmology. Even the probabilities for the different behaviors of human — beings both individually and collectively — could in principle be predicted given a sufficiently accurate description of the individuals, their history, their environment, and their possible modes of behavior. In this way *every* prediction in science could be viewed in terms of a conditional probability in quantum cosmology. Why then do we have separate sciences of astronomy, chemistry, biology, psychology, and so on? The answer, of course, is that it is neither especially interesting nor practical to reduce the predictions of these sciences to a computation in quantum cosmology.

One measure of the difference between the sciences is how sensitive the regularities they study are to the forms of the initial condition of the universe and the fundamental theory of dynamics. The phenomena studied in chemistry, fluid mechanics, geology, biology, psychology, and human history, depend only very little on the particular form of the initial condition. All of these sciences, especially chemistry, depend on the form of the theory of dynamics in some approximation, but as we move through the list we are moving towards in the direction of the study of the regularities of *increasingly specific subsystems of the universe*. Specific subsystems can exhibit more regularities than are implied generally by the laws of dynamics and the initial condition. The explanation of these regularities lies in the origin and evolution of the specific subsystems in question. Naturally these regularities are more sensitive to this specific history than they are to the form of the initial condition and dynamics. That is especially clear in a science like biology. Of course, living systems

TABLE I. Some Differences Between the Sciences

	Length of Coarse-grained Description of Alternatives	Length of Coarse-grained Description of Conditions	Length of Computation of Conditional Probabilities
Classical Physics	Very Short	Short	Very Short
Astronomy	Short	Short	Short — Long
Fluid Mechanics	Short — Long	Short	Short — Long
Chemistry	Short — Long	Short — Long	Long — Very Long
Geology	Long	Long	Long
Biology	Long — Very Long	Long — Very Long	Long — Very Long
Psychology	Very Long	Very Long	Very, Very Long(?)

conform to the laws of physics and chemistry, but their detailed form and behavior depend much more on the frozen accidents of several billion years of evolutionary history on a particular planet moving around a particular star than they do on the details of superstring theory or the "no-boundary" initial condition of the universe.[7] Conversely the phenomena studied by these sciences do not help much in discriminating among different theories of the initial condition and dynamics. It is for such reasons that it is not of pressing interest — either for other areas of science or for quantum cosmology itself — to express the predictions of such phenomena as quantum cosmological probabilities, even though it is in principle possible to do so.

Even if we were to wish to carry out a calculation of the conditional probabilities in quantum cosmology necessary for prediction in the other sciences, an examination of what it would take yields three measures which distinguish the other sciences from quantum cosmology and from one another. To yield a conditional probability the theory requires:

- A description of the coarse-grained alternatives whose probabilities are to be predicted in terms of fundamental quantum fields.
- A description of the circumstances on which the probabilities are conditioned in terms of fundamental quantum fields.
- A computation of the conditional probabilities.

The table above shows some simplistic guesses of the lengths of these three parameters for typical problems in the various sciences. We can discuss a few of these:

By classical physics I simply mean Newton's laws of mechanics and gravity, the laws of continuum mechanics, Maxwell's electrodynamics, the laws of thermodynamics, etc. — in short, the basic laws of physics as they were formulated in the 19th century. (I do not mean some specific application of these laws, as to the breaking of ocean waves.) Classical physics might almost be counted as a science separate from physics, for the laws of classical physics

[7]See, e.g. [2] for more discussion in greater depth and examples from this point of view.

do not hold universally, but only for certain kinds of subsystems in particular circumstances. However, the table shows the reason these laws are usually considered part of the science of physics. There is just a short list of quasiclassical variables (volume averages of fields, densities of energy, momentum, chemical composition, etc.) whose ranges of values define the coarse-grained alternatives of classical physics [7]. It is a somewhat longer business to spell out, in quantum mechanical terms, the circumstances in which classical physics applies. But the derivation of the laws of classical physics can be as short as a journal paper.[8]

As we move down the table to astronomy we encounter more specific classes of physical systems — stars, clusters, galaxies, etc. However, the difficulty of obtaining data on such distant objects prevents us from learning much individual detail. The length of the coarse-grained descriptions of both conditions and alternatives are typically short. The computations utilizing the equations of classical physics, however, range from very short dimensional estimates to long simulations of supernovae explosions.

