COMS 4995-6: Exercise Set #1

Due by Noon on Wednesday, January 29, 2020

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code MKRKK6 to register for COMS 4995-6. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) Refer to the course Web site for the late day policy.

Exercise 1

Recall the room assignment problem and serial dictatorship mechanisms.

Serial Dictatorship Mechanism

1. Each student submits a ranked list (with no limit on the number of entries).
2. The students are ordered in some (possibly randomized) way. (E.g., by draw numbers.)
3. The students are considered in order. When student i is considered, she receives her top-ranked option that is still available.

Prove that a serial dictatorship is *group-strategyproof*, meaning that even when students can collude, honesty is the best policy. Formally, prove that: for every subset S of students, no matter how students are ordered in the second step, if some coordinated misreport of the subset's ranked lists makes a student of S strictly better off, then this misreport also makes at least one student of S strictly worse off.

Exercise 2

Here's a bad alternative to a serial dictatorship, which unfortunately was used to assign kids to elementary schools in a number of major cities for many years.

A Bad Mechanism for One-Sided Markets

1. Each student submits a ranked list (with no limit on the number of entries).
2. The students are ordered in some way. (E.g., by lottery numbers.)
3. The students are considered in this order. When student i is considered, if her top-ranked school is still available, then she is (permanently) assigned to that school. (Otherwise, she is not assigned in this phase.)
4. The still-unassigned students are considered in the same order as before. When student i is considered, if her second-ranked school is still available, then she is assigned to that school. (Otherwise, she is not assigned in this phase.)
5. And so on with the still-unassigned students' third choices, fourth choices, etc.

Discuss in detail what type of strategic behavior (i.e., gaming of the system) you would expect to see from the participants in this mechanism. Do you think the flaws of this mechanism would harm all students equally, or would some demographics be harmed more than others?

Exercise 3

With n students, there are $n!$ different deterministic serial dictatorship mechanisms — one for each possible ordering of the students. (Every probability distribution over these $n!$ orderings defines a randomized serial dictatorship mechanism.)

We saw in lecture that every serial dictatorship is strategyproof and always produces a Pareto optimal outcome. Propose a mechanism that:

(i) Accepts as input a full ranked list (with no ties) from each participant, and based on these computes an outcome.

(ii) Is strategyproof and Pareto optimal.

(iii) Is deterministic (for a given set of student rankings, it always produces the same outcome).

(iv) Is not equivalent to a serial dictatorship. That is, your mechanism M should have the property that, for each of the serial dictatorships D (i.e., for each of the $n!$ ways of ordering the students a priori), there exists a set of student rankings such that M and D compute different outcomes.

[Hint: Rather than starting from scratch, consider slight modifications of serial dictatorships.]

Exercise 4

We mentioned in lecture that the initially proposed mechanism for the National Resident Matching Program was overruled by a group of protesting medical school students in favor of the deferred acceptance algorithm. Here is the original proposal:

A Bad Mechanism for Two-Sided Markets

1. Each student submits a ranked list of hospitals, and each hospital a ranked list of students (with no limits on length).
2. In the first phase, whenever a student s ranked h first and vice versa, s is (permanently) assigned to h. (Note that all such matches can be accommodated.) Call these the "1-1 matches."
3. In the second phase, whenever s and h are currently unmatched, s ranked h first, and h ranked s second, then s is (permanently) assigned to h. Call these the "2-1 matches."

4. In the third phase, whenever s and h are currently unmatched, s ranked h second, and also h ranked s first, then s is (permanently) assigned to h. Call these the "1-2 matches."
5. Subsequent phases consider 2-2 matches, 3-1 matches, 3-2 matches, 1-3 matches, 2-3 matches, 3-3 matches, 4-1 matches, and so on.

Why do you think the medical students rejected this proposal? What were they worried about?

Exercise 5

Recall from lecture the stable matching problem, and that a *matching* is an assignment of students to hospitals. (If the capacity of hospital h is c_h, at most c_h students should be assigned to h.) A matching is *Pareto optimal* if every other matching that makes someone better off (i.e., matches them to a preferred student/hospital) also makes someone else worse off. Is every stable matching also Pareto optimal? Provide a proof or an explicit counterexample.

Exercise 6

Is every Pareto optimal matching also stable? Provide a proof or an explicit counterexample.

Exercise 7

Show by example that the deferred acceptance algorithm is not strategyproof for the hospitals. That is, exhibit a stable matching problem (students, hospitals, and their true ranked lists), a hospital h, and an untruthful preference list for h, such that h is strictly better off in the deferred acceptance algorithm by submitting the untruthful list than the truthful one. (Assume that everyone other than h reports their true lists.)