Incentives in Computer Science (COMS 4995-6): Exercise Set #5

Due by Noon on Wednesday, February 26, 2020

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code MKRKK6 to register for COMS 4995-6. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into 2–3 pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) Refer to the course Web site for the late day policy.

Exercise 25

[Repeat from last week; ignore if you already turned a solution in with HW#4.]

Show that the general VCG mechanism is "individually rational," meaning that a truthful bidder is guaranteed nonnegative utility.[1]

[Hint: prove that $p_i \leq b_i(\omega^*)$, where p_i is the VCG payment by bidder i, ω^* is the outcome chosen by the mechanism, and $b_i(\omega^*)$ is the bid by bidder i for the outcome ω^*.]

Exercise 26

(a) Prove that, for every fixed $r \geq 0$, a Vickrey single-item auction with reserve price r is truthful.

(b) Propose how to incorporate a reserve price into the VCG sponsored search auction while preserving the auction's truthfulness. Specify precisely what the payments (per-click) are. Prove that your proposed mechanism is truthful.

[Hint: to prove truthfulness, you can do it directly, or you could try to reduce the statement to the truthfulness of the standard VCG mechanism (as proved in lecture) when there are k additional bidders each with a valuation (per-click) of r.]

[1] You can assume that all bids are nonnegative.

Exercise 27

Consider the following extension of the sponsored search setting described in lecture. Each bidder i now has a publicly known *quality* β_i, in addition to a private valuation v_i per click. As usual, each slot j has a CTR α_j, and $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$. We assume that if bidder i is placed in slot j, then the probability of a click is $\beta_i \alpha_j$. Thus bidder i derives value $v_i \beta_i \alpha_j$ from an impression in the jth slot.

Describe a truthful and welfare-maximizing auction for this generalized sponsored search setting. Explicitly discuss both how you assign bidders to slots, and how you define the payments. Prove that your mechanism is truthful and welfare-maximizing (assuming truthful bids).

Exercise 28

The point of this exercise is to demonstrate that exact optimization is crucial for the nice properties of the VCG mechanism.

Consider the following error-prone algorithm $\mathcal{A}$ for computing the maximum of a set $\{b_1, \ldots, b_n\}$ of numbers: (i) if the second-highest number is exactly one less than the largest number, then output the former; (ii) otherwise, output the largest number.

Consider the following error-prone version of the Vickrey auction:

1. Accept bids $b_1, \ldots, b_n$.
2. Award the item to the bidder i^* corresponding to the output of $\mathcal{A}(b_1, \ldots, b_n)$.
3. Charge the winning bidder $\mathcal{A}(\mathbf{b}_{-i^*})$, where $\mathbf{b}_{-i^*}$ denotes the bids by bidders other than i^*.

Prove that the error-prone Vickrey auction is neither truthful nor individually rational (as defined in Exercise 25).

Exercise 29

By extending the previous exercise, show by example that substituting a heuristic $\mathcal{A}$ (i.e., a non-exact algorithm) for welfare maximization in the VCG mechanism can result in negative payments (i.e., payments from the mechanism to the bidders).

[Hint: go beyond single-item auctions. For example, consider the setting in Exercise 22.]

Exercise 30

Compute the monopoly price of the following distributions.[2]

(a) The uniform distribution on $[0, a]$ with $a > 0$.

(b) The exponential distribution with rate $\lambda > 0$ (on $[0, \infty)$).

[2]Recall that a *monopoly price* of a distribution is a price p that maximizes $p \times$ (probability of a sale), where the probability of a sale at a price p is the probability that a random valuation from the distribution is at least p.