In fluid mechanics we encounter a wide variety of particular phenomena arising from differential equations of classical physics. One has only to mention laminar flow, turbulence, cavitation, percolation, convection, solitons, shock waves, detonation, superfluidity, clouds, dynamos, internal waves, ocean waves, the weather, etc., to recall something of the richness of phenomena studied in this subject. The coarse-grainings describing the alternative behaviors of fluids can sometimes be long although the description of the conditions is usually shorter. Many of these phenomena can be simulated on computers today by solving the differential equations of classical physics. These calculations could be considered calculations in quantum cosmology were we to append to them a standard description of the alternatives and conditions, together with the computations that justify the use of these approximate equations in terms of the fundamental theory of quantum fields and the initial condition.

The description of the molecules of interest in chemistry can vary from short — as in typical chemical formulae — to long — as in the base sequence in human DNA. There is a similar range of conditions for chemical reactions ranging from a few reagents in a test tube to the interiors of cells. Quantum chemists *can* compute certain chemical properties such as the those of chemical bonds directly from the equations of an effective low-energy theory of the elementary particles but these computations can only be described as long.

In geology we have a science concerned with a very specific system — the earth — observed in considerable detail. A lengthy string is needed to describe the alternative configurations and composition of the material on the surface in the detail that we know it. A long history would have to be described to set the conditions for calculating the probabilities and calculations of these probabilities, even assuming the laws of classical physics, would be very long.

The reader probably needs little convincing that the description of the behavior of a complex biological organism plus its evolutionary history and its present environment in the language of quantum field theory would be a long business indeed! We should not pretend that we are anywhere close to being able to give such a description or to being able to carry out the relevant computations of conditional probabilities in quantum cosmology. Psychology and human history are yet more difficult. We may have a rough idea of how

---

[8]For a one-journal-paper derivation from the quantum cosmological point of view see, *e.g.* Ref. [7].

to describe the action of a bird's beak in the language of quantum field theory, but very little idea of the coarse grainings that describe an individuals thoughts and emotions or the vissitudes of empires.

Dear reader, please do not write the author concerning the inadequacies of the above discussion. He is aware that the boundaries between the sciences are not precisely defined and that there is wide variation in these three parameters within each one. In astronomy, for example, the description of our nearest star — the sun — can be just as complex as that of any phenomena in fluid mechanics (and indeed is a part of fluid mechanics). The smallest self-reproducing biological units may be simulable by conceivable computers [14]. There may be universal principles of mind which derive rather directly from the basics of physics [15]. The important point is, that at a basic level, every prediction in science may be viewed as the prediction of a conditional probability for alternatives in quantum cosmology and that the probabilities relevant to different sciences may be distinguished, in part, by their sensitivity to the theories of the initial condition and dynamics, by the length of the description of the alternatives, by the length of the description of the conditions, and by the length of the computation needed to produce them.

VII. LIMITS TO IMPLEMENTATION

The preceding Section discussed some limitations of practice in our effort to implement the predictions of quantum cosmology for interesting specific subsystems in the universe. These limits were of the general character described in Section IIB. Are there more fundamental and general limits arising from computational intractability? Quantum cosmology provides some examples.

There are physical reasons for computational intractability and mathematical ones. Landauer [16] has raised the issue of whether there are predictions whose computation would require more resources of space, material, and time than are available in the universe. Quantum cosmology may also present an example of what might be regarded as an extreme example of mathematical computational intractability. There is some evidence that the wave function of the universe might be non-computable in the technical mathematical sense.

