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Sources of Predictability*

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Abstract

Sources of predictability in the basic laws of physics are described in the most general theoretical context — the quantum theory of the universe as a whole.

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I. INTRODUCTION

The title of our conference, *Fundamental Sources of Unpredictability*, invites the question: "What are the obstacles and limitations to the predictive power of the laws of physics?" But in a quantum mechanical world characterized by indeterminacy and distributed probabilities, in a world where even classically describable phenomena can be intractable or impossible to compute, and in a finite universe with limited opportunities for observation and induction, we might more straightforwardly ask the question: "What are the fundamental origins of predictability?" More specifically, what features of the basic laws of physics lead to predictable regularities in the universe? This essay is devoted to a few general remarks about this question from the perspective of quantum cosmology — the quantum theory of the universe as a whole.

It seems inescapable that there are limits to the predictive power of the laws of physics. The world appears to be complex. But the basic laws that govern the regularities of the world must be simple to be discoverable, comprehensible, and effectively applicable. It is a logical possibility that every feature of our experience — the shape of each galaxy, the number of planets circumnavigating each star, the character of each biological species, the course of human history, and the result of every experiment — is the output of some short computer program with no input. However, there is no evidence that our universe is so regular. Even the most deterministic classical theories did not claim this. They claimed only to predict future evolution when *given* initial conditions. Not everything that can be observed can be predicted — only the regularities in those observations are the province of science. The regularities of the universe are limited and therefore there are limits to the predictive power of physics. This kind of limit is therefore not a failure of the scientific enterprise. Limits are inherent to that enterprise, and their demarcation is an important scientific question.

To ask about the *fundamental* sources of predictability is to ask for the regularities implied by the basic laws that apply universally to all physical systems — without exception, qualification, or approximation — in the most general theoretical context. In contemporary physics, that is the subject of quantum cosmology — the quantum theory of the initial condition of the universe and its subsequent history. In present thinking this theory is based on two laws:

- The basic theory of dynamics (*e.g.* heterotic superstring theory),
- The theory of the initial condition of the universe (*e.g.* the "no-boundary" wave function of the universe).

There are no interesting predictions that do not depend on these two laws, even if only very weakly. In this essay, we shall describe some specific features of these two basic laws which give rise to predictable general regularities of the universe. We cannot pretend to discuss the origin of all the regularities considered by science in this brief compass, but we shall describe some of the more general ones. We begin in Section II with the general regularities exhibited by any quantum system, and then proceed to the regularities that depend on the particular properties of the laws of dynamics and initial condition that hold in our universe. Specifically, we consider the origin of a realm of approximate classical determinism in Section III and the origin of individual, approximately isolated subsystems in Section IV. Mere existence of such regularities, however, is not enough for science. They must be accessible to inference and amenable to computation. We mention a few properties of the basic laws that allow for these qualities in Sections V and VI.

II. A QUANTUM UNIVERSE

It is an inescapable inference from the physics of the last seventy years that we live in a quantum mechanical universe and that the basic laws of dynamics and initial condition are consistent with the general framework of quantum theory. Let us accept the inference and ask for its implications for predictability.

To keep the discussion manageable let us neglect gross quantum fluctuations in the geometry of spacetime such as would be expected to occur in the first 10^{-43} sec. after the big bang. The quantum mechanics of a universe of matter fields moving in a fixed spacetime geometry exhibits the determinism of the Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \ . \tag{2.1}$$

The basic theory of dynamics supplies the Hamiltonian H, the theory of the initial condition supplies the initial quantum state Ψ_0. The only regularities exhibited with certainty by all quantum systems, independent of the specific natures of H and Ψ_0, are paraphrases of the prediction that at any time t the universe is in the state $\Psi(t)$ that evolved deterministically from the initial state Ψ_0 by the Schrödinger equation.

But it is not merely these certain and general regularities of the Schrödinger equation which are of interest. We are interested in the much broader class of regularities which are not certain but occur with high probability as a consequence of the *particular* properties of the laws of dynamics and initial condition. The approximate regularities of classical physics are the most prominent example.

Most generally quantum mechanics predicts the probabilities of particular *histories* of the universe in sets of alternative possible histories. Examples of sets of alternative histories are the alternative histories of the abundances of the elements over time, the alternative histories of the motion of the earth around the sun, the alternative histories of outcomes of a laboratory experiment testing quantum mechanics, the alternative evolutionary tracks of biological species on earth, and the alternative histories of the S&P stock average. Probabilities for the members of each of these sets of alternative histories can in principle be calculated from the initial state of the universe and the basic dynamical law.

