CS Theory (Fall '25)	Assigned: Sept 9, 2025

Homework 1

Instructors: *William Pires and Toniann Pitassi* — Due: Sept 18, 2025 at 11am

Collaboration: Collaboration with other students in the class homework is allowed. However, you must write up solutions by yourself and understand everything that you hand in. You may also consult other reference materials, but you may not seek out answers from other sources or use AI tools. **You are required to list who you collaborated with on each problem, including any TAs or students you discussed the problems with in office hours. Also list any reference materials consulted other than the lectures and textbook for our class.** See the course webpage for more details.

Formatting: Write the solution to each part of each problem on a separate page. Be sure to correctly indicate which page each problem appears on in GradeScope. Please do not write your name on any page of the submission; we are using anonymous grading in GradeScope.

Grading: There are 80 total possible points, not including the Bonus problem which is optional and for extra credit. For all problems except for the Bonus problem, you have the option of answering "Don't know" on any parts of the question, and you will receive 20 percent of the total marks for those parts.

0 Exercises (not graded)

Here are some recommended exercises. You should not turn in your solutions for these, and we will not grade them.

(a) From the Sipser textbook: Exercises 1.1, 1.2, 1.3, 1.5

(b) 1.6 (Give NFAs for the languages described in Exercises 1.4 b,c,k,n)

(c) 1.7 part a

(d) Are the following languages regular or not? (Just say "yes" or "no". Later we will show how to prove that a language is not regular.)

- $L = \{w \mid w = w^R\}$ (Notation: w^R is the string w written in reverse. For example if $w = 1101000$ then $w^R = 0001011$.)
- $L = \{0^n \mid n \text{ is even}\}$
- $L = \{0^n 1^m 0^n \mid m, n \geq 2\}$

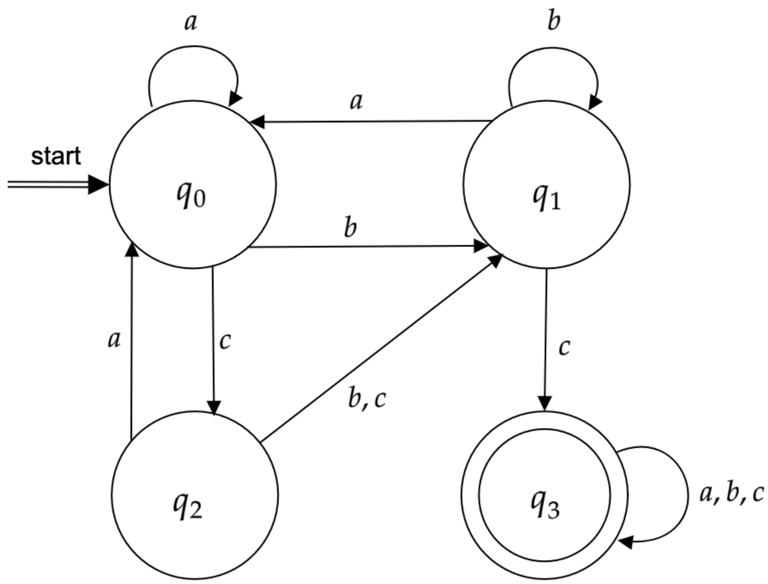
(e) Draw an NFA using 5 states or less for the language L over $\Sigma = \{0, 1, 2\}$ where:

$$L = \{w \mid w \text{ contains the substring } 210\}.$$

1 Problems

Problem 1: A DFA with four states (20 points)

Let D denote the following DFA over the alphabet $\{a, b, c\}$.



(a) Write out the formal definition of D (as a 5-tuple). Describe the transition function in a table.

(b) What is the language recognized by D? Give a short English description, then explain in a paragraph why your description is correct. Please give an explanation that is as short and simple as possible!

Problem 2: Constructing DFAs and NFAs (30 points)

For each question below, you may give your DFA (NFA) either as a 5-tuple or by drawing the deterministic (nondeterministic) finite automata for the language. No justification is required. For all questions try to design your DFA (NFA) with the minimal number of states.

(a) Give a DFA for the language L over $\Sigma = \{0, 1\}$ consisting of all strings *not* containing the substring 111.

