COMS 4995-6: Exercise Set #3

Due by Noon on Wednesday, February 12, 2020

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code MKRKK6 to register for COMS 4995-6. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into 2–3 pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) Refer to the course Web site for the late day policy.

Exercise 14

Recall the definition of a competitive equilibrium from lecture.

(a) Suppose there is only one good (and multiple buyers, each with a valuation for it). Give a complete description of all of the competitive equilibria in such a market, along with a brief justification.

(b) Repeat the same exercise for a market with k identical copies of a good and at least $k+1$ buyers (e.g., several new copies of a book). Assume that each buyer has a valuation for a copy of the good—the same for each copy—and does not want two or more copies.

Exercise 15

Recall the First Welfare Theorem for the model covered in lecture (with $r_j = 0$ for every good j): if $(M, \mathbf{p})$ is a competitive equilibrium (where M is a matching and $\mathbf{p}$ is a price vector indexed by the goods), then

$$\sum_{i=1}^{n} v_{iM(i)} \geq \sum_{i=1}^{n} v_{iM^*(i)}$$

for every matching M^*. (Reminders: $M(i)$ denotes the good assigned to i in M or its outside option, as appropriate; outside options have value 0; and v_{ij} denotes the valuation of buyer i for good j.)

Use the First Welfare Theorem to prove that every competitive equilibrium $(M, \mathbf{p})$ is a Pareto optimal outcome. That is, prove that for every other matching M' and price vector $\mathbf{q}$, if some buyer or seller is strictly better off in $(M', \mathbf{q})$ than in $(M, \mathbf{p})$, then some other buyer or seller is strictly worse off in $(M', \mathbf{q})$ than in $(M, \mathbf{p})$. (By definition, a buyer i is better/worse off if $v_{iM'(i)} - q_{M'(i)}$ is bigger/smaller than $v_{iM(i)} - p_{M(i)}$; the seller of a good j is better/worse off if q_j is bigger/smaller than p_j.)

Exercise 16

Now suppose that the seller of a good j is allowed to have an arbitrary nonnegative reserve price r_j.[1]

(a) Redefine a competitive equilibrium for this more general setting.

(b) Prove an analog of the First Welfare Theorem for this more general setting.

Exercise 17

In the "market for lemons" example in lecture, there were two types of cars (good and bad). Now suppose there are three types: good, medium, and lemons. Every seller knows what kind of car they have, but buyers cannot distinguish between different types. The fraction of used cars of each type is $\frac{1}{3}$ and buyers know this. Assume that good cars are worth 8 to sellers and 9 to buyers, medium cars are worth 5 to sellers and 8 to buyers, and lemons are worth 1 to sellers and 4 to buyers. If you wish, you can assume that there are more buyers than sellers.

(a) Is there an equilibrium in the used-car market in which all types of cars are sold? Explain briefly.

(b) Is there an equilibrium in the used-car market in which only medium quality cars and lemons are sold? Explain briefly.

(c) Is there an equilibrium in the used-car market in which only lemons are sold? Explain briefly.

Exercise 18

In lecture we mentioned that one common view in economics is that the value of higher education is primarily as a signaling device—allowing workers who will be highly productive to distinguish themselves as such. An opposing view is that the benefit of education stems primarily from learning and skill-building. Choose a side and argue for it (in 2-3 paragraphs), backing up the main points of your argument with sources/data when possible. (To keep things simple, focus on the U.S.)

To get started, you might want to think about/research some of the following questions:

- What are the correlations between each pair of { lifetime wages, education level, IQ scores }? Is there any causal evidence among these?
- Why are many employers more inclined to hire Ivy League graduates than graduates from other universities?
- How much knowledge and skills have you retained from the courses you took your freshman year?
- To what extent is "education" the same as the knowledge gained and the skills acquired in courses?
- Would you rather have a Columbia degree without the Columbia education, or a Columbia education without the Columbia degree?

[1] For part (b), you might want to also interpret r_j as a seller's valuation for its own good.

Exercise 19

Here are four online platforms which are still extant but seem to have jumped the shark:

1. Craigslist
2. OkCupid
3. TripAdvisor
4. Yelp

For each, list at least one way in which adverse selection could potentially be a problem on the platform. Then, for each one, speculate in one short paragraph on whether adverse selection has been one of the first-order causes of waning popularity (and explain your reasoning).