One idea for a theory of the wave function of the universe is the "no-boundary" proposal [17]. To understand a little of this idea assume, for simplicity, that the universe is spatially closed and that gravity is the only quantum field. A cosmological wave function is then a function of the the possible geometries of three-dimensional space. The "no-boundary" idea is that the value of the wave function of the universe Ψ at one particular spatial geometry is a sum over all locally Euclidean four-dimensional geometries which have this three-dimensional space as a boundary and *no other boundaries*. Each four-dimensional geometry $\mathcal{G}$ in the sum is weighted by $\exp\left(-I[\mathcal{G}]\right)$ where $I[\mathcal{G}]$ is the classical action for the geometry. Mathematically a geometry is a specification of a notion of distance (a metric) on a space such that any small region can be smoothly mapped into a to a region of flat Euclidean space (a manifold). A sum over geometries would therefore naturally include a sum over manifolds as well as a sum over metrics. By suppressing two of the four dimensions we can give a crude pictorial representation of this double sum as shown in Figure 3.

The mathematics of quantum gravity has not been developed to the point that we have a precise mathematical formulation of what the relation schematically represented in Figure

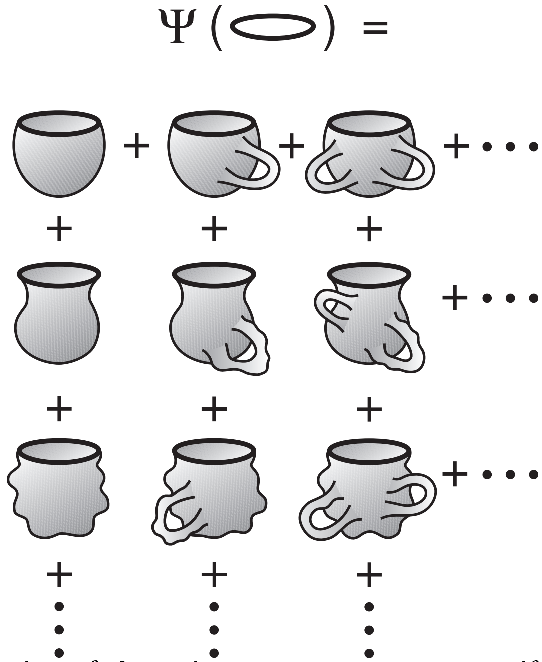

FIG. 3. *The wave function of the universe as a sum over manifolds and metrics. This figure uses two-dimensional analogs to illustrate some of the ideas that enter into the construction of the "no-boundary" wave function of the universe. That wave function is a function of three-dimensional spatial geometries one of which is represented here in two fewer dimensions by the heavy circular curve. For that given three-geometry, the "no-boundary" wave function is a sum over Euclidean four-geometries that have it as one boundary and* no other boundary*. This sum can be divided into a sum over four-manifolds and a sum over different four-metrics on those manifolds. The two-dimensional analog of this sum is shown above. The surfaces in each column represent different metrics on the* same *manifold. The manifolds in each column are the same because the surfaces can be smoothly deformed into one another by changing their shape. The metrics are different from one surface to another in a given column because the distance between two points is generally different from one shape to another. For example, the overall surface area may differ from one shape to another. The two-dimensional surfaces in different columns are different manifolds because they have different numbers of handles, and surfaces with different number of handles cannot be smoothly deformed into one another. A sum over manifolds is thus analogous to the sum over columns. A sum over metrics is analogous to the sum over different surfaces in each column.*

3 might mean. One idea for making it precise is to approximate each term in the sum by a manifold constructed of flat four-simplices — the four-dimensional analogs of triangles in two-dimensions and tetrahedra in three-dimensions. The two-dimensional analog of such a simplicial manifold would be a surface made up of triangles like a geodesic dome as illustrated in Figure 4. To calculate in four-dimensions the sum crudely pictured in Figure 3 one would proceed as follows: Choose a large number of four-simplices N. Find all possible manifolds that can be made by joining these four-simplices together. Choose *one* such assembly to represent each manifold in the sum. Integrate $\exp(-I[\mathcal{G}])$ over the edge lengths of the simplices that are compatible with the triangle and similar inequalities to approximate the sum over metrics. Sum the result over all manifolds. Take the limit as $N \to \infty$. That is one possible way the sum over geometries in the "no boundary" proposal for the wave function of the universe might be implemented.[9]

