1. Satisfiability, Validity, Unsatisfiability. For each of the following formulas/sequents, specify whether the formula/sequent as valid/tautology, satisfiable (but not valid), or unsatisfiable and prove your answer.
   (a) $(P \wedge (Q \vee R)) \rightarrow ((P \wedge Q) \vee (P \wedge R))$.
   (b) $(P \vee Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P)$
   (c) $\neg P \wedge \neg Q \rightarrow \neg(P \vee Q)$

2. For the formulas from Exercise 1 that are valid, give a PK proof. (Recall that if A is valid, then a PK proof of A is a PK proof of the sequent $\rightarrow A$.)

3. For the formulas from Exercise 1 that are valid, give a Resolution refutation of the negation of the formula. (Note that if A is valid, then $\neg A$ is unsatisfiable. So you first have to show how to convert $\neg A$ into an equivalent CNF formula, using new variables as discussed in class and in the notes.)

4. (Completeness) Let A be a propositional formula and let Φ be a set of formulas. What happens when you run the derivational completeness algorithm to try to obtain a Φ-PK proof of A in the following scenarios. In particular you should specify the following for each scenario: (i) does the algorithm halt and output a Φ-PK proof of A? (ii) If the algorithm does not halt, what happens and why?
   (a) Φ is empty and A is a (propositional) formula that is not a tautology (so A is not valid). For example, $A = (x \vee y)$.
   (b) Φ is finite, and Φ does not logically imply A.

5. Exercise 11 of Lecture Notes on Propositional Calculus (Prove equivalence of the 3 forms of compactness)

6. Prove that a theory Σ is consistent if and only if Σ has a model.

7. Exercises 4, page 25 of notes on Predicate Calculus

8. Exercise 6, page 25 of notes on Predicate Calculus

9. Exercises 7, page 25 of notes on Predicate Calculus

10. Exercise 10, Herbrand, Equality, Compactness, p.50

    Let A be a first order sentence over the language with one binary predicate symbol R and equality. Suppose that for each $n \geq 3$, A has a model consisting of a directed cycle with n nodes, where R represents the edge relation of a directed graph. Prove that A has a model M whose universe includes an infinite path. (a set of distinct elements $v_0, v_1, \ldots$ such that $R^M(v_i, v_{i+1})$ holds for all $i \geq 0$.)

11. Exercise 13, Herbrand, Equality, Compactness, p.51 Use Exercise 11 above to show that $Th(s)$ (the theory of successor) is not finitely axiomatizable.

12. Exercise 15, Herbrand, Equality, Compactness, p.53

    Let $\mathcal{L}$ be a language which includes an infinite list $c_1, c_2, \ldots$ of constant symbols. Let Γ be the set of sentences $c_i \neq c_j$ for all $i < j$. Let A be a sentence such that A is a logical consequence of Γ. Prove that A has a model with a finite universe.