Probabilities of single events or single histories are not necessarily definite predictions. From a theory that a coin is unbiased, we calculate the probability of the outcome of a single toss as 1/2, but we cannot be said to have made a definite prediction of the outcome. The definite predictions of a theory of the initial state and dynamics are the sets of histories in which only one member has a probability very close to unity and the rest have probabilities close to zero. Among the examples of sets of alternative histories mentioned above, only the first — the alternative histories of the abundances of the elements — has this character. Current theories of the initial state and the dynamics predict a high probability of primordial abundances of approximately 75% hydrogen, 25% He^4, small but definite abundances of He^3, deuterium, lithium, and negligible percentages of the other elements. There are similarly definite predictions for some other large scale features of the universe, but the probabilities for the other sets of alternatives mentioned above are likely to be highly distributed and so are not definite predictions. We expect only a few definite predictions from the basic laws of dynamics and initial condition alone, unaugmented by further information. The largest and most useful general regularities of this kind are those summarized by the deterministic laws of classical physics. It is to these we now turn.

III. THE QUASICLASSICAL REALM

Classical deterministic laws approximately govern the regularities in time of a wide range of phenomena over a broad span of time, place, and scale in the universe. The domain of applicability of these classical laws is the quasiclassical realm of everyday experience. In a quantum mechanical universe these classical equations can be but approximations to unitary evolution by the Schrödinger equation and reduction of the state vector. To what do we owe the validity of this classical approximation over such an extensive realm of phenomena, time, place, and scale?

The answer is certainly not merely the determinism of the Schrödinger equation. The regularities in time that it summarizes generally relate quantities that are nothing like those of classical physics. The simple example of Ehrenfest's theorem for the quantum mechanics of a single particle moving in one dimension suggests what is necessary for classical behavior. Ehrenfest's theorem relates the acceleration of the expected position to the expected value of the force:

$$m\,\frac{d^2\langle x\rangle}{dt^2} = -\left\langle \frac{\partial V(x)}{\partial x}\right\rangle \ . \tag{3.1}$$

This is an exact consequence of the determinism of the Schrödinger equation, but not a classical equation of motion because it is not a differential equation for $\langle x\rangle$. It becomes a deterministic equation in the approximation that the expected value of the force may be replaced by the force evaluated at the expected position

$$m\frac{d^2\langle x\rangle}{dt^2} \approx -\frac{\partial V(\langle x\rangle)}{\partial x} \ . \tag{3.2}$$

There are at least four requirements for the validity of this approximation. First, the Hamiltonian must be of the form that can give rise to an equation of motion. Second, the approximation is true *only for certain states*, typically narrow wave packets. Third, the right variables must be followed, in this case position. Finally, there must be coarseness in how they are followed, here average rather than exact position.

Only certain states and certain coarse-grained descriptions exhibit patterns of classical correlations in this simple model. Similarly, but more generally, we cannot expect a quasiclassical realm in a universe with a generic initial condition and a generic Hamiltonian. The approximate quasiclassical realm that extends over most of cosmological space and time in this universe owes its existence to an appropriate coarse-grained description in terms of the variables of classical physics and the particular properties of the initial quantum state and dynamical law. Classical predictability is an emergent feature of the universe's particular initial condition and dynamical law.

IV. REGULARITIES OF SUBSYSTEMS

Most of science is not concerned with the regularities exhibited universally by all physical systems but rather with the particular regularities of classes of individual, localized, subsystems. Stars, atoms, oceans, biological species and individual human behaviors are just a few of the myriad examples. In interesting circumstances such subsystems approximately preserve their identity over significant periods of time and can be treated as approximately isolated. Their properties and evolution can thus be studied individually rather than as a feature of the evolution of the whole universe. There is no particular reason to believe that a universe with a generic Hamiltonian and a generic initial state would exhibit such approximately isolated subsystems. To what features of our particular basic laws do we owe the existence of such subsystems and what is the origin of their regularities?

The locality of the effective interactions between the elementary particles that is a consequence of the basic dynamical law is certainly necessary for the approximate isolation of localized subsystems. However, properties of the law of initial condition also play a role. There were no individual subsystems in the early universe. The evidence of the observations is that the early universe was nearly smooth, featureless, homogeneous, isotropic, and far from equilibrium. Today's subsystems have evolved from the quantum fluctuations away from this smoothness that are properties of the initial quantum state primarily by the universal action of gravitational attraction. The existence of individual galaxies, for example, is a consequence of the gravitational collapse of primordial fluctuations in the uniform density of matter.