A computer program to carry out this task would first have to try all possible ways of assembling N four-simplices together and reject those which do not give a manifold. This is already a formidable mathematical problem and it has only been recently proven that an algorithm exists to carry out this computation for four-dimensional manifolds [19,20]. The next step would be for the computer to take this list of four-manifolds and eliminate duplications. However, it is known that the issue of whether two simplicial four-manifolds are identical is undecidable.[10] More precisely, there does not exist a computer program which, *for any* N, can compare two input assemblies of N four-simplices making up manifolds, and halt after having printed out "yes" if the manifolds are identical and "no" if they are not.

This suggests that the wave function of the universe defined by a sum over geometries that includes a sum over manifolds is a non-computable number.[11] However, appearances can be deceptive. Whether a number is non-computable or not is a property of the number and not of the way it is represented. Merely exhibiting one non-computable representation like the series in Figure 3 does not establish that there is not some other representation in which it *is* computable. Demonstrating non-computability in such cases is likely to be a difficult mathematical problem.

This suggested non-computability of sums over topologies has been taken as motivation for modifying the theory of the initial condition so that it is clearly computable [22,23]. But suppose that the wave function of the universe were non-computable. What would be the implications for science? Bob Geroch and I analyzed the implications of non-computability for physics in 1986 [24]. Our conclusion was that the prediction of non-computable numbers would not be a disaster for physics. That is because at any one time one needs theoretical predictions only to an accuracy consistent with experimental possibilities. Suppose, for example, it was sufficient for comparison with present observations to know the wave function of the universe to an accuracy of 10%. Suppose further it could be shown that to achieve this accuracy only simplicial manifolds with less than 100 four-simplices need be included

[9] For more details and references to the earlier literature see, *e.g.* Ref. [18].

[10] For a review see [21].

[11] We are specifically assuming the Turing model of computability.

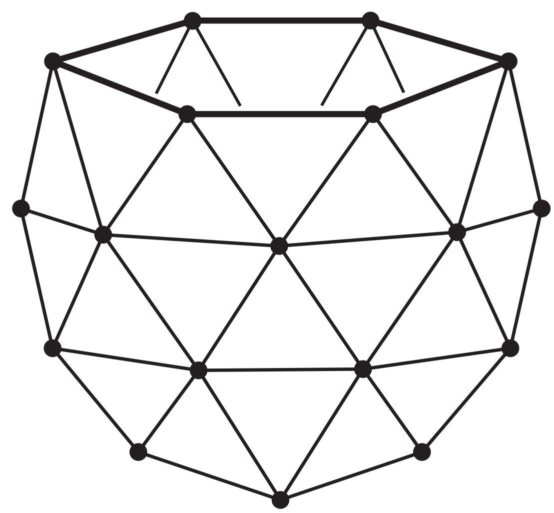

FIG. 4. *A smooth two-dimensional surface may be approximated by an assembly of flat triangles like a geodesic dome. In an analogous way, a curved four-dimensional geometry may be approximated by an assembly of four-simplices — the four dimensional analog of two-dimensional triangles or three-dimensional tetrahedra. The four-simplices must be assembled so as to make a manifold — a space such that any small region can be smoothly mapped to a region of Euclidean space. Suppose we are given N four-simplices. Assembling them in different ways can give different manifolds (the different vertical columns in Figure 3). Different assignments of lengths to the edges give different sizes and shapes on a given manifold, that is, a different metric (the different shapes within each column in Figure 3). A sum over geometries, which is a sum over manifolds and metrics, may therefore be approximated by choosing one assembly to represent each manifold in the sum, integrating over its possible edge-lengths, and summing over all manifolds. The resulting sum may be a non-computable number because there is no algorithm for deciding when two simplicial four-manifolds are identical.*

