CS Theory (Fall '25)	Assigned: Oct 7, 2025

Homework 3

Instructors: *William Pires and Toniann Pitassi* — Due: Oct 16, 2025 at 11am

Collaboration: Collaboration with other students in the class homework is allowed. However, you must write up solutions by yourself and understand everything that you hand in. **You may not use the web or AI tools when working on homework questions, and you are required to list who you collaborated with on each problem, including any TAs or students you discussed the problems with in office hours. List any reference materials consulted other than the lectures and textbook for our class. See the course webpage for more details.**

Formatting: Write the solution to each part of each problem on a separate page. Be sure to correctly indicate which page each problem appears on in GradeScope. Please do not write your name on any page of the submission; we are using anonymous grading in GradeScope. If we can't find your solution to any problems because it was not properly tagged with the page, or if handwriting is not legible, you will receive 0 percent on these problems.

Grading: There are 60 total possible points. For all problems you have the option of answering "Don't know" on any parts of the question, and you will receive 20 percent of the total marks for those parts. *You will also be graded on clarity and brevity of your answers. For each question below, we have added guidelines for the length of your solution.*

0 Exercises

Here is a recommended exercise. This will not be graded, so you should not turn in your solution.

(a) Give a streaming algorithm $\mathcal{A}$ (give the 5-tuple) for recognizing the following language L of minimal space complexity. State the space usage of your algorithm in big-O notation, as a function of n, the length (number of digits) of the input. $L = \{x \in \{0,1\}^* \mid |x| \text{ is a power of } 2\ \}$.

1 Asymptotic Analysis (10 points)

For each of the following answer T (true) or F (false). No explanation necesssary. [1] Whenever the base of a log isn't specified, we mean $\log_2$.

(a) $2^n = O(n^n)$.

(b) $\log_e(n) = O(\log_{100}(n))$.

(c) $n^2 \log\log(n) = \Omega(n \log(n))$.

(d) $n^3 = n^{\Omega(5)}$.

[1] We will not grade explanations but it is important that you can explain your answer so we will be providing explanations/proofs in the solutons.

(e) $\log_2(n) = \Omega(\log_2(n^{100}))$.

(f) $n^{1/\log n} = \text{poly}(n)$.

(g) $16^{\log_2(n)} + 20n^2 + \sqrt{n} + 45 = \text{poly}(n)$.

(h) $\frac{n}{\log\log\log n} = o(n)$.

(i) $100^{-100} = o(1)$.

(j) For all functions $f, g : \mathbb{N} \to \mathbb{R}^+$, if $f = o(g(n))$, then $\frac{1}{2}(f(n) + g(n)) = o(g(n))$.

2 Nonregular Languages (10 points)

Let FlipFlop $\subseteq \{0,1\}^*$ be the following language: $w \in \{0,1\}^*$ is in FlipFlop if and only if w can be written as $u \cdot \overline{u}$ where $\overline{u}$ is the string obtained by replacing all 0's in u by 1's, and replacing all 1's in u by 0's. For example, $w = 0100010111$ is in FlipFlop since $w = u\overline{u}$ where $u = 01000$ and $\overline{u} = 10111$.

Prove that FlipFlop is not regular by giving a $\Omega(n)$ one-way communication lower bound for the associated communication problem. Do not use the Pumping Lemma. However you may use anything from the course notes on streaming/communication complexity. Your answer should be half a page or less.

3 Streaming Algorithms (20 points)

Let Sum $= \{w \in \{-1, 1, 0\}^* \mid$ the digits in w sum to $0\}$.

(a) Give a streaming algorithm $\mathcal{A}$ (give the 5-tuple) for recognizing Sum with space complexity $O(\log n)$ where n is the input length. Give a brief explanation of your algorithm (1-2 sentences) and a brief justification of its space complexity (1-2 sentences). [2]

(b) Prove that this language is not regular either by a direct argument (for example, the direct proof of Theorem 2 from Oct 6/7 notes) or by proving that it does not have constant one-way communication complexity. Do not use the Pumping Lemma. However, you may use anything from the course notes on streaming/communication complexity. Your answer should be less than one page.

4 Fun with Polynomials (10 points)

Let $f : \mathbb{N} \to \mathbb{R}^+$, $g : \mathbb{N} \to \mathbb{R}^+$ be functions.
Use the formal definition of poly(n) (see notes) to prove that if $f = \text{poly}(n)$ and $g = \text{poly}(n)$, then $f(g(n)) = \text{poly}(n)$. Your answer should be half a page or less.

5 So far away... (10 points)

In this problem we consider the following language

$$\text{Far} = \{w \in \{a, b, c\}^* \mid \text{ there are at most } \lfloor |w|/4 \rfloor \text{ symbols after the last } a \text{ in } w\}.$$

Examples:

[2] Your explanation should be here to help us understand your algorithm and what you're keeping track of.

(i) The string $w = bcbaa \in$ Far since there are 0 symbols after the last a;

(ii) $w = abcbcabb \in$ Far since there are $2 \leq \lfloor |w|/4 \rfloor$ symbols after the last a;

(iii) $w = bbbbbbcc \in$ Far since there's no a in w (so the condition to be in L is trivially true);

(iv) $w = ab \notin$ Far since there's $1 > \lfloor |w|/4 \rfloor$ symbols after the last a;

(v) $w = ccccabbb \notin$ Far since there's $3 > \lfloor |w|/4 \rfloor$ symbols after the last a.

Give a streaming algorithm $\mathcal{A}$ (give the 5-tuple) for Far of space complexity $O(\log n)$ where n is the input length. Give a brief explanation of your algorithm (2-3 sentences) and a brief justification for its space complexity (2-3 sentences). [3]

[3]Your explanation should be here to help us understand your algorithm and what you're keeping track of.