There are several possible sources for the regularities among members of certain classes of approximately isolated subsystems. The simplest is a common historical origin as with individual members of a biological species. But the laws of dynamics and initial condition can also be a source of regularities. Einstein's theory of gravity ensures that one black hole is similar to any other no matter how different were the situations in which they were formed. The regularities among galaxies arise from their origin in similar circumstances across the universe as mandated by an initial condition of close to perfect homogeneity and isotropy.

Thus the specific regularities of particular subsystems that are the subject of much of science outside of physics arise in part from special properties of the basic laws of dynamics and the initial condition of the universe.

V. INFERENCE AND COMPUTATION

In the previous sections we have discussed how the fundamental laws of dynamics and the initial condition permit prediction in this quantum mechanical universe. But these laws are not presented to us by revelation. They are arrived at by a process of induction from observation, theoretical development, and experimental test. Further, exhibiting the predictions of these laws is not simply a matter of displaying their form. Their consequences must be computed in particular circumstances. No treatment of the sources of predictability would be complete without discussion of those features of the two fundamental laws that permit the processes of induction and application. Space does not permit an examination of these questions that is worthy of their depth. We can only list a few features of the fundamental laws that help us in our task.

The locality of the fundamental law of dynamics, and the separation of phenomena by scale that it permits, have both aided its discovery. Locality enables us to extrapolate the regularities of familiar scales both much larger and smaller domains. Assuming locality, from gravitational phenomena on earth we are able to infer the rules governing the evolution of the planets, galaxies, and eventually the universe itself. Assuming locality, from the electromagnetic forces between macroscopic bodies we are able to infer those between the constituents of atoms.

Separation of phenomena by scale has permitted a kind of progression in discovery of the fundamental dynamical law — each stage being characterized by a discoverable effective theory of limited validity. Thus we moved from the motion of the planets in the solar system, to the motion of electrons in an atom, to the motion of protons and neutrons in the nucleus, to the motion of quarks in the proton and neutron, *etc., etc.* Thus classical physics was discovered before quantum mechanics, Newtonian mechanics before special relativity, and Newtonian gravity before general relativity, *etc.* The similarity of phenomena on different scales has been advanced [1] as the reason for the "unreasonable effectiveness of mathematics in the natural sciences."

Locality of effective interactions and separation of phenomena by scale also reduce the computation of certain predictions to manageable tasks. Locality has already been mentioned as a prerequisite for approximately isolated subsystems. For these the application of the dynamical law leads to computations which are more tractable than those arising in its application to the universe as a whole. The separation of phenomena by scale permits the computation of the evolution of a wide range of phenomena using simpler, approximate, effective forms of the dynamical law.

More generally it is important for practical predictability that the known laws of physics predict computable numbers extractable in particular circumstances by the application of standard algorithms. This is not a trivial statement because there are infinitely many more non-computable numbers than computable ones.

The phenomenon of chaos is commonly mentioned as a source of unpredictability in physics. But in an essay on sources of predictability it is more appropriate to focus on the existence of non-chaotic, integrable systems whose evolution can be tractably computed. The source of this integrability lies in the conservation laws of the dynamical theory and these arise from its symmetries — exact or approximate. These same conservation laws are at the heart of the effectiveness of statistical mechanics in separating out quantities that approach equilibrium slowly enough to be governed by phenomenological equations of motion. In these ways symmetries of the dynamical law can be a source of predictability.

VI. CONCLUSION

Predictable regularities in this specific quantum universe arise from particular features of its fundamental laws of dynamics and the initial condition. As observers of the universe we rely on the existence of a quasiclassical realm, the locality of effective interactions, the separation of phenomena by scale, and the existence and regularities of approximately isolated subsystems — all properties which depend on the form of the basic laws of dynamics and initial condition. One could ask: "Why do the fundamental laws exhibit these features which are of such use to us?" That, however, is the wrong way of putting the question. Individually and collectively we are complex adaptive systems within the universe that evolved to exploit the emergent regularities that the universe presents. Ask rather how or whether such systems would have evolved if the fundamental laws did not display the features we have described. While much beyond our power to answer at present, this question does show that our adaptation to the regularities of our specific universe is another source of predictability in physics.

VII. FURTHER READING

A *Scientific American* introductory article to quantum cosmology is [2]. The author has expanded on many of the topics in this brief essay in [3] where a more extensive list of references may be found.

ACKNOWLEDGMENTS

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