in the series defining the wave function. The theorem concerning the non-existence of an algorithm for deciding the identity of simplicial four-manifolds refers to an algorithm that would work for *any* N. It does not rule out establishing the identity of two four-manifolds with less than 100 four-simplices. Indeed, being a problem that involves a finite number of specific cases, one imagines it could be solved with sufficient work on those cases. What the theorem ensures is that, if observations improve, and the wave function is later needed to an accuracy of 1%, requiring manifolds with a larger number of four-simplices (say 10,000), a new intellectual effort will be required to compute it. The algorithms that worked for manifolds assembled from less than 100 four-simplices are unlikely to work for manifolds assembled from less than 10,000 four-simplices.

Thus, the prediction of non-computable numbers would not mean the end of comparison between theory and observation. It would mean that the process of computing the predictions could be as conceptually challenging a problem as posing the theory itself.

VIII. LIMITS TO VERIFICATION

Quantum mechanics predicts the probabilities of alternative histories of the universe. We cannot interpret these probabilities as predictions of frequencies which are accessible to test, for we have access to but a single universe and but a single history of it. Our ability to test the theory or to infer the theory from empirical data is therefore limited.

An example of current interest is "cosmic variance" in the predictions of temperature fluctuations in the cosmic background radiation. The observed pattern of temperature determines the correlation function $C(\theta)$ between the temperature fluctuations δT at two different directions $\vec{n}_1$ and $\vec{n}_2$ on the sky separated by an angle θ:

$$C(\theta) = \left\langle \frac{\delta T(\vec{n}_1)}{T} \; \frac{\delta T(\vec{n}_2)}{T} \right\rangle \ , \tag{8.1a}$$

where $\langle\cdot\rangle$ denotes an average over all directions $\vec{n}_1$ and $\vec{n}_2$ such that $\vec{n}_1 \cdot \vec{n}_2 = \cos\theta$. This correlation function can be conveniently expanded in spherical harmonics $P_\ell(\cos\theta)$:

$$C(\theta) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} \ C_\ell P_\ell(\cos\theta) \ . \tag{8.1b}$$

The coefficients C_ℓ so defined are the way the data from observations are are usually quoted and are the objects of theoretical prediction.[12]

The probabilities of temperature fluctuations in the cosmic background radiation are predicted from a spectrum of fluctuations implied by the initial quantum state. The probabilities of these fluctuations are thus a detailed prediction of quantum cosmology that stem directly from the initial condition. They are not conditional probabilities requiring other information. The theory does not predict high probabilities for particular fluctuations in temperature at particular locations on the sky. Rather, it predicts distributed probabilities

[12]See *e.g.*, Ref. [25] for a detailed review.

for these fluctuations (See *e.g.*, Ref. [26]), or equivalently the probabilities for various values of the C_ℓ. The expected value and the standard deviation of this distribution is shown in Figure 5. The width of the distribution is "cosmic variance".

We cannot test these probabilistic predictions for the cosmic background temperature fluctuations by measuring these fluctuations in a large number of identical cases. We have only one universe and only one set of observed temperature fluctuations! An observed distribution of C_ℓ's inside this "cosmic variance" would be confirmation of the theory of the initial condition. An observed distribution outside it would be evidence against it. However, observations will not distinguish two theories of the initial condition whose "cosmic variance" both surround the observed distribution. Thus, for such probabilistic predictions we are inevitably limited in our ability to test a theory of the initial condition.

More generally, as mentioned above, a theory of the initial condition can be tested only through predictions whose probabilities are so near certain that we would reject the theory if they were not observed. The sets of histories which lead to near certain predictions are just a small set of those for which probabilities are predicted.

