Incentives in Computer Science (COMS 4995-6): Exercise Set #10

Due by Noon on Wednesday, April 15, 2020

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code MKRKK6 to register for COMS 4995-6. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into 2–3 pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) Refer to the course Web site for the late day policy.

Exercise 51

For exercises 51–54, assume that there is an odd number of voters (and an arbitrary number of alternatives). For a set of votes (i.e., ranked lists) over a set A of alternatives, we say that alternative a *beats* b if more than half of the voters rank a somewhere above b in their lists. A *Condorcet winner* is an alternative that beats every other alternative.

Show by example that there is not always a Condorcet winner.

Exercise 52

A voting rule satisfies the *Condorcet condition* if it elects a Condorcet winner whenever one exists.

Does the plurality rule satisfy the Condorcet condition?[1] Provide either a proof that it does or a counterexample (i.e., a set of votes where there is a Condorcet winner a and the rule chooses an alternative different from a).

Exercise 53

Does ranked-choice voting satisfy the Condorcet condition? Provide either a proof that it does or a counterexample.

[1] Throughout this exercise set, assume that ties are broken in some consistent way, such as lexicographically.

Exercise 54

Does the Borda count satisfy the Condorcet condition? Provide either a proof that it does or a counterexample.

Exercise 55

Recall the basic model and definitions of fair division from lecture. Show that, with three players, a proportional allocation need not be envy-free.