Incentives in Computer Science (COMS 4995-6): Exercise Set #4

Due by Noon on Wednesday, February 19, 2020

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code MKRKK6 to register for COMS 4995-6. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into 2–3 pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) Refer to the course Web site for the late day policy.

Exercise 20

Consider a single-item auction with at least three bidders. Prove that awarding the item to the highest bidder, at a price equal to the third-highest bid, yields an auction that is *not* truthful.

Exercise 21

Prove that for every false bid $b_i \neq v_i$ by a bidder in a second-price auction, there exist bids $\{b_j\}_{j \neq i}$ by the other bidders such that i's utility when bidding b_i is strictly less than when bidding v_i.

Exercise 22

Suppose there are k identical copies of an item and $n > k$ bidders. Suppose also that each bidder can receive at most one item. What is the analog of the second-price auction? Prove that your auction is truthful.

Exercise 23

Exhibit an equilibrium of a GSP sponsored search auction that is not social welfare-maximizing. In other words, come up with bidder valuations $v_1, \ldots, v_n$ and (non-welfare-maximizing) bids $b_1, \ldots, b_n$ so that, for each bidder i, no bid b_i' would give the bidder a strictly higher utility that it receives when bidding b_i (holding other bidders' bids fixed).

Exercise 24

Consider extending the sponsored search auction model with click-through rates α_{ij} that can depend arbitrarily on the advertiser i and slot j.[1] Assume that, for each bidder i, higher slots are better: $\alpha_{i1} \geq \alpha_{i2} \geq \cdots \geq \alpha_{ik}$.

Consider the following greedy algorithm:

1. For $j = 1, 2, \ldots, k$:

   (a) Among all bidders not yet assigned to a slot, assign to slot j the bidder i with the highest value of $v_i \alpha_{ij}$.

Show by example that (assuming truthful bids) this greedy assignment does not always maximize the social welfare. (In this context, the social welfare of an assignment is $\sum_{i=1}^{k} \alpha_{is(i)} v_i$, where $s(i)$ is the slot to which i is assigned, and where we interpret $\alpha_{is(i)} = 0$ if i does not receive a slot.)

Exercise 25

Show that the general VCG mechanism is "individually rational," meaning that a truthful bidder is guaranteed nonnegative utility.[2]

[Hint: prove that $p_i \leq b_i(\omega^*)$, where p_i is the VCG payment by bidder i, ω^* is the outcome chosen by the mechanism, and $b_i(\omega^*)$ is the bid by bidder i for the outcome ω^*.]

[1] This is related to the problem that the Facebook ad auction faces, with different advertisers bidding on different events and therefore having different CTRs.

[2] You can assume that all bids are nonnegative.