There are limits, therefore, to the process of inferring the initial state of the universe from observation. If $C_{\rm obs}$ is the operator describing the entirety of our collective observations then strictly speaking all we can conclude about an initial state $|\Psi\rangle$ is that it is not such that

$$C_{\rm obs}|\Psi\rangle = 0\ . \tag{8.2}$$

That is not much of a restriction. For example, suppose that the projection $P_{\rm pres.\ data}$ represents all our present data including our records of the past history. It is not possible on the basis of either present or future observations to distinguish an initial state $|\Psi\rangle$ from that defined by

$$|\Psi'\rangle = \frac{P_{\rm pres.\ data}|\Psi\rangle}{||P_{\rm pres.\ data}|\Psi\rangle||}\ . \tag{8.3}$$

Retrodictions of the past from present data and $|\Psi\rangle$ could differ greatly from those from the same data and $|\Psi'\rangle$.[13] But retrodictions are not accessible to experimental check and therefore do not distinguish the two candidate initial states. The two initial conditions $|\Psi\rangle$ and $|\Psi'\rangle$ could differ greatly in complexity if the description of $|\Psi\rangle$ is short but that of $P_{\rm pres.\ data}$ is long, and we may choose between these physically equivalent possibilities on the basis of simplicity. The search for a theory of the initial condition must therefore rely on the principles of simplicity and connection with the fundamental dynamical theory in an essential way.

Why is it that the fundamental dynamical theory — the Hamiltonian of the elementary particle system — seems so much more accessible to experimental test and so much easier to infer from observational data than the theory of the initial condition? Strictly speaking

---

[13]Unlike classical physics, where the past can be retrodicted from sufficiently precise present data alone, retrodiction in quantum theory requires present data *and* the initial condition of the system in question. For further discussion see, *e.g.* [5], Section II.3.1.

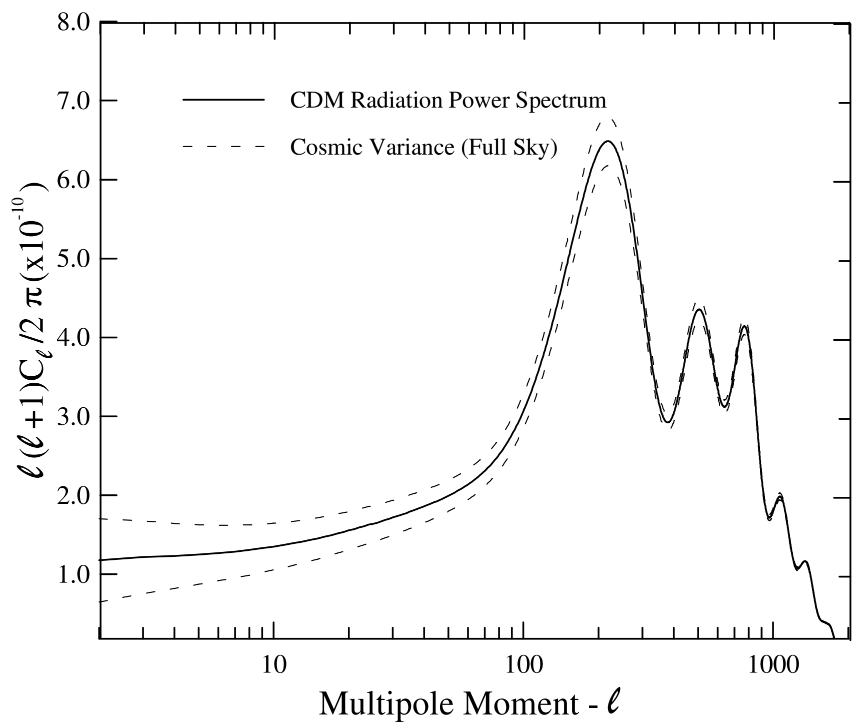

FIG. 5. *Cosmic Variance. The heavy line on this figure shows the expected value of the multipole moments of the two point correlation function defined by eq. (8.1) for temperature fluctuations in the cosmic background radiation as predicted from the probabilities of these fluctuations arising from a simple theory of the universe's initial condition. The dotted lines show the standard deviation of the predicted distribution called the "cosmic variance". Observations of our single universe yield the correlation function and one particular distribution of observed multipole moments. These observations will not distinguish two theories of the initial condition whose "cosmic variance" both surround the observed distribution. [Graph by J. Gundersen.]*