(b) Draw an NFA for the language L over $\Sigma = \{0, 1, 2\}$ where

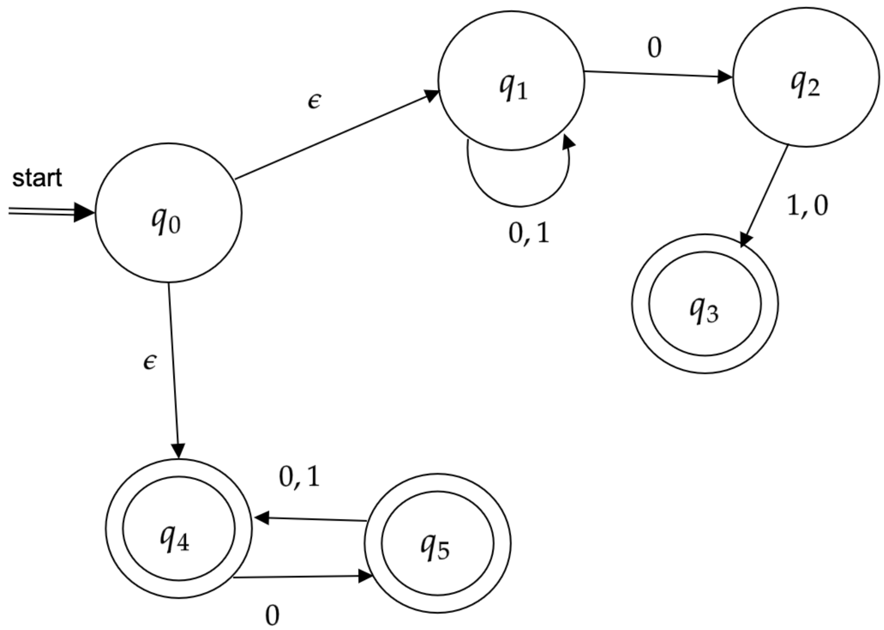
$$L = \{w \mid w \text{ has length divisible by 3 } \textit{or } w \text{ contains the symbol 2 exactly once }\}.$$

Your NFA should have 7 states or less.

(c) Give an NFA that accepts the set of all strings over $\Sigma = \{0, 1\}$ with at least three symbols and such that the third symbol from the right end is 1.

Problem 3: An NFA (15 points)

Let D denote the following NFA over the alphabet $\{0, 1\}$.



- What is the language recognized by D. Give a short English description, then explain in a paragraph why your description is correct. Please give an explanation that is as short and simple as possible!

Problem 4: Missing symbol (15 points)

Consider the following language over $\Sigma = \{0, 1, 2\}$:

$$L = \{w \in \{0, 1, 2\}^* \mid \text{ at least one of the alphabet symbols does not occur in } w.\}$$

For instance $00, 0111$ and 21122 are in L, but $0012, 1201$ aren't in L.

(a) Give a DFA as a 5-tuple or by drawing it for the above language.

(b) Give a short explanation (2 to 4 sentences) explaining why your construction is correct. [1]

2 Bonus Problem

Let L_1, L_2 be languages over $\Sigma = \{0, 1\}$ such that: M_1 is a minimal-state DFA for L_1 with s_1 states and M_2 is a minimal state DFA for M_2 with s_2 states. Since regular languages are closed under union, there

[1] To make the explanation easier, your state's names should reflect what information you're keeping track of. I.e. you could have names like $q_{\text{last symbol seen }=1}$,

is a generic construction of a DFA for $L_1 \cup L_2$ using at most $s' = s_1 \cdot s_2 + 2$ states.(See Theorem 1.25 in Sipser book.)

(a) Give an example where the generic construction is not the best possible. That is, give an example of languages L_1,L_2 over $\Sigma = \{0, 1\}$ such that $L_1 \cup L_2$ has a 1-state DFA, but L_1 requires at least two states. For full credit, describe L_1, L_2; give a 1-state DFA for $L_1 \cup L_2$, and prove that any DFA for L_1 requires at least two states.

(b) Generalize your construction from part (a) to show: for any $s > 1$, there exists L_1, L_2 such that $L_1 \cup L_2$ has a 1-state DFA, but L_1 requires at least s states. You do not need to formally prove that L_1 requires at least s states.