CS 4995 **Problem Set 2** **Fall, 2022**

Due: Tuesday, Nov 29, 11:59pm

1. (5 points) Prove that a theory Σ is consistent if and only if Σ has a model.

2. (15 points) Are each of the following languages (i) recursive, (ii) r.e. but not recursive, (iii) not r.e. Prove your answer. Do not use the S-m-n theorem.

   (a.) (5 points) Let $\mathcal{L}$ be the set of all numbers x such that x codes a TM program, and 10 is in the range of the function computed by the program.

   (b.) (5 points) Let $\mathcal{L}$ be the set of all numbers z such that z encodes a pair of numbers $< x, y >$, where x encodes a TM program that halts within y steps on all inputs.

   (c.) (5 points) Let $\mathcal{L}$ be the set of all numbers x such that x encodes a TM program, and where the program coded by x halts on only finitely many inputs.

3. (5 points) (Exercise 4, Incompleteness Notes) Give a formula which represents the relation $y = 2^x$.

4. (10 points) In class we showed that there is an $\exists\Delta_0$ formula $A(x, y)$ that represents the r.e. language K. Now suppose that $A(x)$ is an $\exists\Delta_0$ formula which represents the r.e. set K in **PA**. Show that there is a consistent extension Σ of **PA** such that $A(x)$ does *not* represent K in Σ.

5. (10 points) Prove that the set of true Δ_0 sentences is recursive and therefore arithmetical. Show that the set of true $\exists\Delta_0$ sentences is r.e., and therefore arithmetical.