it is not. Were the Hamiltonian of the elementary particle system to vary on cosmological scales — to be a function of spacetime position of the form $H(x)$ — then inferring H would be just as difficult a process as inferring the initial $|\Psi\rangle$. However, we *assume* the principle that the elementary particle interactions are local in space and time. With that assumption the Hamiltonian describing these interactions becomes accessible to many local tests on all sorts of scales ranging from those accessible in particle accelerators to the expansion of the universe itself. The problem of inferring the initial $|\Psi\rangle$ therefore is not so very different from that of inferring H in making use of the theoretical assumptions. It is just that the assumption of locality is so well adapted to the quasiclassical realm of familiar experience that many more tests can be devised on small scales of a theory of H than we are ever likely to find of a theory of $|\Psi\rangle$ on cosmological scales.

IX. CONCLUSIONS: THE NECESSITY OF LIMITS TO SCIENTIFIC KNOWLEDGE

If the world is complex and the laws of nature are simple, then there are inevitable limits to science. Not everything that is observed can be predicted; only certain regularities of those observations can be predicted. Even given a theory, computational intractability or observational difficulty may limit our ability to predict. In a world of finite observations, there are inevitable limits to our ability to discriminate between different theories by the process of induction and test.

Quantum cosmology — the most general context for prediction in science — exhibits examples of all three kinds of limits to scientific knowledge. There are only a very few predictions of useful probabilities that are conditioned solely on simple theories of dynamics and the universe's initial condition. There is a far richer variety of useful probabilities conditioned on further empirical data that are the basis for most of the predictions in science. There are some indications that the "no boundary" initial wave function is non-computable in the technical sense of yielding non-computable numbers. That does not limit our ability to extract predictions from the theory in principle, but may be an indication that predictions sensitive to the topological structure of spacetime on small scales could be conceptually challenging to compute. Finally, it is possible to exhibit different theories of the initial condition with identical present and future predictions which can only be discriminated between by an appeal to principles of simplicity and harmony with fundamental dynamical laws.

We should not conclude a discussion of limits in science without mentioning that science is useful *because* of its limits. Complex adaptive systems are successful in evolution and individual behavior because they identify and exploit the regularities that the universe exhibits. Scientific theories predict what these regularities are and explain their origin. Theories can be used to estimate how tractable these predictions are to compute or practical to measure. By comparing different theories induced from the same data an idea can be gained of the reliability of our predictions. The existence of limits of the kind we have discussed therefore does not represent a failure of the scientific enterprise. Limits are inherent in the nature of that enterprise, and their demarcation is an important scientific question.

ACKNOWLEDGMENTS

The author is grateful to M. Gell-Mann for discussions of a number of the issues discussed in Section VIII over many years. Thanks are also due to H. Morowitz and R. Shepard for instruction in certain aspects of biology and psychology respectively, and to J. Gundersen for supplying several of the figures. This work was supported in part by NSF grants PHY90-08502, PHY95-075065, and PHY94-07194.

REFERENCES

[1] N. Bohr, *Atomic Physics and the Description of Nature*, Cambridge University Press, Cambridge, UK (1934).

[2] M. Gell-Mann, *The Quark and the Jaguar*, W. Freeman San Francisco (1994).

[3] J. Halliwell, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, eds. S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg, World Scientific, Singapore (1991) pp. 65-157.

[4] C.H. Woo, *Phys. Rev.*, **D39**, 3174, (1989), *Found. Phys.*, **19**, 57, (1989), and in *Complexity, Entropy, and the Physics of Information*, ed. by W. Zurek (Addison Wesley, Reading, MA, 1990).

[5] J.B. Hartle, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, ed. by S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg, World Scientific, Singapore (1991) pp. 65-157.

[6] J.B. Hartle, in *Directions in General Relativity, Volume 1: A Symposium and Collection of Essays in honor of Professor Charles W. Misner's 60th Birthday*, ed. by B.-L. Hu, M.P. Ryan, and C.V. Vishveshwara, Cambridge University Press, Cambridge (1993). gr-qc/9210006.

[7] M. Gell-Mann and J.B. Hartle, *Phys. Rev.*, **D47**, 3345, (1993). gr-qc/9210010.

[8] R. Griffiths, *J. Stat. Phys.*, **36**, 219 (1984).

[9] R. Omnès, *J. Stat. Phys.*, **53**, 893 (1988), *ibid*, **53**, 933 (1988); *ibid*, **53**, 957 (1988); *ibid*, **57**, 357 (1989); *Rev. Mod. Phys.*, **64**, 339 (1992); *Interpretation of Quantum Mechanics*, Princeton University Press, Princeton, (1994).

[10] M. Gell-Mann and J.B. Hartle, in *Complexity, Entropy, and the Physics of Information, SFI Studies in the Sciences of Complexity*, Vol. VIII, ed. by W. Zurek, Addison Wesley, Reading, MA or in *Proceedings of the 3rd International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology* ed. by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura, Physical Society of Japan, Tokyo (1990).

[11] E. Joos and H.D. Zeh, *Zeit. Phys.*, **B59**, 223 (1985).

[12] W. Zurek, *Phys. Rev. D*, **24**, 1516 (1981), *ibid.* **26**, 1862 (1982).

[13] J.B. Hartle, in *Proceedings of the Cornelius Lanczos International Centenary Conference*, North Carolina State University, December 1992, ed. by J.D. Brown, M.T. Chu, D.C. Ellison, R.J. Plemmons, SIAM, Philadelphia, (1994). gr-qc/9404017; also J.B. Hartle in the Proceedings of the SFI conference on *Fundamental Sources of Unpredictability* to be held March 28–30, 1996.

[14] C.M. Fraser et. al., *Science*, **270**, 397 (1995).

[15] R.N. Shepard, *Psychonomic Bulletin & Review*, **1**, 2 (1994), *World and Mind* (to be published).

[16] R. Landauer, *IEEE Spectrum*, **4**, 105 (1967); in *Proceedings of the 3rd International Symposium on Foundations of Quantum Mechanics in the Light of New Technology*, ed. by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura, Physical Society of Japan, Tokyo (1990); and *Physics Today*, **44**, 23 (1990).

[17] J.B. Hartle and S.W. Hawking, *Phys. Rev.*, **D28**, 2960, (1983).

[18] J.B. Hartle, *J. Math. Phys.*, **26**, 804, (1985a).

[19] H. Rubenstein, *The Solution to the Recognition Problem for* S^3, unpublished lectures, Haifa, Israel (1992).

[20] A. Thompson, *Math. Res. Lett.*, **1** 613 (1994).
[21] W. Haken, in *Word Problems*, ed. by W.W. Boone, F.B. Cannonito and R.C. Lyndon, North Holland, Amsterdam, (1973).
[22] J.B. Hartle, *Class. & Quant. Grav.*, **2**, 707, 1985b.
[23] K. Schleich and D. Witt, *Nucl. Phys.*, **402**, 411, 1993; *ibid.*, **402**, 469, 1993.
[24] R. Geroch and J.B. Hartle, *Found. Phys.*, **16**, 533 (1986).
[25] J.R. Bond, in *Cosmology and Large Scale Structure, Proceedings of the Les Houches School, Session LX, August 1993*, ed. by R. Schaefer, Elsevier Science Publishers, Amsterdam (1995).
[26] J. Halliwell and S.W. Hawking, *Phys. Rev. D*, **31**, 1777 (1985).